NESBIT'S AND LITTLE'S

TREATISE

ON

PRACTICAL

GAUGING.

TO THE

HONOURABLE COMMISSIONERS

OF

His Majesty's Revenue

OF

EXCISE,

THIS

NEW AND IMPROVED SYSTEM

OF

GAUGING

IS,

WITH THE MOST PROFOUND RESPECT,

HUMBLY DEDICATED,

BY THEIR MOST OBEDIENT,

AND MUCH OBLIGED SERVANTS,

ANTHONY NESBIT.

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PREFACE.

Scientific Pursuits have, in all ages, been considered of the greatest utility to mankind in general; and those individuals who have laboured successfully to elucidate the principles of Mathematics, or to simplify their application to the Practical affairs of life, have always been ranked among the most useful and honourable members of society.

Honourable indeed they must appear to all those who have learned to set due estimation on intellectual attainments; as, next to vital religion, they certainly constitute the true honour and dignity of man.

Riches and titles may be our legal possessions, by birthright; however, in a meritorious point of view, they can scarcely be denominated our own; but the treasures of a well-stored mind can only be acquired by diligent study, and assiduous application; and are therefore our natural and unalienable property.

Mental acquirements not only enrich and dignify the possessor; but are productive of the most happy effects to the public in general. Like the common source of light, they shed beams of lustre on all around; and dispel the clouds of darkness, ignorance, and superstition on every side.

To the great improvements made in the Arts and Sciences we are indebted for most of the comforts and conveniences of life; and it may be affirmed, without fear of contradiction, that the advancement of no branch of

human knowledge has been attended with more beneficial consequences to the community, than the cultivation of the Mathematics.

Arithmetic and Geometry are the two grand pillars that support every other branch of science; and to their Practical application, together with that of Mechanics, we may atttibute the production of every machine now in use, from the stupendous and powerful steam engine, down to the elegant clock and watch.

By the assistance of Arithmetic, we are enabled to make every calculation that business or science requires; and from the principles of Geometry we derive Practical Rules by which we can estimate the areas, solidities, and capacities of all kinds of Geometrical Figures.

The application of these Rules to finding the areas of figures, is called Mensuration of Superficies; their application to finding the contents of bodies, is called Mensuration of Solids; and when they are applied in finding the capacities of vessels, in ale and wine gallons, and malt bushels, the process is denominated GAUGING.

This Art is of such general utility in the common affairs of life, that there are few persons who do not occasionally want its assistance; and to Victuallers, Common Brewers, Maltsters, Distillers, Wine-Merchants, Spirit-Merchants, Soap-Makers, Starch-Makers, and Glass-Makers, it is of considerable moment; for without its aid, they could not ascertain with accuracy the quantity of those articles which they manufacture.

There is, however, another class of persons to whom this Science is indispensably necessary; namely, to those who intend to become Candidates for the Excise; and as so large a portion of His Majesty's Revenue falls under the inspection of the Officers of the Excise, it is certainly of the first importance that they should be made fully acquainted with the Principles and Practice of Gauging, before they are appointed to discharge so momentous a duty as that with which they are intrusted.

Such being the utility of this Science, it is no wonder that so many persons have written on the Subject; but it

is certainly matter of surprise, that of all the Treatises which have yet made their appearance, not one of them contains the Theory of Gauging, as it ought to be taught in Schools, united with the Practice, as adopted in the Excise.

We have Books of Arithmetic, Mensuration, Land-Surveying, Trigonometry, Navigation, Astronomy, Algebra, Fluxions, and Mechanics, which contain numerous Examples for the Exercise of the Learner; but not one Work on Gauging, that contains more than *One Example* in each Problem; and that Example wrought out at length.

Now, as Schools are the proper places in which the principles of every Science ought to be taught; why not have a Treatise on Gauging adapted for the use of Schools, and at the same time containing every necessary information relating to the Science, as it is *practised* by the Officers of the Excise?

Such are the reasons that have induced the Authors to write the following Work; and whatever may be its merits, it is certainly not a hasty production. It is nearly twenty years since one of its Authors first laid down the Plan, having found the want of such a Book in his avocation of instructing Youth; and both he and his Colleague have employed all their leisure moments for upwards of five years, in bringing the Plan to maturity.

The execution of the Work has cost them much more labour than they at first anticipated; and whoever reads it with attention, and solves all the Questions it contains, will not, they think, charge them with the crime of *book-making*, so much practised in the present day.

Having said thus much concerning the advantages of Mathematical Studies, and the motives that led to the production of the Work in question; we will now proceed to give some account of each of the Seven Parts into which it is divided.

Part the First contains a clear and concise view of Vulgar and Decimal Fractions; Square and Cube Roots;

with the application of the two latter to the Solution of various Mathematical Problems. A thorough knowledge of these subjects being indispensably necessary in all kinds of measurements; they form the most appropriate Introduction to a Treatise on Practical Gauging.

PART THE SECOND contains the description of the Sliding Rule; directions for finding any number upon it; and the application of the different lines to Multiplication, Division, the Rule of Three Direct and Inverse; and the Extraction of the Square and Cube Roots.

PART THE THIRD contains such Definitions, Problems, and Theorems in Geometry, as we conceived to be necessary in a Treatise on Practical Gauging. Those who desire to see the subject of Geometry more fully elucidated, are referred to the Elements of Simpson, Emerson, Bonnycastle, Keith, Playfair, and Leslie; to Simson's Euclid, Hutton's Course of Mathematics, and Reynard's Geometria Legitima. The last Work is well adapted to the capacities of Youth; and contains a number of Quæstiones Solvendæ, at the end of each Book; to which an excellent Key has lately been published by the Author.

PART THE FOURTH contains the Mensuration of Superficies applied to Gauging. Besides giving Rules for finding the areas of regular and irregular figures; we have treated largely on the method of finding the areas of oval figures, which are not true ellipses, by means of equidistant ordinates. The Honourable Board of Commissioners have, by various General Letters, enjoined the Officers of the Excise to gauge all oval vessels by this method; and it certainly far excels every other, for finding the areas of curvilineal figures, when the nature of the curves cannot be determined.

In this Part we have adopted a method entirely different from that of any of our Predecessors; for by considering every plane figure as the base of some vessel whose sides are perpendicular, we have entered immediately on the Practical Part of Gauging, by multiplying the area of the base by the perpendicular depth; and thus we obtain the content of the vessel in ale and wine gallons, and malt bushels. This method has a decided advantage over that

of merely finding the areas of figures; as the Learner immediately perceives the real use of the Science.

Part the Fifth contains the Mensuration of Solids, applied to finding the capacities of vessels of every description. Besides giving Rules for determining the contents of all regular vessels, of a known form; we have given the method of finding the contents of circular vessels, with curved sides; when the nature of the curves cannot be ascertained. This we have done by means of equidistant parallel sections, founded on the method of equidistant ordinates. This Process is entirely new in a Book of Gauging; having never before appeared, except in Nesbit's 'Treatise on Practical Mensuration.' We have also given Rules for finding the contents of all cylindrical, pyramidical, and conical ungulas, that can possibly be formed, by placing, in various positions, vessels containing fluids. This has long been a *desideratum* in every Treatise on Gauging; and although these figures seldom occur in Practice, yet we are of opinion that every person who professes the art of Gauging, ought to be able to determine the capacity of any vessel or figure that can possibly be devised. This Part also contains One Hundred Miscellaneous Questions, which will serve to exercise the ability of the Learner; and prove his knowledge of the Theorems and Rules given in the former part of the Work.

Part the Sixth is divided into Seven Sections; and comprehends the whole System of Practical Gauging, as adopted in the Excise. The first Section contains the method of *gauging* and *fixing* Victuallers' Utensils; the second that of *gauging* and *inching* Common Brewers' Utensils; and the third comprehends the method of *gauging, inching,* and *ullaging* Casks. As no part of the Science is attended with such difficulty as that of finding the contents of close vessels; we have bestowed much attention upon this part of the subject; and to the method of obtaining the contents of casks, according to their several varieties, we have added those of finding their contents, by General Rules, without paying any regard to their varieties.

Cask Gauging by the Callipers, Bung Rod, and Head *Rod*, as practised by the Port Gaugers of the Excise and

Customs, is also given in this Section. In this Part of the Work we have been *ably* assisted by a Gentleman who has had considerable Practice in the Excise Department of Port Gauging, at Newcastle-upon-Tyne. He also very kindly furnished us with a Manuscript containing many *excellent* observations on the method of taking the dimensions, making the allowances, casting the contents, &c. &c. For these favours we offer him our most unfeigned thanks.

The fourth Section contains the method of *gauging* and *fixing* Maltsters' Utensils; and the fifth that of *gauging* and *inching* the Utensils of Distillers. The sixth Section comprehends the method of *gauging* and *inching* the Utensils of Soap-Makers, Starch-Makers, and Glass-Makers.

The seventh Section contains the Method of finding the areas and contents of vessels in Irish malt bushels and liquid gallons; and of reducing Irish to English, and English to Irish measure; thus the Work is adapted for Ireland, as well as for England and Scotland. This Section also contains a Table of the Specific Gravities of bodies; the method of finding the magnitude of a body from its weight, or the weight of a body from its magnitude; and likewise the process of determining the tonnage of Ships, according to the Parliamentary Rules.

Part the Seventh is divided into three Sections. The first contains a Table of Ale Areas; the second a Table of Wine Areas; and the third a Table for reducing Ale Gallons to Victuallers' Barrels.

Besides the regular subjects enumerated in the Table of Contents, the Work is interspersed with numerous Notes, Remarks, and Observations, which will tend greatly to elucidate the Subject. The leading features of these Remarks we will just notice in a cursory manner. Part the Sixth contains copious directions for taking the dimensions of all kinds of vessels that can possibly be met with in the Practice of Gauging. This is of the greatest importance; for it is evident that if the dimensions be improperly taken, the results must always be incorrect; notwithstanding the greatest accuracy may be used in finding the areas and contents.

Every possible attention has been paid to the method of gauging and *fixing* the Utensils of Victuallers; and to gauging and *inching* those of Common Brewers; and numerous Examples have been given to illustrate the method of entering the dimensions and areas; and forming the Table Books, as practised by Officers of the Excise. In Cask Gauging we have made several Remarks, Observations, and Comparisons, which will tend to elucidate that subject; and point out the impropriety of gauging all casks as belonging to the first variety.

Malt Gauging is treated of very extensively; and every information has been given, that we thought necessary on that subject. The method of gauging Stills, and Still-Heads is rendered as simple as possible; and that of gauging and inching oval Wash-backs, and Jack-backs, by means of equidistant ordinates, is treated in a clear and perspicuous manner. The methods of gauging Soap and Glass have never before appeared in any Work on this subject; and will therefore be found very acceptable to all those who are concerned in the manufacturing of these articles.

In finding the areas and contents of vessels, we have not only used the Pen, and the Sliding Rule; but also the Tables of Ale and Wine Areas given in Part the Seventh. As the Sliding Rule is much used by Officers of the Excise, we have given clear and perspicuous Rules for its application, whenever it could be introduced with success. The Tables of Ale and Wine Areas we have taken from Mr. Moss's Gauging; they being more correct than any other that have fallen under our notice. They generally give the areas of circles rather more than the Rule by the Pen; but sufficiently correct for every Purpose of Practical Gauging.

The Questions given in each Problem, for the Exercise of the Learner, form one of the most prominent features of the Work; and completely distinguish it from all others on the same subject. These Questions will be found of infinite service in Schools; and are what Teachers have long desired; for it is impossible to make young persons comprehend any subject properly, without giving them Line upon Line, and Precept upon Precept.

Notwithstanding a person may possess the most bri ant talents; that he may comprehend general arts a sciences in all their ramifications; and that long pract may have made him familiar with particular subjects; it will readily be admitted that he will find some difficu in retaining every *minutia* relating even to those scien with which he is most intimately connected. Taki these things into consideration, we are persuaded that t following Work will be found useful, not only to Learne but also to those persons who are already acquainted wi the subject of Gauging; and we hope it is executed such a manner, and treats so extensively of the Science, all its Departments, that even Experienced Officers of t Excise and Customs will not find it unworthy of the Notice, as a Book of General Reference.

Having taken a survey of the subsequent Work, an made such introductory observations as we thought nece sary; it now only remains for us to solicit the indulgen of the Public, for any errors that may have escaped ou notice.

As the Work is entirely new, we do not expect that will be found quite free from imperfections; however, w can assure our Readers, that neither labour nor pains hav been spared in order to produce such a Treatise on Prac tical Gauging, as may be found worthy of Genera Patronage.

A. NESBIT.
W. LITTLE.

Bradford, Yorkshire,
June, 1822.

P. S. A Key to the Practical Gauging is in the Press; and will be published in a few Weeks.

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AN

EXPLANATION

OF THE

Principal Mathematical Characters.

THE sign or character = (called equality) denotes ıat the respective quantities, between which it is laced, are equal; as 282 cubic inches = 8 pints = quarts = 1 ale gallon.

The sign + (called *plus*, or more) signifies that the umbers between which it is placed, are to be added ogether; as 9 + 6 (read 9 *plus* 6) = 15. Geometrical lines are generally represented by capital letters; hen AB + CD, signifies that the line AB is to be ıdded to the line CD.

The sign − (called *minus*, or less) denotes that the quantity which it precedes, is to be subtracted; as 15 − 6 (read 15 *minus* 6) = 9. In geometrical lines also, AB − CD, signifies that the line CD is to be subtracted from the line AB.

The sign $\times$ denotes that the numbers, betwe which it is placed, are to be multiplied together ; 5×3 (read 5 multiplied by 3) $= 15$.

The sign $\div$ signifies division ; as $15 \div 3$ (read divided by 3) $= 5$. Numbers placed like a vulg fraction, also denote division ; the upper number bei the dividend, and the lower the divisor ; as $\frac{15}{3} = 5$

The signs : :: : (called proportionals) deno proportionality ; as 2 : 5 :: 6 : to 15, signifying, th the number 2 bears the same proportion to 5, as 6 do to 15 : or, in other words, as 2 is to 5, so is 6 to 15.

The sign $\overline{}|$ (called *vinculum*) is used connect several quantities together ; as $\overline{9 + 3}|\overline{- 6}|$ $2 = \overline{12 - 6}| \times 2 = 6 \times 2 = 12$.

The sign ^{2}, placed above a quantity, represents th square of that quantity ; as $\overline{5 + 3}| = 8^2 = 8 \times$ $= 64$.

The sign ^{3}, placed above a quantity, denotes th cube of that quantity ; as $\overline{\overline{9 + 3}| - 8}|^3 = \overline{12 - 8}^3$ $= 4^3 = 4 \times 4 \times 4 = 64$.

The sign $\sqrt{}$ or $\sqrt[2]{}$, placed before a quantity, denotes the square root of that quantity : as $\sqrt{9 \times 4} = \sqrt{36}$ $= 6$.

The sign $\sqrt[3]{}$, placed before a quantity, represents the cube root of that quantity ; as $\sqrt[3]{\overline{6 \times 4 \times 3}| - 8}$ $= \sqrt[3]{\overline{24 \times 3}| - 8} = \sqrt[3]{72 - 8} = \sqrt[3]{64} = 4$.

X Denotes Strong Beer.
T Denotes Table Beer.

$$\left.\begin{array}{l}\text{A.G.}\\ \text{W.G.}\\ \text{M.B.}\\ \text{G.P.}\\ \text{S.G.P.}\\ \text{C.G.P.}\end{array}\right\}\text{Denote}\left\{\begin{array}{l}\text{Ale Gallons.}\\ \text{Wine Gallons.}\\ \text{Malt Bushels.}\\ \text{The Gauge Point.}\\ \text{The Square Gauge Point.}\\ \text{The Circular Gauge Point.}\end{array}\right.$$

A

TREATISE

ON

PRACTICAL GAUGING.

AUGING is the Art of finding the Capacities or Con-
nts of all sorts of Vessels used by Maltsters, Brewers,
stillers, Wine Merchants, Victuallers, &c. &c.; such as
sterns, Couches, Coppers, Coolers, Tuns, Vats, Stills,
sks, &c. &c.

ow, as all Instruments used in Gauging are decimally
ded; and as the Dimensions of all Vessels are taken
Inches and Tenths, it is evident that the most appro-
iate Introduction to a Treatise on Practical Gauging,
st be a clear and concise View of Decimal Fractions.

PART I.

lgar and Decimal Fractions, and Square and Cube Roots.

FRACTIONS.

DEFINITIONS.

1. A VULGAR FRACTION is a broken number, repre-
ted by two figures placed one above the other, with a
e drawn between them.

2. The upper figure of a fraction is called the *numera-*
, and the lower figure the *denominator*.

3. The denominator shows into how many equal parts

the whole, or unity, is supposed to be divided; and the numerator, the number of those parts taken.

4. When the numerator is less than the denominator, the fraction is less than unity, and is called a *proper fraction;* as $\frac{1}{4}$ one-quarter; $\frac{1}{2}$ one-half; $\frac{3}{4}$ three-quarters; $\frac{6}{8}$ six-eighths, &c.

5. When the numerator is greater than the denominator, the fraction is greater than unity, and is called an *improper fraction;* as $\frac{3}{2}$ three-seconds; $\frac{5}{3}$ five-thirds; $\frac{12}{9}$ twelve-ninths, &c.

6. A *simple fraction* is that which is expressed singly, or without any reference to others, as $\frac{4}{5}$, $\frac{6}{8}$, &c.

7. A *compound fraction* is the fraction of a fraction, as $\frac{1}{3}$ of $\frac{3}{4}$; $\frac{4}{5}$ of $\frac{6}{8}$, &c.

8. The *common measure* of a fraction is a number that will divide both the numerator and denominator without a remainder.

9. A *decimal fraction,* is that which has 10, on some power of 10, as 100, 1000, &c. for its denominator; and instead of *writing* the denominator under the numerator, it is *expressed* by pointing off, from the right of the numerator, as many figures as there are cyphers in the denominator; thus, $\frac{2}{10}$, $\frac{24}{100}$, $\frac{364}{1000}$, $\frac{5648}{1000}$, are respectively equal to .2, .24, .364, 5.648.

10. Cyphers placed on the right of a decimal make no alteration in its value; thus, .2, .20, .200, or $\frac{2}{10}$, $\frac{20}{100}$, $\frac{200}{1000}$, are all equal to each other; but if they be placed on the left of the decimal, they decrease its value in a ten-fold proportion; thus, .2, .02, .002 are respectively equal to $\frac{2}{10}$, $\frac{2}{100}$, $\frac{2}{1000}$.

11. Decimals have the same properties as whole numbers, and are subject to the same rules; for in the notation of both, the values of the figures *decrease* in a ten-fold proportion from the left to the right.

12. The first figure of a decimal fraction, or that next the decimal *point,* is called tenths, the second hundredths, the third thousandths, &c.; thus, the fraction .234 is read, 2 tenths, 3 hundredths, and 4 thousandths of a unit. *(See the following table.)*

13. A *mixed number* is composed of a whole number and a decimal, which are separated from each other by a point; thus 25.6 are equal to 25 $\frac{6}{10}$.

14. *Compound numbers* are such as consist of several denominations, as pounds, shillings, and pence; hundred weights, quarters, and pounds, &c.

TABLE.

Whole numbers.							*Decimal parts.*					
7	6	5	4	3	2	1	.2	3	4	5	6	7
Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred thousandths.	Millionths.

Note. Decimals are only vulgar fractions of a particular descrip-on, and were introduced in order to lessen the trouble which, in any cases, attends the use of the latter. They are much used the *practical affairs of life*, particularly in Mensuration and auging.

ADDITION OF DECIMALS.

RULE.

Place the figures in such a manner that those of the me denomination may stand under each other; add em together, as in whole numbers, and point off as any figures, for decimals, as are equal to the greatest umber of decimal places in any of the given numbers.

Note. It has already been observed, that decimals and whole num-rs are both subject to the same rules; but in all calculations by former, great care must be taken to point off the proper number decimal places in the results, or the truth of the operations will be mpletely destroyed.

EXAMPLES.

1. What is the sum of 54.646+3.95+46.3905+968.202 +7.0264+.2064+2.0463+5.646?

```
   54.646
    3.95
   46.3905
  968.202
    7.0264
     .2064
    2.0463
    5.646
 ---------
 1088.1136 Ans.
```

2. Find the sum of 367.60+4678.3609+869.563+.2003+7.5964+42.67. *Ans.* 5965.9906.

3. Add 53.7+2943+1.2+2.0073+1.47+637. *Ans.* 3638.3773.

4. Required the sum of 124.1+.3492+84.02+6.349+.00879+71.2. *Ans.* 286.02699.

SUBTRACTION.

RULE.

Place the figures of the same denomination under each other; then, beginning at the right-hand subtract as in whole numbers, and point off the decimals as in addition.

EXAMPLES.

1. What is the difference between 52.73 and 2.676?

```
 52.73
  2.676
 ------
 50.054 Ans.
```

2. From 2.18 take .814. *Ans.* 1.366

3. From .794 take .0981. *Ans.* .6959

4. What is the difference between .0943 and .09281? *Ans.* .00149.

5. Required the difference between 374.901 and 68.14. *Ans.* 306.761.

MULTIPLICATION.

RULE.

Place the figures under each other, and multiply them together, as in whole numbers; and point off as many decimal places in the product as there are in the multiplier and the multiplicand together.

Note 1. When there are not so many figures in the product as there re decimals in the multiplier and multiplicand together, cyphers must annexed to the *left* of the product, that the decimal places may be operly represented.

2. When a decimal is to be multiplied by 10, 100, 1000, &c., it is ly necessary to remove the decimal point so many places to the ht, as there are cyphers in the multiplier; thus, 4.27 × 1 = 042.7; d 379 × 100 = 37.9.

3. If a decimal be multiplied by a decimal, the product will be less an either the multiplier or multiplicand; and if a whole or a mixed mber be multiplied by a decimal, the product will always be less an the multiplicand.

EXAMPLES.

1. What is the product of 24.73 multiplied by 7.325?

```
     24.73
     7.325
     -----
     12365
    4946
   7419
 17311
 ---------
 181.14725 Ans.
```

2. Multiply .00741 by .00054. *Ans.* .0000040014.

3. Multiply .3141 by 20.5. *Ans.* 6.43905.

4. Multiply .35426 by .025. *Ans.* .00885650.

5. What is the product of 9268.456 multiplied by .389? *Ans.* 7844719.20538.

DIVISION.

RULE.

Divide as in whole numbers, and point off from the ht-hand of the quotient, as many figures for decimals, the decimal places in the dividend exceed those in the isor.

Note 1. When the figures in the quotient are too few to make up the proper number of decimals required by the rule, the defect must be supplied by prefixing cyphers to the left of the quotient.

2. When the dividend does not contain as many decimals as the divisor, cyphers must be placed on the right of the dividend, until they are made equal, previously to beginning the operation; and the quotient, to that extent, will be a whole number.

3. If there be a remainder after division, you may continue the quotient to any extent that may be thought necessary, by subjoining a cypher continually to the last remainder.

4. When it is required to divide a decimal by 10, 100, 1000, &c., remove the decimal point so many places to the left as there are cyphers in the divisor; thus, 36.4 ÷ 10 = 3.64; and .5864 ÷ 100 = .005864.

5. If a decimal be divided by a decimal, the quotient will be greater than either the divisor or dividend; and if a whole, or a mixed number, be divided by a decimal, the quotient will be greater than the dividend; but if a decimal be divided by a whole, or a mixed number, the quotient will be less than the dividend.

EXAMPLES.

1. Divide .2843701 by .147.

```
.147).2843701(1.9344 Ans.
      147
      ----
      1373
      1323
      ----
        507
        441
        ---
         660
         588
         ---
          721
          588
          ---
          133 remainder.
```

2. What is the quotient of 741 divided by .325? *Ans.* 2280.

3. Divide 839 by 5.2. *Ans.* 161.3461.

4. What is the quotient of .074 divided by 36? *Ans.* .00205.

5. Divide 48324.36 by 24.05. *Ans.* 2009.32.

6. Divide 5.6569 by 265.686. *Ans.* .02203.

REDUCTION.

CASE I.

To reduce a vulgar fraction to a decimal of the same value.

RULE.

Place cyphers, at pleasure, on the right of the numerator, as decimals; then divide by the denominator, and the quotient will be the decimal required.

Note 1. A compound fraction may be reduced to a simple one, by multiplying all the numerators together for a new numerator, and all the denominators together for a new denominator; thus, $\frac{2}{3}$ of $\frac{3}{4}$ $= \frac{6}{12} = \frac{1}{2}$.

2. An improper fraction may be reduced to a mixed number, by dividing the numerator by the denominator; thus, [illegible] $= 29$[illegible] $= 27$[illegible].

3. A mixed number may be reduced to an improper fraction, by multiplying the whole number by the denominator of the fraction, adding the numerator to the product; and placing the sum over the denominator; thus, $28\frac{4}{5} = \frac{28\times5+4}{5} = \frac{144}{5}$.

4. A whole number may be expressed like a fraction, by putting 1 for its denominator.

EXAMPLES.

1. Reduce $\frac{1}{4}$ to a decimal.

$$4)1.00$$
$$.25 \ \textit{Ans.}$$

2. Reduce $\frac{1}{2}$ to a decimal. *Ans.* .5.
3. Reduce $\frac{3}{4}$ to a decimal. *Ans.* .75.
4. What is the decimal of $\frac{3}{8}$? *Ans.* .375.
5. Reduce $\frac{18}{25}$ to an equivalent decimal. *Ans.* .72.
6. What is the decimal of $\frac{24}{38}$? *Ans.* .631578.
7. Let $\frac{35}{722}$ be reduced to a decimal. *Ans.* .048476.
8. Express $\frac{48}{144}$ by decimals. *Ans.* .333333.
9. What is the decimal of $\frac{382}{4693}$. *Ans.* .0813631.
10. Let $\frac{5869}{85367}$ be expressed in decimals. *Ans.* .687502.
11. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{9}$ to a decimal. *Ans.* .277.
12. Reduce $\frac{16}{18}$ of $\frac{22}{25}$ of $\frac{38}{48}$ to a decimal. *Ans.* .9533.

CASE II.

To reduce numbers of different denominations, as mo weights, measures, &c. to their equivalent decimal value

RULE.

Reduce the given numbers to the lowest denominat mentioned, for a dividend; also reduce the integer to same denomination, for a divisor; then annex cyphers the dividend, divide as in whole numbers, and the qu tient will be the decimal required.

Note. Tables of Ale and Beer Measure, Wine Measure, Dry M sure, and Avoirdupois Weight, may be seen in Part IV.

EXAMPLES.

1. Reduce 15*s.* 6¾*d.* to the decimal of a pound ste ling.

```
                 s.   d.
                 15.  6¾
                 12
                 ---
                 186
       £.  Far.   4
                 -----
        1=960)747.000000(.778125 Ans.
              6720
              ----
               7500
               6720
               ----
                7800
                7680
                ----
                 1200
                  960
                 ----
                 2400
                 1920
                 ----
                 4800
                 4800
                 ====
```

2. Reduce 10*s.* 9¾*d.* to the decimal of a pound.
Ans. .540625.

3. Reduce 9*s.* 3¼*d.* to the decimal of a pound.
Ans. .4635416.

4. Reduce $10\frac{1}{2}d.$ to the decimal of a shilling. *Ans.* .875.

5. Reduce 19s. $11\frac{3}{4}d.$ to the decimal of a pound. *Ans.* .9989583.

6. Reduce 2 pints of wine to the decimal of a gallon. *Ans.* .25.

7. Reduce 189 gallons of wine to the decimal of a tun. *Ans.* .75.

8. Reduce 8 gallons of ale to the decimal of a firkin. *Ans.* .888888.

9. Reduce 56 gallons of beer to the decimal of a butt. *Ans.* .518518.

10. Reduce 3 pecks of malt to the decimal of a bushel. *Ans.* .75.

11. Reduce 4 quarters, 5 bushels, and 2 pecks of bar-y, to the decimal of a last. *Ans.* .46875.

12. Reduce 12 ounces avoirdupois, to the decimal of a und. *Ans.* .75.

13. Reduce 22 pounds, 9 ounces of candles, to the cimal of a quarter. *Ans.* .8058035.

14. Reduce 3 quarters, 14 pounds, and 8 ounces of llow, to the decimal of a hundred weight. *Ans.* .8794.

15. Reduce 15 hundred weight, 2 quarters, and 21 unds of soap to the decimal of a ton. *Ans.* .784375.

CASE III.

find the value of a decimal fraction in the known parts of an integer.

RULE.

Multiply the given decimal by the number of parts ntained in the next inferior denomination; and, from right of the product, point off as many figures as re are places in the given decimal. Multiply the cimals thus pointed off, by the parts in the next less nomination; reserving as many places to the right, as fore. Proceed in this manner through all the denomi-tions to the last; then the several figures on the left of decimal points, will be the answer required.

EXAMPLES.

1. What is the value of .7362 of a pound sterling?

$$\begin{array}{r} .7362 \\ 20 \\ \hline 14.7240 \\ 12 \\ \hline 8.6880 \\ 4 \\ \hline 2.7520 \\ \hline \end{array}$$

Ans. 14*s.* 8½*d.*

2. What is the value of .8649 of a shilling? *Ans.* 10¼*d.*
3. Find the value of .92846 of a pound. *Ans.* 18*s.* 6¾*d.*
4. Required the value of .8694 of a hogshead of wine. *Ans.* 54 *gal.* 3 *qt.*
5. What is the value of .73828 of a barrel of beer? *Ans.* 26 *gal.* 2 *qt.*
6. Required the value of .5694 of a quarter of malt. *Ans.* 4 *bush.* 2 *pk.*
7. What is the value of .68328 of a last of barley? *Ans.* 6 *qr.* 6 *bush.* 2½ *pk.*
8. Find the value of .9326 of a hundred weight of tallow. *Ans.* 3 *qr.* 20 *lb.* 7 *oz.*

RULE OF THREE.

RULE.

State the question as in the common Rule of Three; reduce the inferior denominations of such of the terms as are compound, to the decimal parts of their integer; multiply the second and third terms of the proportion together, and divide the product by the first term, and the quotient will be the answer required, which must, if necessary, be reduced to its integral value.

Note 1. In solving questions in the Rule of Three, proper attention must be paid to the rules given in multiplication and division, for pointing off the decimals.

2. As Arithmeticians differ in their opinions with regard to the best method of stating questions in the Rule of Three, we shall here give both Rules; and the learner may use that of which he most approves.

RULE I.

nsider which of the three given terms is of the same kind as nswer, or number sought, and put it down as the third term of the ortion.

ien, if it appears, from the nature of the question, that the er will be greater than this number, make the greater of the other umbers the second term, and the less the first; but if it will be make the less number the second term, and the greater the

luce the first and second terms of the proportion to the same ination, and the third to the lowest denomination mentioned.

tiply the second and third terms together, and divide the pro- y the first term; and the quotient will be the answer, in the lenomination to which the third term was reduced.

e. The answer or the fourth term, must, when necessary, be ht again, to the highest denomination of which it admits, in order may be expressed in a proper form.

RULE II.

te down that number which is of the same name as the answer, second term in the proportion.

erve, from the nature of the question, whether the answer will ater or less than this number.

t appears that it will be greater, make the less of the two re- g numbers, the first term, and the greater the third; but if swer will be less than the second term, make the greater number t term, and the less the third.

uce the first and third terms of the proportion to the same deno- on, and the second to the lowest denomination mentioned.

tiply the second and third terms together, divide the product by st term; and the quotient will be the answer, in the same deno- on as the second term.

e 1. In stating questions in the Rule of Three, the word As, is lly placed before the first term, and the signs of proportion en each of the other terms, thus, As 4 lb. : 12 lb. :: 5s. : 15s. by st Rule.

Or,

As 4 lb. : 5s. :: 12 lb. : 15s. by the second Rule.

Some persons object to the second Rule, on this ground, that no tion whatever can subsist between 4 lb. and 5s. or between 12 lb. 5s. The first Rule is, in our opinion, more scientific; but we clined to think, that the second will be more easily comprehended rners; and, according to it, the numbers are always alternately rtional.

Questions in the Rule of Three are of two kinds, namely, *direct* verse; but both the foregoing Rules are *general*.

Direct Proportion, is when more requires more, or less requires as in the following examples: If 6 men can dig a trench 48 yards

in length, in a certain time; how many yards can 12 men dig in the same time? Here it is obvious, that the more men there are employed, the more work will they perform; and, therefore, in this case, more requires more.

Again, If 6 men can dig 48 yards, in a given time; how many yards can 3 men dig in the same time? Here less requires less; for the less number of men there are employed, the less work will there be performed by them.

All questions that are of this class, are said to be in the Rule of Three Direct.

5. *Inverse Proportion*, is when more requires less, or less requires more; thus, If 6 men can dig a certain quantity of trench in 12 hours; how many hours will it require 12 men to dig the same quantity? Here more requires less: for 12 men being more than 6, it is manifest, that they will require less time to perform the same work.

Again, If 6 men perform a piece of work in 10 hours; how many hours will 3 men be in performing the same work? Here less requires more; for the number of men being less, they will require more time to do the same quantity of work.

All questions of this kind are said to be in the Rule of Three Inverse.

EXAMPLES.

1. If 5.75 lb. of candles cost 4.25*s.* what will be the price of 17.25 lb.?

By Rule I.

lb.		lb.		s.		s.	s.	d.	
As 5.75	:	17.25	::	4.25	:	12.75 =	12	9	*Ans.*

```
           17.25
            4.25
           -----
            8625
           3450
          6900
          ------
5.75)73.3125(12.75
     575          12
     ----       ----
     1581       9.00
     1150
     ----
      4312
      4025
      ----
       2875
       2875
       ====
```

By Rule II.

lb.		s.		lb.		s.	s.	d.	
As 5.75	:	4.25	::	17.25	::	12.75 =	12	9	*Ans.*

2. If 17.25 lb. of candles cost 12.75*s.* what will 5.75 lb. cost?

By Rule I.

	lb.		lb.		s.		s.	s. d.
As	17.25	:	5.75	::	12.75	:	4.25=4	3 *Ans.*

By Rule II.

	lb.		s.		lb.		s.	s. d.
As	17.25	:	12.75	::	5.75	:	4.25=4	3 *Ans.*

. What will 28.5 quarts of wine cost, if 9.75*s.* be given for 3.25 quarts? *Ans.* £.4 5*s.* 6*d.*

4. If $\frac{1}{2}$ cwt. of tobacco cost £.6 15*s.*; what will be the price of 3 cwt. 2 qr. 7 lb? *Ans.* £.48 1*s.* 10$\frac{1}{2}$*d.*

. If £.3 5*s.* be given for 1 cwt. of tallow; what will the price of 5 cwt. 3 qr. 14 lb. at the same rate? *Ans.* £.19 1*s.* 10$\frac{1}{2}$*d.*

. What is the duty of 243$\frac{1}{4}$ barrels of ale, when . 6*d.* is paid for 3 barrels? *Ans.* £.111 9*s.* 9$\frac{1}{2}$*d.*

. If the duty of 3 barrels of table beer, amounts to 6*d.*; what will be the duty of 18$\frac{3}{4}$ barrels? *Ans.* £.1 14*s.* 4$\frac{1}{2}$*d.*

. If the duty of $\frac{3}{4}$ of a barrel of ale comes to 10$\frac{1}{2}$*d.*; what will be the duty of 16$\frac{1}{4}$ barrels? *Ans.* £.7 8*s.* 11$\frac{1}{4}$*d.*

. What is the duty of $\frac{7}{8}$ of a 1000 of polished bricks, when the duty of $\frac{3}{4}$ of a 1000 amounts to 9*s.* 7$\frac{1}{2}$*d.*? *Ans.* 11*s.* 2$\frac{3}{4}$*d.*

10. If the duty of $\frac{1}{20}$ of a 1000 of common bricks comes to 3$\frac{1}{2}$*d.*; what will be the duty of $\frac{1}{4}\frac{2}{8}$ of a 1000? *Ans.* 5*s.* 8$\frac{1}{2}$*d.*

11. What is the duty of $\frac{4}{5}$ of a 1000 of common bricks, when the duty of $\frac{1}{40}$ of a 1000 is 1$\frac{3}{4}$*d.*? *Ans.* 4*s.* 8*d.*

12. If the duty of 5 pounds of hard soap comes to *d.*; what is the duty of 42364 pounds? *Ans.* £.397 3*s.* 3*d.*

SQUARE ROOT.

To extract the Square Root, is to find such a number as being multiplied by itself will produce the given number. Thus the Square Root of 25, is 5; because 5 squared, or multiplied by 5=25, the proposed number.

RULE.

Divide the given number into periods of two figures each, by placing a point above every second figure; beginning at units place, and proceeding to the left in integers, and to the right in decimals.

Find the greatest square root contained in the first period on the left-hand, and place it on the right-hand of the given number, after the manner of a quotient figure in division.

Subtract the square of this root from the said period; and to the remainder bring down the two figures of the next period, for a dividend.

Double the root above mentioned, for a divisor; find how often it is contained in the dividend, exclusive of the right-hand figure; and place the result both in the quotient and the divisor.

Multiply the divisor thus augmented, by the last quotient figure; subtract the product from the dividend; and bring down the next period to the remainder, for a new dividend.

Repeat the same process for each period, and the quotient thus obtained will be the root required.

A Table of Squares and Roots.

Squares	1.	4.	16.	25.	36.	49.	64.	81.
Roots	1.	2.	4.	5.	6.	7.	8.	9.

Note 1. The root will consist of as many integers and decimals as there are periods of each in the given number.

2. When the decimals in the given number do not consist of an even number of figures, they must be made even, by placing a cypher on the right; and when all the periods are brought down, the operation may be continued at pleasure, by annexing two cyphers to each remainder.

3. If the square root of a vulgar fraction be required, reduce it to a decimal, and then extract the root.

The best method of doubling the root, to form new divisors, add the last quotient figure to the last divisor.

The method of proof is to multiply the root into itself, add the ainder, if any, to the product, and the sum will be equal to the number, if the work be right.

EXAMPLES.

What is the square root of 642.1156?

```
        642.1156(25.34 Ans.
        4
   45)242
      225
  503)1711
      1509
 5064) 20256
       20256
```

Proof.

```
   25.34
   25.34
  101.36
  7602
 12670
 5068
 642.1156
```

What is the square root of 1048576? *Ans.* 1024.

Required the square root of 983. *Ans.* 31.352.

What is the square root of 8104.2624? *Ans.* 90.023.

Find the square root of 744.326. *Ans.* 27.282.

Extract the square root of .0000178929. *Ans.* .00423.

What is the square root of $\frac{202}{314}$? *Ans.* .802.

Extract the square root of $\frac{26896}{38416}$. *Ans.* .8367.

What is the square root of $\frac{18}{20}$ of $\frac{25}{26}$ of $\frac{35}{45}$ of $\frac{42}{56}$? *Ans.* .7099.

SQUARE ROOT,

APPLIED IN SOLVING USEFUL MATHEMATICAL PROBLEMS.

PROBLEM I.

To find a mean proportional between two given numbers.

RULE.

Multiply the two given numbers together, and the square root of their product will be the mean proportional sought.

EXAMPLES.

1. What is the mean proportional between 20 and 45?

$$\begin{array}{l} 45 \\ \underline{20} \\ 900(30\ \textit{Ans.} \\ \underline{9} \\ \underline{\underline{00}} \end{array}$$

2. There are three numbers in geometrical progression; the first is 48, and the third 243, what is the middle number? *Ans.* 108.

3. In a pair of false scales, a body weighed 36 lb. in one scale, but only 16 lb. in the other; required its true weight; and the ratio of the lengths of the two arms of the balance on each side of the point of suspension.

Ans. The true weight of the body is 24 *lb. and the arms of the balance are to each other as* 16 *to* 24, *or as* 24 *to* 36, *or as* 2 *to* 3.

PROBLEM II.

To find the side of a square equal in area to any given superficies.

RULE.

Extract the square root of the number expressing the

erficies of the given figure; and it will be the side of uare equal in area.

te. The definitions of Geometrical Figures, and the methods of ding their areas may be seen in Parts III. and IV.

EXAMPLES.

The area of a rectangular floor is 2025 square feet; t will be the side of a square floor of equal area?

Ans. 45 *feet.*

The area of the base of a triangular cistern is 324 re feet; required the side of a square cistern that be equal in area. *Ans.* 18 *feet.*

A maltster has a rectangular kiln whose area is 144 re feet; what must be the side of a square one, h he intends to build, so that the latter may dry times as much malt at a time as the former?

Ans. 24 *feet.*

PROBLEM III.

find the diameter of a circle, when the area is given.

RULE.

ivide the area by .7854, and the square root of the ient will be the diameter required.

EXAMPLES.

The area of a circle is 8824.7 square inches; what e diameter? *Ans.* 106 *inches, nearly.*

The area of the head of a cask is 254.5 square in-; required the head diameter. *Ans.* 18 *inches.*

PROBLEM IV.

find the hypothenuse of a right-angled triangle, when the base and perpendicular are given.

RULE.

'o the square of the base add the square of the per-dicular, and the square root of the sum will be the othenuse.

EXAMPLES.

1. The base of a right-angled triangle is 105, and perpendicular 56 inches; what is the hypothenuse?

Ans. 119 *inc*

2. The length of a cistern in the form of a parallelo pedon is 112, and its breadth 84 inches; what is the d gonal of its bottom? *Ans.* 140 *inch*

3. The side of a cubical mash-tun is 72.5 inche what is the diagonal of its bottom? *Ans.* 102.53 *inch*

PROBLEM V.

Given the hypothenuse of a right-angled triangle, and eith of the legs, to find the other leg.

RULE.

From the square of the hypothenuse subtract th square of the given leg, and the square root of the re mainder will be the required leg.

EXAMPLES.

1. The hypothenuse of a right-angled triangle is 153 and the base 135 inches; what is the perpendicular?

Ans. 72 *inches*

2. The diagonal of a cylindrical vessel is 60, and the diameter of its bottom 36 inches; what is its perpendicular depth? *Ans.* 48 *inches.*

3. The length of a guile-tun, in the form of a parallelopipedon, is 78, the longest diagonal that can be measured within it 125, and the diagonal of its bottom 100 inches; required its depth and breadth.

Ans. Its depth is 75, *and its breadth* 62.577 *inches.*

PROBLEM VI.

Given the head and bung diameters and length of a cask, to find the diagonal or distance between the centre of the bung-hole and that point where the middle of the opposite staff and head of the cask intersect each other.

RULE.

To the square of half the sum of the diameters, add

e square of half the length of the cask; and the square
ot of the sum will be the diagonal.

EXAMPLES.

1. The length of a cask is 30, the bung diameter 24,
d the head diameter 18 inches; required the diagonal.
Ans. 25.8 *inches.*

2. The bung and head diameters of a cask are 32 and
and the length is 40 inches; what is the diagonal?
Ans. 34.4 *inches.*

PROBLEM VII.

en the diagonal and diameters of a cask, to find its length.

RULE.

From the square of the diagonal take the square of
f the sum of the diameters; and twice the square root
the remainder will be the length of the cask.

EXAMPLES.

1. The bung diameter of a cask is 24, the head dia-
er 18, and the diagonal 25.8 inches; what is the
gth of the cask? *Ans.* 30 *inches, nearly.*

2. The bung diameter of a cask is 35, the head dia-
ter 29, and the diagonal 40 inches; required the
gth. *Ans.* 48 *inches.*

CUBE ROOT.

To extract the Cube Root is to find such a number as
ng multiplied by itself, and that product again by the
ne number, will produce the given number. Thus the
be Root of 125, is 5; because 5 squared, or multiplied
5, and that product by 5=125, the proposed number.

RULE.

Divide the given number into periods of three figures
ch, by placing a point above every third figure; be-

ginning at unit's place, and proceeding to the left in integers, and to the right in decimals.

Find the greatest cube root contained in the first period, on the left-hand, and place it on the right-hand of the given number, after the manner of a quotient figure in division.

Subtract the cube of this root from the first period; to the remainder annex the three figures in the following period; and call this number the resolvend.

Under the resolvend, write three times the root above found, and also three times its square; placing the latter one figure to the left of the former; and call their sum the divisor.

Find how often this divisor is contained in the resolvend, exclusive of the place of units; and set the result in the quotient, to the right of the root already found.

Under the divisor write the cube of the last quotient figure; its square multiplied by three times the former figure; and its triple multiplied by the square of the former figure; placing each one figure to the left; and call their sum the subtrahend.

Subtract the subtrahend from the resolvend, and to the remainder bring down the next period, for a new resolvend, with which proceed as before; always repeating the same process for each period, and the quotient thus obtained will be the root required.

Note 1. The last figure of the root, in each operation, must be so taken that the subtrahend may be less than the resolvend.

2. When the decimals in the given number do not form exact periods, the defect must be supplied by placing cyphers on the right; and when all the periods are brought down, the operation may be continued at pleasure, by subjoining three cyphers to each remainder.

3. The cube root of a vulgar fraction may be obtained by reducing it to a decimal, and then extracting the root.

A Table of Cubes and Roots.

Cubes	1,	8,	27,	64,	125,	216,	343,	512,	729.
Roots	1,	2,	3,	4,	5,	6,	7,	8,	9.

EXAMPLES.

1. What is the cube root of 52734375.

52734375(375 *Ans.*
27

25734 resolvend.

9 triple of 3.
27 triple square of 3.

279 divisor.

343 cube of 7.
441 square of 7 × by the triple of 3.
189 triple of 7 × by the square of 3.

23653 subtrahend.

2081375 second resolvend.

111 triple of 37.
4107 triple square of 37.

41181 second divisor.

125 cube of 5.
2775 square of 5 × by the triple of 37.
20535 triple of 5 × by the square of 37.

2081375 second subtrahend.

Required the cube root of 63044792. *Ans.* 398.
What is the cube root of 958585256? *Ans.* 986.
Required the cube root of 633.839779. *Ans.* 8.59.
What is the cube root of 1006.012008? *Ans.* 10.02.
Required the cube root of .898632125. *Ans.* .965.
What is the cube root of $\frac{4}{3}\frac{?}{?}$? *Ans.* .967.

CUBE ROOT,

APPLIED IN SOLVING USEFUL MATHEMATICAL PROBLEMS.

PROBLEM I.

Find the side of a cubical vessel that shall be equal in content to any given vessel whose form is that of a parallelopipedon, a cylinder, a prism, a cone, &c. &c.

RULE.

Extract the cube root of the content of the given ves-

sel, and it will be the side of the cubical vessel required.

Note. When the content of the given vessel is in ale or wine gallons, or malt bushels, it must be brought into inches, by multiplying by 282 for ale gallons, 231 for wine gallons, and 2150.42 for malt bushels; these being the cubic inches contained in one gallon, and one bushel respectively.

EXAMPLES.

1. The content of a vessel in the form of a parallelopipedon is 216000 cubic inches; what must be the side of a cubical vessel that shall be equal in content?

Ans. 60 *inches.*

2. A cylindrical vessel contains 43 gallons of wine; what must be the side of a cubical vessel that will contain the same quantity of rum? *Ans.* 21.496 *inches.*

3. A maltster has a cistern in the form of a square prism, that will contain 48 bushels of barley; what must be the side of a cubical cistern that will hold the same quantity? *Ans.* 46.908 *inches.*

PROBLEM II.

Given the dimensions of any vessel, to find the dimensions of another similar vessel, that shall be any number of times greater or less than the given vessel.

RULE.

Multiply the cubes of the dimensions of the given vessel by the given ratio, if the required vessel is to contain more, but divide the said cubes by the ratio, if it is to contain less; and the cube roots of the products or quotients will be the dimensions required.

EXAMPLES.

1. The length of a vessel in the form of a parallelopipedon is 216, its breadth 125, and its depth 64 inches; required the dimensions of a similar vessel that shall contain 8 times as much.

Ans. The length is 432, *the breadth* 250, *and its depth* 128 *inches.*

2. The depth of a cylindrical vessel is 64, and its diameter 27 inches; required the depth and diameter of a similar vessel that shall contain 27 times as much.

Ans. The depth is 192, *and the diameter* 81 *inches.*

The depth of a cylindrical vessel is 343, and its eter 125 inches; what must be the dimensions of a ar vessel that shall contain no more than ⅛ of the tity?

s. *The depth is* 171.5, *and the diameter* 62.5 *inches*.

The length of a cask is 30, the bung diameter 24, he head diameter 18 inches; required the dimens of a similar cask that shall contain three times as

. *The length is* 43.267, *the bung diameter* 34.614, *e head diameter* 25.96 *inches*.

PROBLEM III.

the dimensions and content of any vessel, to find the imensions of a similar vessel of a given content.

RULE.

Euclid's Elements, book the 11th, proposition the and book the 12th, propositions the 12th and 18th, content of any solid, is to the cube of its dimenso is the content of any similar solid, to the cube limensions. (See Theo. 20, Part III.)

EXAMPLES.

The content of a vessel in the form of a parallellon, is 2625 cubic inches, its length 25, its breadth d its depth 7 inches; required the dimensions of a vessel whose content shall be 21000 cubic inches.

. *The length is* 50, *the breadth* 30, *and the depth* 14

The diameter of a cylindrical vessel is 30 inches, pth 50, and its content 125.33 ale gallons; what be the diameter and depth of a similar vessel that contain only 68.5 gallons?

ns. *The diameter is* 24.52, *and the depth* 40.88 *inches*.

The bottom diameter of a vessel in the form of the um of a cone, is 46 inches, the top diameter 62, the 60, and the content 599.45 wine gallons; what

must be the dimensions of a similar vessel that shall contain 946 gallons?

Ans. The bottom diameter is 58.55, the top diameter 72.18, and the depth 69.85 inches.

4. The length of a cask is 30 inches, the bung diameter 24, the head diameter 18, and its content 38.75 ale gallons; required the dimensions of a similar cask that shall contain 35.5 gallons.

Ans. The length is 33.82, the bung diameter 27.05, and the head diameter 20.28 inches.

PART II.

A DESCRIPTION of the SLIDING RULE; with Directions how to find any Number upon it; and also the Application of the different Lines to Multiplication, Division, the Rule of Three, and the Extraction of the Square and Cube Roots.

DESCRIPTION OF THE SLIDING RULE.

THIS Instrument is in the form of a parallelopipedon; has four sliding pieces, which run in grooves, and is commonly made of box. It was invented by Mr. Thomas Everand, about the year 1683, and is generally called Everand's Sliding Rule.

It is of various lengths, as 6, 9, 12, 18 inches, &c.; but 12 inches is the most common length.

This instrument is much used in Gauging, in consequence of the ease and expedition with which calculations may be made by it.

Lines on the first Face of the Rule.

Upon the first face of the Rule, is a line of numbers, marked A, which is called Gunter's Line, from its inventor, Mr. Edmund Gunter; and is numbered from the left to the right with the figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10;

e spaces between each of these figures are gradua-
o subdivisions.
2150.42, is a brass pin marked M B, signifying
ubic inches in a bushel of malt; and at 282 is
er brass pin, marked A, denoting the number of
inches in a gallon of ale.
e second line on this face is upon the slide, and is
l by the letter B. It is divided exactly in the same
er as that marked A. There is also another slide
ich is used with the former; the two brass ends
en placed together, in which position they form a
e radius numbered from the left to the right.
231, on the second slide or radius, is a brass pin
ed W, denoting the cubic inches in a gallon of
; and at 3.1416 is another brass pin, marked C, sig-
g the circumference of a circle whose diameter is

these lines, multiplication, division, proportion, &c.,
be performed; and the manner of reading and
them will be described hereafter.
on the back of the first slide, marked B, are placed
ivisors for ale, wine, mash-tun gallons, malt-bushels,
starch, dry starch, hard soap hot, hard soap cold,
s in the Table of Factors, Divisors, &c., Part IV;
he back of the second slide, marked B, contains the
-points corresponding to these divisors, where S
s for squares, and C for circles.
on the same face of the Rule, is another line mark-
D, signifying malt-depth; and is numbered from
ght to the left, with the figures, 3, 4, 5, 6, 7, 8, 9,

e 1 or 10 on this line is placed directly opposite to
on the line A; and if it be called 1000, the last
on at the left-end of the rule, will denote 2150.42,
ubic inches in a Winchester bushel; but if 10
lled 1, the last division at this end will be 2.15,
me as the first division at the right-end of the Rule.
s line is used with the line A, and the slides B or
malt gauging.

Lines on the second Face of the Rule.

n the second face of the Rule, or that opposite to

the one already described, is a line marked D. This line begins on the upper edge of the Rule, and is numbered from the left to the right with the figures, 1, 2, 3, 31, 32, which is at the right-end of the Rule; the line is then continued from the left-end of the lower edge, 32, 4, 5, 6, 7, 8, 9, 10, which is the last division.

At 17.15 is a brass pin, marked W G, signifying the circular gauge-point for wine gallons; and at 18.95, the circular gauge-point for ale gallons, is another brass pin marked A G.

There is also another brass pin at 46.37, marked M S, which is the square guage-point for malt bushels; and at 52.32, is another marked M R, denoting the round or circular gauge-point for malt bushels.

On this line the gauge-points for ale gallons, wine gallons, &c. may also be found although they are not pointed out by letters and brass pins.

The second line on this face of the Rule, is marked C, which is upon the slide, and is numbered and divided in the same manner as the lines A and B. Belonging to the Rule is also another slide C, which is used with the former, in the same manner as the two slides marked B.

The line D, and the two slides C, are used in finding the contents of vessels whose form is that of a cube, a parallelopipedon, a cylinder, &c. &c.

By these lines the square root of any number may also be readily extracted; for if 1 on C be set to 1 on D, we have against any proposed number on C, its root on D.

The back of the first slide, marked C, is divided, next the upper edge, into inches and tenths; and numbered from the left to the right, with 1, 2, 3, 4, 5, &c.; the second line is marked *spheroid*, and the third line, *second variety*, and both are numbered from the left to the right with the figures 1, 2, 3, 4, 5, &c., and the spaces between each of these figures are divided into ten equal parts.

These three lines are used for finding a mean diameter between the head and bung diameters of casks of the first and second varieties, which is performed in the following manner: Find the difference between the bung and head diameters, on the first line, or line of inches, then against it, for each variety, is a number, which being added to the head, will give the mean diameter sought; hence the cask is reduced to a cylinder.

: back of the second slide, marked C, contains
l multipliers for reducing goods of one denomina-
) their equivalent values in those of another: Thus
o T 5. | signifies that to reduce strong beer at 10*s.*
rrel, to small beer at 2*s.* per barrel, you must mul-
he number of gallons of strong beer by 5; and the
:t will be the equivalent number of gallons of
)eer. | T to X .2 | signifies that small beer at 2*s.*
rrel, must be multiplied by .2, in order to reduce
trong beer at 10*s.* per barrel. | C to X 1.62 |
: that to reduce cyder at 1*s.* 10*d.* per hogshead, to
beer at 10*s.* per barrel, the number of gallons of
nust be multiplied by 1.62; and to reduce strong
cyder, the multiplier is .62.

In consequence of the various changes that have, at different
ken place in the duties upon beer, cyder, &c. &c., scarcely
ng Rules can be met with that are correct in these multipliers.
:oblem on this subject, in Part VI.)

Lines on the third Face of the Rule.

third face of the Rule contains a line marked Seg.
ꞌ S, signifying segments standing; and is numbered
ie left to the right on the upper edge of the Rule,
2, 3, 4, 5, 6, 7, 8; it is then continued on the
dge, 8, 9, 10, 20, 30, &c. to 100.
line is used with the two slides, marked C, in
the *ullage* of a standing cask, or the quantity of
it contains when it is not full.

Lines on the fourth Face of the Rule.

n the fourth face of the Rule, or the one opposite to
st described, is a line marked Seg. Ly. or S L,
ıg segments lying. This line is numbered nearly
same manner as the last, and is used with the slides
llaging lying casks.
se are all the lines that are upon this Rule, which,
our opinion, better adapted for practice, than any
vith which we are acquainted.

There are various kinds of Sliding Rules, some of which
ine marked E, for extracting the cube roots of numbers, and
line for the third variety of casks; they are however, all upon the

same principle, and may be easily understood from the foregoing description.

There is also a rule called Branan's Rule, which has lines of numbers upon it similar to those described on the sliding rule; and may be used in most cases with the same advantage. (See Cask Gauging.)

THE METHOD OF ESTIMATING THE VALUES OF THE DIVISIONS ON THE SLIDING RULE.

The lines marked A, B, C, and D, on this instrument are all logarithmic lines, and are divided into spaces which are proportional to the logarithms of the numbers placed at their ends.

These lines were first placed on Rules by Mr. Edmund Gunter, who in the year 1624, first made the discovery of applying logarithms to extension; and of performing, with great facility, by means of a pair of compasses, and the line of numbers, the business of Multiplication, Division, and all Arithmetical operations, where the Rule of Proportion is required. The use of compasses, however, being found both troublesome and inaccurate, Mr. Thomas Everand, about the year 1683, made a very essential improvement in the application of the line of numbers, by contriving one line to slide by another; and the Rule has since been further improved by Mr. Verie.

In order to obtain a proper knowledge of the line of numbers, it is necessary that you make yourself well acquainted with the values of the different divisions.

Whatever value you assign to 1, at the beginning of the line, whether 1, 10, 100, or 1000, the following integral numbers 2, 3, 4, &c. will represent twice, thrice, four-times, &c. as much; consequently, the second 1 will be 10 times the value of the first, and the third 1 will express 100 times the value of the first, or 10 times the value of the second. Thus the values of the integral divisions being known, those of the subdivisions may be easily ascertained; being always the quotient obtained by dividing the difference of any two adjoining integral numbers, by the number of parts contained between them.

)ivisions on the lines A, B, and C.

between the figures 1 and 2, on the line A, into 10 larger divisions, and 50 smaller we call one at the beginning of this line, will the following figures denote two, three, d as the difference of the integral divisions value of each of the larger subdivisions will by $\frac{1}{10}$ or .1; and the value of each of the visions by $\frac{1}{50}$ or .02; consequently the va- t, 2nd, 3rd, 4th, 5th, &c. divisions, from 1, essed by $1\frac{1}{50}$, $1\frac{2}{50}$, $1\frac{3}{50}$, $1\frac{4}{50}$, $1\frac{5}{50}$, &c. or in 2, 1.04, 1.06, 1.08, 1,1, &c.

1 at the beginning of the line 10, then will figures be read 20, 30, 40, &c.; and the 1st, 2nd, 3rd, &c. divisions, from 10, be $10\frac{10}{50}$, $10\frac{20}{50}$, $10\frac{30}{50}$, $10\frac{40}{50}$, $10\frac{50}{50}$, &c. or in 2, 10.4, 10.6, 10.8, 11.0; hence it appears the larger divisions will be a *unit*, when the d 10; and many of the new Rules, are uch a manner as to read 10, 11, 12, 13,

been already advanced on this subject will mprehended by inspecting the following

2, the tenths are divided into five parts, each ng two hundredths; therefore,

	.1	Then will	.01	And	.002
	1.	the larger	.1	the	.02
	10.	divisions	1.	smaller	.2
	100.	represent	10.	divisions	2.

o 5, the tenths are divided into two parts, ıtaining five hundreths; therefore,

.2	Then will	.01	And	.005
2.	the larger	.1	the	.05
20.	divisions	1.	smaller	.5
200.	represent	10.	divisions	5.

o 10, the tenths are not divided at all; con- he hundredths must be guessed at; there-

If the figure 5 represent	.5	The tenths will represent	.01
	5.		.1
	50.		1.
	500.		10.

Divisions on the line D.

From 1 to 2, the tenths are divided into ten parts, ca ed hundredths; therefore,

If the figure 1, at the beginning of the line represent	.1	Then will the larger divisions represent	.01	And the smaller divisions	.0
	1.		.1		.0
	10.		1.		.1
	100.		10.		1.

From 2 to 3, the tenths are divided into five parts, ea part containing two hundredths; therefore,

If the figure 2, represent	.2	Then will the larger divisions represent	.01	And the smaller divisions	.00
	2.		.1		.02
	20.		1.		.2
	200.		10.		2.

From 3 to the end of the line D, the tenths are or divided into two parts, each part containing five hu dredths; therefore,

If the figure 3 represent	.3	Then will the larger divisions represent	.01	And the smaller divisions	.00
	3.		.1		.05
	30.		1.		.5
	300.		10.		5.

From what has been said on this subject, it will n be difficult to find the point upon any of the lines, which any given number is represented; for examp let it be required to find 18.95 on the line D.

For the first figure 1, in the given number, call 1 the beginning of the line, 10; for the second figure count *eight* of the larger divisions as *units*, and you w have the point where 18 stands; then, from this poi count *nine* of the smaller divisions for .9, and *half* of t next division for .05; and you will find the *brass p* marked A G, which is placed at 18.95, the circular *gau point* for ale gallons.

;ain, Suppose it be required to find .001895, .01895, ;, 1.895, 18.95, 189.5, and 1895; they will all be l at the same point; but the figure 1, at the be- ing of the line, must be conceived to be of a differ- alue, in finding each number; viz. .001, .01, .1, 1, 00, and 1000.

this method of counting, you will find 17.15, at the marked W G; 46.87 at M S; and 52.32 at M R. on the line A, you will find 2150.42 at the point ed M B, and 282, at that marked A.

stly, Let it be required to find 305, on the line A. ie figure 3, on this line, must be counted 300; and e second figure, in the given number, is a cypher, t *no* tenths; but for the last figure 5, count *half* a 1, or *one* of the smaller divisions, and you will have equired number 305.

THE USE OF THE SLIDING RULE.

PROBLEM I.

Multiplication by the lines A and B.

RULE.

As unity on A is to the multiplier on B, so is the mul- licand on A to the product on B.

Or,

As unity on B is to either of the factors, on A, so is the ier factor on B to the product on A.

Note 1. It will sometimes happen that when the multiplier on B, set to unity on A, the multiplicand cannot, according to the true meration of the Rule, be properly expressed on A; in such cases, er having set one of the factors on B, to unity on A, divide the ier factor by some power of 10,—viz. 10, 100, 1000, &c. so that e quotient may be found on A, opposite to some division, or oduct, on B; then multiply this product by the same power of 10 which the given factor was divided, and the number thus obtained ill be the product required. Or, which is the same thing, after ving set one of the factors on B, to unity on A, in finding the other

factor on A, call *unity* 10, 100, 1000, &c. as the case may require; and in estimating the product on B, whatever number you called unity, in valuing the factor on that line, you must increase it in the same proportion as you have done unity on A.

2. When the product consists of more than three or four figures, it is difficult to value the latter figures on the Rule; but in this case, it will be easy to bear one or two in the memory: Thus, if it were required to multiply 56 by 38, the product will be 2128; for the last figure is known to be an 8, by multiplying 6 by 8, mentally; and it is evident, from the Rule, that the product is more than 2118, and less than 2138.

3. If both the multiplicand and multiplier are whole numbers, the product will be a whole number; if one be a whole number, and the other a mixed number, or both mixed numbers, the product will be a mixed number; and if they both be decimals; the product will be a decimal. Also, the product will always have as many decimal places in it, as there are in both the factors.

EXAMPLES.

1. What is the product of 9 multiplied by 7?
Set 7 *on* B *to* 1 *on* A, *then against* 9 *on* A, *is* 63 *on* B.

Or, by Proportion,

As 1 *on* A : 7 *on* B :: 9 *on* A : 63 *on* B; *Or, As* 1 *on* B : 7 *on* A :: 9 *on* B : 63 *on* A.

2. Required the product of 15 multiplied by 8. *Ans.* 120.
3. It is required to multiply 64 by 24. *Ans.* 1536.
4. What is the product of 265 by 128? *Ans.* 33920.
5. Multiply 8.5 by 5.3. *Ans.* 45.05.
6. Multiply 4.7 by 3.5. *Ans.* 16.45.
7. Multiply 3.5 by 1.7. *Ans.* 5.95.
8. Multiply 6.2 by 2.4. *Ans.* 14.88.
9. Multiply 1.8 by .9. *Ans.* 1.62.
10. Multiply 64.7 by 32.8. *Ans.* 2122.16.
11. Multiply 86.3 by .32. *Ans.* 27.616.
12. Multiply .562 by .238. *Ans.* 133756.

PROBLEM II.

Division by the lines A and B.

RULE.

As the divisor on B is to unity on A, so is the divi- on B to the quotient on A.

Or,

As the divisor on A is to unity on B, so is the divi- on A to the quotient on B.

1. When the divisor on B is set to unity on A, if the dividend be properly expressed on B, according to the true numeration of e, you must divide it by some power of 10, and multiply the t by the same number by which you divide the dividend. (See : Note in the last Problem.)

f the divisor and dividend be both whole numbers, and the less than the dividend, the quotient will be a whole, or a number; but if the divisor be greater than the dividend, the it will be a decimal. Also, if the divisor be a whole number, e dividend a decimal, the quotient will be a decimal; if the be a decimal, and the dividend a whole number, the quotient either a whole, or a mixed number; and if both the divisor idend be decimals, the quotient will be a decimal when the is greater than the dividend; but when it is equal to, or less he dividend, the quotient will be either a whole, or a mixed r.

When the divisor is greater than the dividend, you must conceive petent number of cyphers to be placed on the right of the id, as decimals; and you must always remember that there as many decimal places in the quotient as the dividend contains than the divisor.

It is sometimes difficult to know how many whole numbers, and many decimals there should be in the quotient; but this may be ascertained by inspection: Thus, if it be required to divide 3; it is evident that there will be two whole numbers in the ent, and the rest of the figures will be decimals.

ain, let it be required to divide 32 by 46. Here it is evident that must be a cypher placed on the right of the dividend, before it ontain the divisor; hence, all the quotient will be decimals.

stly, let it be required to divide 22 by 365. Here it appears that yphers must be affixed to the dividend, before we can divide it e divisor; hence all the quotient will be decimals, and the first e on the left will be a cypher.

EXAMPLES.

1. What is the quotient of 8 divided by 2?
Set 2 *on* B *to* 1 *on* A, *then against* 8 *on* B, *is* 4 *on* A;

Or, by Proportion,

As 2 *on* B : 1 *on* A :: 8 *on* B : 4 *on* A; *Or, As* 2 *on* A : 1 *on* B :: 8 *on* A : 4 *on* B.

2. Divide 42 by 7. *Ans.* (
3. Required the quotient of 96 divided by 8. *Ans.* 1
4. Divide 1536 by 24. *Ans.* 6
5. What is the quotient of 33920, divided by 128? *Ans.* 26
6. Divide 45.05 by 5.3. *Ans.* 8.
7. What is the quotient of 16.45 divided by 4.7? *Ans.* 3.
8. Divide 14.88 by 2.4. *Ans.* 6.
9. Required the quotient of 1.62 divided by .9. *Ans.* 1.
10. What is the quotient of 2122.16 divided by 32.8? *Ans.* 64.
11. Divide 35 by 125. *Ans.* .2
12. What is the quotient of 15 divided by 632? *Ans.* .023

PROBLEM III.

To find a fourth proportional to three numbers; or to perform the Rule of Three by the lines A and B.

RULE.

As the first term on A, is to the second on B; so is the third term on A, to the fourth on B.

Note. The method of finding a third proportional is exactly the same; the second number being twice repeated: Thus, if a third proportional be required to 80 and 60; it will be as 80 on A : 60 on B :: 60 on A : 45 on B, the third proportional sought.

EXAMPLES.

1. If 12 yards of cloth cost 56*s.* what will 18 yards cost at the same rate?

yds. *s.* *yds.* *s.*
As 12 *on* A : 56 *on* B :: 18 *on* A : 84 *on* B.
f 44 gallons of ale cost 56*s.* what will 8 gallons
Ans. 10*s.*
f 24 candles weigh a pound, what is the weight of
andles? *Ans.* 150 *lb.*
That is the fourth proportional to the three num-
, 24, 36? *Ans.* 72.
f $3\frac{1}{4}$ bushels of malt cost 28*s.* 6*d.* what will $8\frac{3}{4}$
cost? *Ans.* 76.73*s.*
a gallon of rum cost 19*s.* 9*d.* what will $45\frac{1}{2}$ gallons
Ans. 898.6*s.*

PROBLEM IV.

Inverse Proportion by the lines A and B.

RULE.

e third term on A, is to the first on B, so is the
on A, to the fourth on B.

EXAMPLES.

20 workmen can build a brew-house in 180 days;
ny workmen can build it in 90 days?
da. *men.* *da.* *men.*
90 *on* A : 20 *on* B :: 180 *on* A : 40 *on* B.
ow many gallons of rum at 16*s.* per gallon, are
value to 8 gallons of brandy at 20*s.* per gallon?
Ans. 10 *gallons.*
8 men can do a piece of work in 18 days, in
ny days can 48 men do it? *Ans.* 3 *days.*
2 Excise Officers can gauge and fix a certain
of utensils in 12 days, in how many days can 6
gauge and fix them? *Ans.* 4 *days.*

PROBLEM V.

uce a vulgar fraction to a decimal, by the lines A and B.

RULE.

he denominator on A, is to unity on B, so is the
tor on A, to the required decimal on B.

EXAMPLES.

1. Reduce $\frac{1}{4}$ to a decimal fraction.

Denom. *Unity.* *Num.* *Decimal.*
As 4 *on* A : 1 *on* B :: 1 *on* A : .25 *on* B.

2. Reduce $\frac{1}{2}$ to a decimal. *Ans.* .5.
3. Reduce $\frac{3}{4}$ to a decimal. *Ans.* .75.
4. Reduce $\frac{24}{96}$ to a decimal. *Ans.* .25.

PROBLEM VI.

Having a divisor given, to find, by the lines A and B, a factor that shall perform the same by Multiplication as the divisor would do by Division.

RULE.

As the given divisor is to unity, so is unity to the factor sought.

EXAMPLES.

1. If the divisor be 125, what will be the factor to that number?

As 125 *on* A : 1 *on* B :: 1 *on* A : .008 *on* B, *the factor sought.*

2. If the divisor be 282, what will be the factor?
Ans. .003546.
3. The divisor is 231, what is the factor?
Ans. .004329.
4. What is the factor to the divisor 2150.42?
Ans. .000465.

PROBLEM VII.

Having a factor given to find a divisor by the lines A and B.

RULE.

As the factor is to unity, so is unity to the divisor.

EXAMPLES.

1. Find a divisor to the factor .008.
As .008 *on* A : 1 *on* B :: 1 *on* A : 125 *on* B, *the divisor sought.*
2. The factor is .003546, what is the divisor? *Ans.* 282.
3. If a factor be .004329, what is the divisor? *Ans.* 231.
4. Find a divisor to the factor .000465. *Ans.* 2150.42.

PROBLEM VIII.

To square any number by the lines C and D.

RULE.

Set 1 on D to 1 on C, then against the given number on D, is its square on C.

EXAMPLES.

1. What is the square of 3?
Set 1 *on* D *to* 1 *on* C; *then against* 3 *on* D, *is* 9 *on* C.
2. Required the square of 12. *Ans.* 144.
3. What is the square of 17.5? *Ans.* 306.25.
4. Find the square of 125.8. *Ans.* 15825.64.

PROBLEM IX.

To extract the Square Root of any number by the lines C and D.

RULE.

Set 1 on C to 1 on D, then against the given number on C, is its root on D.

EXAMPLES.

1. What is the square root of 9?
Set 1 *on* C *to* 1 *on* D; *then against* 9 *on* C, *is* 3 *on* D.

2. Required the square root of 144. *Ans.* 12.
3. What is the square root of 306.25? *Ans.* 17.5.
4. Find the square root of 15825.64. *Ans.* 125.8.

Note. When the Rule is set as directed in the last two Problems, the lines C and D form a table of squares and their roots.

PROBLEM X.

To find a geometrical mean proportional between two numbers, or to extract the Square Root of their product by the lines C and D.

RULE.

Set one of the given numbers on C, to the same on D; then against the other number on C, will be the mean on D.

EXAMPLES.

1. What is the mean proportional between 9 and 16?
Set 9 *on* C *to* 9 *on* D; *then against* 16 *on* C, *is* 12 *on* D.
2. What is the mean geometrical proportion between 48 and 243? *Ans.* 108.
3. Required the proportional between 18 and 32. *Ans.* 24.
4. Find the mean proportional between 96 and 486. *Ans.* 216.

PROBLEM XI.

To cube any number by the lines C and D.

RULE.

Set 1 on D, to the root or given number on C; then against the root on D, is the cube on C.

EXAMPLES.

1. What is the cube of 4?
Set 1 *on* D, *to* 4 *on* C; *then against* 4 *on* D, *is* 64 *on* C.
2. Required the cube of 12. *Ans.* 1728.

at is the cube of 18? *Ans.* 5832.
quired the cube of 35. *Ans.* 42875.

PROBLEM XII.

ct the Cube Root of any number by the lines C and D.

RULE.

the slide either way, until 1 at the beginning of D, 10 at the end of the line D, and the given on C, cut the same number on the opposite lines; number will be the root required.

EXAMPLES.

at is the cube root of 8?
1 on the line C, *then move the slide until* 1 *on* D, 8 *d* 10 *on* D, *cut the same number on the opposite ich you will find to be* 2, *the cube root required.*
quired the cube root of 64. *Ans.* 4.
at is the cube root of 120? *Ans.* 4.93.
quired the cube root of 200. *Ans.* 5.84.
hat is the cube root 1728? *Ans.* 12.
hat is the cube root of 1000? *Ans.* 10.

f the Sliding Rule has the line E upon it, set unity at the of D, to unity at the beginning of E; then against any n E, is its cube root upon D, and *vice versâ.*

PART III.

itions, Problems, and Theorems, in Geometry.

METRY originally signified the art of measuring th, or any distance or dimensions upon or within : it is now used for the science of quantity, exten- r magnitude, abstractedly considered.

METRY is divided into several parts, as *elementary*, *ical*, and *practical.*

Elementary Geometry treats of the properties and proportions of right lines, and right lined figures; and also of the circle and its several parts.

Theoretical Geometry has for its object the demonstration of certain geometrical theorems.

Practical Geometry is the performance of certain geometrical operations, such as the construction of figures, and the drawing of lines in certain positions, as parallel, perpendicular, &c. to other given lines.

GEOMETRICAL DEFINITIONS.

1. A point is considered as having neither length, breadth, nor thickness.

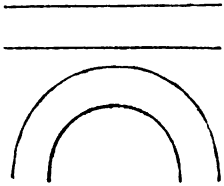
2. A line has length, but is considered as having neither breadth, nor thickness.

3. Lines are either right, curved, or parallel.

4. A right or straight line lies wholly in the same direction between its extremities, and is the shortest distance between two points.

5. A curved line continually changes its direction between its extremities.

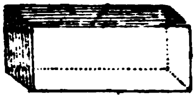
6. Parallel lines always remain at the same distance from each other, though infinitely produced.



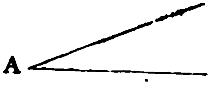
7. A superficies has length and breadth, but is considered as having no thickness.

8. A superficies may be contained within one curved line, but cannot be contained within fewer than three straight lines.

9. A solid is a figure of three dimensions; namely, length, breadth, and thickness.

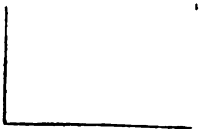


10. An angle is the inclination or opening of two lines, having different directions, and meeting in a point, which is called the angular point, as at A; and when three letters are used, the middle one denotes that point.

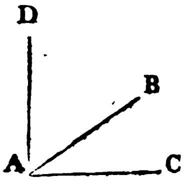


11. Angles are of three kinds; viz. right, acute, and obtuse.

12. A right angle is made by two right lines which are perpendicular to each other.

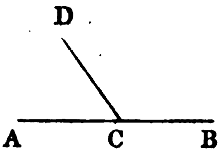


13. An acute angle is less than a right angle, as C A B.



14. The complement of an angle is what it wants to complete a right angle, as the angle D A B is the complement of the angle C A B.

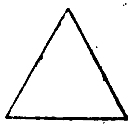
15. An obtuse angle is greater than a right angle, B C D.



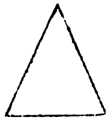
16. The supplement of an angle is what it wants two right angles, as the angle A C D, is the suppleme of the angle B C D.

17. A triangle is a figure or superficies bounded l three right lines, and admits of three varieties; viz. equ lateral, isosceles, and scalene.

18. An equilateral triangle has all its sides equal.



19. An isosceles triangle has only two of its side equal.

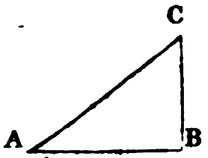


20. A scalene triangle has all its sides unequal.

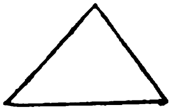
21. Triangles are also right-angled, acute-angled, and obtuse-angled.

22. A right-angled triangle has one right angle, the side opposite to which is called the hypothenuse, the

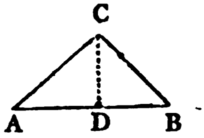
other two being termed legs, or one the perpendicular, and the other the base; thus A C is the hypothenuse, B C the perpendicular, and A B the base.



23. An acute-angled triangle has all its angles acute.



24. An obtuse-angled triangle has one of its angles obtuse, as A C B.

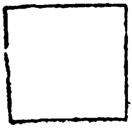


25. The base of any figure is that side upon which it is supposed to stand, or upon which a perpendicular is let fall from the vertex or opposite angle; and the altitude of a figure is its perpendicular height. In the last figure A B is the base, and C D the perpendicular.

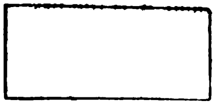
26. A figure of four sides and angles is denominated a quadrangle or quadrilateral figure.

27. A parallelogram is a quadrilateral figure, having its opposite sides parallel and equal, and admits of four varieties; viz. the square, the rectangle, the rhombus, and the rhomboid.

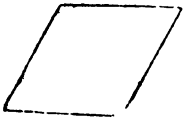
28. A square is an equilateral parallelogram, having all its angles right angles.



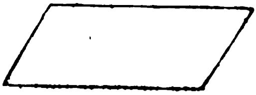
29. A rectangle is a parallelogram, having its opposite sides equal, and all its angles right angles.



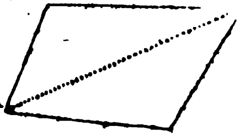
30. A rhombus is an equilateral parallelogram, having its opposite angles equal; two of which are acute, and two obtuse, which, in a regular rhombus, are 60 and 120 degrees.



31. A rhomboid or rhomboides is a parallelogram, having its opposite sides and angles equal; two of its angles being acute, and two obtuse; and when the figure is regular, the angles are 60 and 120 degrees.

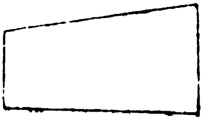


32. A trapezium is a quadrilateral figure, whose opposite sides are not parallel to each other.



33. A diagonal is a right line joining the opposite angles of a quadrilateral figure, as A B.

34. A trapezoid is a quadrilateral figure, having two of its opposite sides parallel.

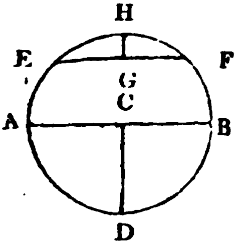


35. Plane figures having more than four sides, are generally called polygons; and receive their particular denominations from the number of their sides or angles.

36. A pentagon is a polygon of five sides; a hexagon of six; a heptagon of seven; an octagon of eight; a nonagon of nine; a decagon of ten; an undecagon of of eleven; and a duodecagon of twelve.

37. A regular polygon has all its sides and angles equal; when they are unequal, the polygon is irregular.

38. A circle is a plane figure, bounded by a curved line, called the circumference, which is every where equidistant from a certain point within it, called the centre.



39. The diameter of a circle is a right line drawn through the centre, and terminating in the circumference on each side, as A B.

40. The radius of a circle is half the diameter, or it is a right line drawn from the centre to the circumference, as C D.

41. An arc of a circle is any part of the circumference, as the arc E H F.

42. A chord is a right line joining the extremities of an arc, as the line E F; and the versed sine is part of the diameter cut off by the chord, as G H.

43. A segment is any part of a circle, bounded by an arc and its chord.

44. A semicircle is half of a circle, or a segment cut off by the diameter, as A D B.

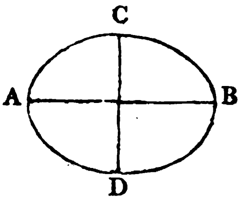
45. A quadrant is the fourth part of a circle, as A D C.

46. A sector is any part of a circle bounded by an arc and two radii.

47. The circumference of every circle is supposed to be divided into 360 equal parts, called degrees; each degree into 60 equal parts, called minutes; and each minute into 60 equal parts, called seconds, &c.

48. The arc of a quadrant contains 90 degrees, which is the measure of a right angle.

49. An ellipse is a plane figure bounded by a curved line, called the circumference; but as the figure is not a circle, it is described from two points in the longest diameter, called the foci, or focuses.



50. The longest diameter that can be drawn within an ellipse, is called the transverse diameter, as A B; and the shortest, the conjugate diameter, as C D.

Sometimes these diameters are termed axes.

51. A right-lined figure is inscribed in a circle, or the circle circumscribes it, when all the angular points of the figure are in the circumference of the circle.

52. A right-lined figure circumscribes a circle, or the circle is inscribed in it, when all the sides of the figure touch the circumference of the circle.

53. Identical figures are such as have all the sides and all the angles of one, respectively equal to all the sides and all the angles of the other.

54. Similar figures are such as have all the angles of one, equal to all the angles of the other, each to each; and the sides about the equal angles proportional.

55. A Proposition is something proposed either to be done, or to be demonstrated; and is either a Problem or a Theorem.

56. A Problem is something proposed to be done.

57. A Theorem is something proposed to be demonstrated.

58. A Demonstration is a certain and convincing proof of the truth of some proposition.

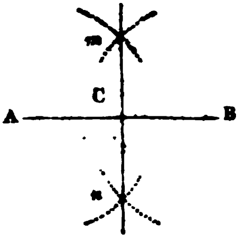
59. A Corollary is a consequent truth, gained immediately from some preceding truth or demonstration.

60. A Scholium is a remark or observation made upon some preceding Problem, Theorem, or Demonstration.

GEOMETRICAL PROBLEMS.

PROBLEM I.

To bisect a given line A B.

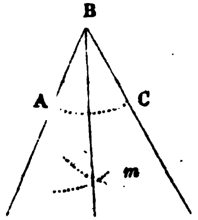


From A and B as centres, with any radius, greater than half A B, in your compasses, describe arcs cutting each other in m and n. Draw the line m C n, and it will bisect A B in C.

Note. In bisecting lines, drawing parallels, erecting perpendiculars, &c. &c. in the real Practice of Gauging, when the radius is too great to be taken in a pair of compasses, a measuring-tape, or a chalk-line must be used; and the intersections or arcs may be made with a pencil, or with chalk finely pointed, held at the end of the tape or line. The perpendiculars, parallels, &c. must also be struck by means of a chalk-line.

PROBLEM II.

To bisect a given angle A B C.

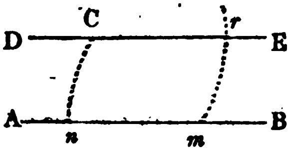


From the point B, with any radius, describe the A C. From A and C, with the same, or any other radi make the intersection *m*. Draw the line B *m* and it w bisect the angle A B C, as required.

PROBLEM III.

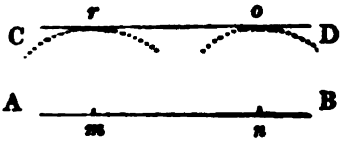
To draw a line parallel to a given line A B.

Case 1. When the parallel line is to pass through given point C.



From any point *m*, in the line A B, with the radius *m* C describe the arc C *n*. From the centre C, with the sam radius, describe the arc *m r*. Take the distance C *n* in th compasses, and apply it from *m* to *r*. Through C and r draw the line D E, and it will be the parallel required.

CASE 2. When the parallel line is to be at a given distance from A B.

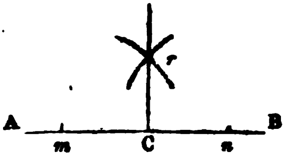


From any two points *m* and *n*, in the line A B, with a radius equal to the given distance, describe the arcs *r* and *o*. Draw the line C D, to touch these arcs, without cutting them, and it will be the parallel required.

Note. This Problem may be more easily performed by means of a parallel ruler, which may also be used to advantage in several operations in Practical Geometry.

PROBLEM IV.

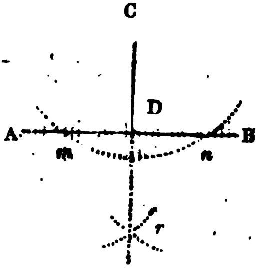
To erect a perpendicular from a given point C, in a given line A B.



On each side of the point C, take any two equal distances, C *m* and C *n*. From *m* and *n*, as centres, with any radius greater than C *m* or C *n*, describe two arcs cutting each other in *r*. Draw the line C *r*, and it will be the perpendicular required.

PROBLEM V.

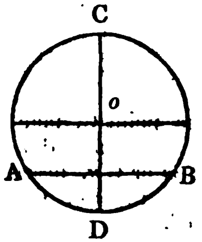
From a given point C, to let fall a perpendicular upon given line A B.



With C as a centre, and any radius a little exceedin the distance of the given line, describe an arc cutting A in *m* and *n*. With the centres *m* and *n*, and the same o any other radius exceeding half their distance, describ arcs intersecting each other in *r*. Draw the line C *r*; an CD will be the perpendicular required.

PROBLEM VI.

To find the centre of a given circle, or one already de scribed.

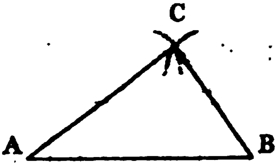


Draw any chord A B, and bisect it perpendicularly with C D, which will be a diameter. Bisect C D in the point *o*, which will be the centre required.

PROBLEM VII.

To make a triangle with three given lines, any two of which, taken together, must be greater than the third. (Euclid I. 22.)

Let the given lines be A B=12, A C=10, and B C=8.



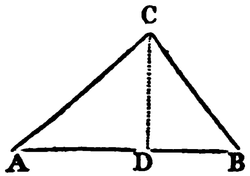
From any scale of equal parts (which is to be understood as employed likewise in the following Problems) lay off the base A B. With the centre A, and radius A C, describe an arc. With the centre B, and radius B C, describe another arc, cutting the former in C. Draw the lines A C and B C, and the triangle will be completed.

Note. A trapezium may be constructed in the same manner; having the four sides and one of the diagonals.

PROBLEM VIII.

Having given the base, the perpendicular, and the place of the perpendicular upon the base, to construct a triangle.

Let the base A B=12, the perpendicular C D=6, and the distance A D=7.



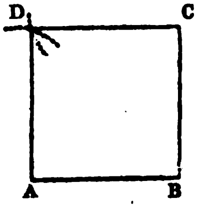
Make A B equal to 12, and A D equal to 7. At D erect the perpendicular D C, which make equal to 6. Join A C and B C, and the figure will be completed.

Note. A trapezium may be constructed in a similar manner, having one of the diagonals, the two perpendiculars let fall the from the opposite angles, and the places of these perpendiculars u the diagonal; and a trapezoid may be constructed by drawing the parallel sides perpendicularly to their base or given distance.

PROBLEM IX.

To describe a square whose side shall be equal to a gi line.

Let the given line A B=8.

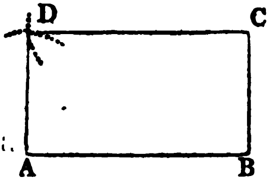


Upon one extremity B, of the given line, erect tl perpendicular B C, which make equal to A B. With . and C as centres, and the radius A B, describe ar cutting each other in D. Join A D and C D and th square will be completed.

PROBLEM X.

To describe a rectangular parallelogram, whose length an breadth shall be equal to two given lines.

Let the length A B=12, and the breadth B C=6.



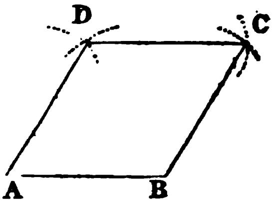
At B erect the perpendicular B C, which make equal to 6. With A as a centre, and the radius B C, describe an

arc; and with C as a centre, and the radius A B, describe another arc, cutting the former in D. Draw the lines A D and C D, and the rectangle will be completed.

PROBLEM XI.

Upon a given right line to construct a regular rhombus.

Let the given line A B=8.

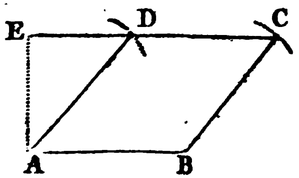


Draw the line A B equal to 8. With A and B as centres, and the radius A B, describe arcs cutting each other in D; then with B and D as centres, and the same radius, make the intersection C. Draw the lines A D, D C, and B C, and the rhombus will be completed.

PROBLEM XII.

To construct an irregular rhombus, having given its side and perpendicular height.

Let the side = 8, and the perpendicular = 6.



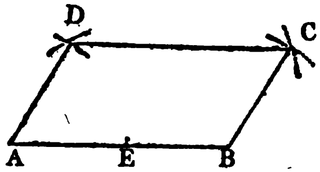
Draw A B equal to 8; at A erect the perpendicular A E, which make equal to 6; and draw E C parallel to

A B. With the radius A B, and A as a centre, make intersection D; and with the same radius, and B a centre, make the intersection C. Join A D, D C, C B, and the figure will be completed.

PROBLEM XIII.

Having any two right lines given, to construct a regu rhomboid.

Let the given lines A B=12, and B C=6.

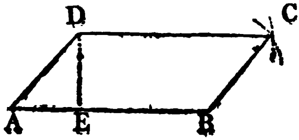


Draw the line A B equal to 12. Take in your compasses the line B C, and lay it from A to E. With A and E as centres, and the radius A E, make the intersection D. Then with B as a centre, and the same radius, describe an arc; and with D as a centre, and the radius A B describe another arc, cutting the former in C. Draw the lines A D, D C, and B C, and the rhomboid will be completed.

PROBLEM XIV.

Having given the base, the perpendicular, and the place of the perpendicular upon the base, to construct an irregular rhomboid.

Let the base A B=15, the perpendicular D E=6, and the distance A E=5.



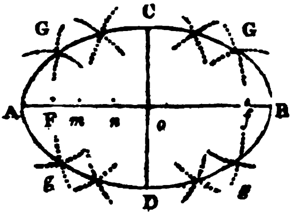
Make A B equal to 15, and A E equal to 6. At E erect the perpendicular E D, which make equal to 6, and join A D. With the radius A B, and D as a centre, describe an arc; and with B as a centre, and the radius A D, describe another arc, cutting the former in C. Draw the lines D C and B C, and the figure will be completed.

Note. The sum of all the interior angles of any quadrilateral figure, is equal to four right angles.

PROBLEM XV.

Having the transverse and conjugate diameters given, to construct an ellipse.

Let the transverse diameter A B=14, and the conjugate diameter C D=8.

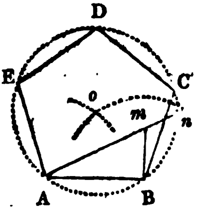


Draw the two diameters to bisect each other perpendicularly in the centre *o*. With the radius A *o*, and the centre D, intersect A B in F and *f*: these two points will be the foci of the ellipse. Take any point *m*, in the transverse diameter, and with F and *f* as centres, and the radius A *m*, describe the arcs G, G, *g*, *g*. Then with the same centres, and the radius B *m*, describe arcs cutting the former in the points G, G, *g*, *g*. Thus you will have four points in the circumference of the ellipse. Again, take a second point *n*, in the transverse diameter, and proceeding as before, you will determine other four points. By the same method you may determine as many more as you please; through all of which, with a steady hand, draw the circumference of the ellipse.

Note. An ellipse may also be constructed as follows: Having found the foci F, *f*, as before, take a thread equal in length to the transverse diameter A B, and fasten its ends, with two pins, in the points F, *f*: then stretch the thread to its greatest extent; and by moving a pencil round, within the thread, keeping it always tight, you will trace out the curve of the ellipse. (See the 2nd Property of the Ellipse, Part V.)

PROBLEM XVI.

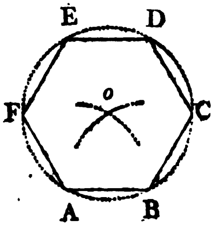
Upon a given line A B, to make a regular pentagon.



Make B *m* perpendicular to A B, and equal to half of it. Draw A *m*, and produce it till *m n* be equal to B *m*. With the radius B *n*, and A and B as centres, describe arcs cutting each other in *o*, which will be the centre of the circumscribing circle. From the point *o*, with the same radius, describe the circle A B C D E; and apply the line A B five times to the circumference, marking the angular points, which connect with right lines, and the figure will be completed.

PROBLEM XVII.

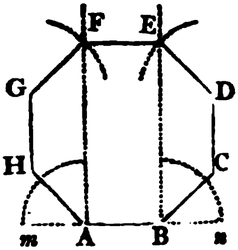
Upon a given line A B, to make a regular hexagon.



With A and B as centres, and the radius A B, describe arcs intersecting each other in *o*; and with *o* as a centre, and the same radius, describe the circle A B C D E F. Apply the line A B, six times to the circumference, and it will form the hexagon required.

PROBLEM XVIII.

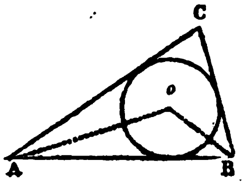
Upon a given line A B, to construct a regular octagon.



On the extremities of the given line A B, erect the infinite perpendiculars A F and B E. Produce the line A B, both ways to *m* and *n*; and with the lines A H and B C, each equal to A B, bisect the angles *m* A F and *n* B E. Draw C D and H G parallel to A F or B E, and each equal to A B. With D and G as centres, and the radius A B, describe arcs intersecting A F and B E, in the points F and E. Join G F, F E, and E D, and the figure will be completed.

PROBLEM XIX.

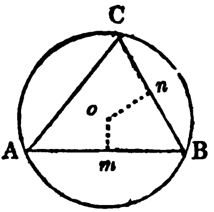
In a given triangle A B C, to inscribe a circle.



Bisect the angles A and B, with the lines A o, B o, and o will be the centre of the required circle; and its radius will be the nearest distance to any one of the sides; hence the circle may be described.

PROBLEM XX.

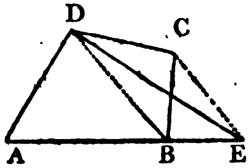
About a given triangle A B C, to circumscribe a circle, or to describe the circumference of a circle through three given points A, B, C.



Bisect the sides A B, B C, with the perpendiculars $m\,o$ and $n\,o$; and o will be the centre of the circle, and its radius will be A o, B o, or C o.

PROBLEM XXI.

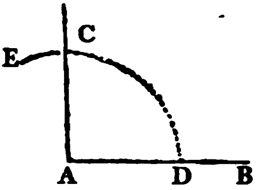
To make a triangle equal to a given trapezium A B C D.



Draw the diagonal D B, and parallel to it draw C E, meeting A B produced in E. Join the points D E, so shall the triangle A D E be equal to the trapezium A B C D.

PROBLEM XXII.

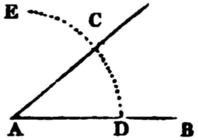
'o make a right angle by the line of chords on the plane scale.



Draw the unlimited line A B; then take in your com-sses 60° from the line of chords, and with A as a ntre, describe the arc E D. Take 90° from the same ale, and set off that extent from D to C. Draw the e A C, and C A D will be the angle required.

PROBLEM XXIII.

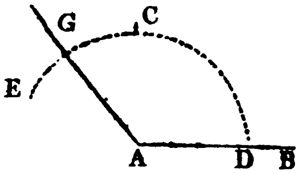
make an acute angle that shall contain any number of degrees, suppose 35° 30′.



)raw the unlimited line A B; then take 60° in your npasses, and with A as a centre, describe the arc E D. off the angle 35° 30′, from D to C. Draw the line ', and C A D will be the angle required.

PROBLEM XXIV.

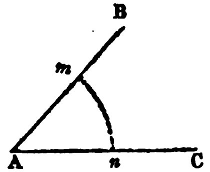
make an obtuse angle that shall contain any number of degrees, suppose 128° 35′.



Draw the indefinite line A B; and with the cho 60° in your compasses describe the arc D E. Set of from D to C; and from C to G set off the excess a 90°, which is 38° 35′. Draw the line A G, and G will be the required angle.

PROBLEM XXV.

To find the number of degrees contained in any given BAC.

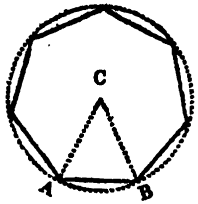


With the chord of 60°, and A as a centre, describe arc *m n*. Take the distance *m n* in your compasses, apply it to the line of chords, and it will shew the num of degrees required.

Note. Angles may be more expeditiously laid down or measure means of a semi-circle of brass, called "a Protractor," the arc of w is divided into 180 degrees.

PROBLEM XXVI.

Upon a given line A B, to make a regular polygon of proposed number of sides.

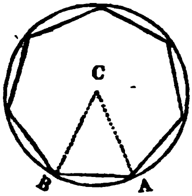


Divide 360° by the number of sides, and subtract t quotient from 180°; and divide the difference by tw

Make the angles A B C, and B A C each equal to the quotient last found; and the point of intersection C, will be the centre of the circumscribing circle. With the radius A C or B C, describe the circle; and apply the chord A B to the circumference the proposed number of times, and it will form the polygon required.

PROBLEM XXVII.

In any given circle to inscribe a regular polygon of any proposed number of sides; or to divide the circumference into any number of equal parts.

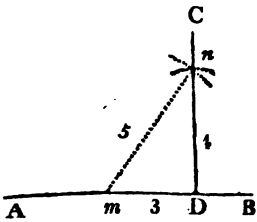


Divide 360° by the number of sides, and make the angle A C B, at the centre, equal to the number of degrees contained in the quotient; and the arc A B will be one of the equal parts of the circumference; hence the polygon may be formed.

Note. The sum of all the interior angles of any polygon, whether regular or irregular, is equal to twice as many right angles, wanting four, as the figure has sides.

PROBLEM XXVIII.

To raise a perpendicular from any point D, in a given line A B, by a scale of equal parts.



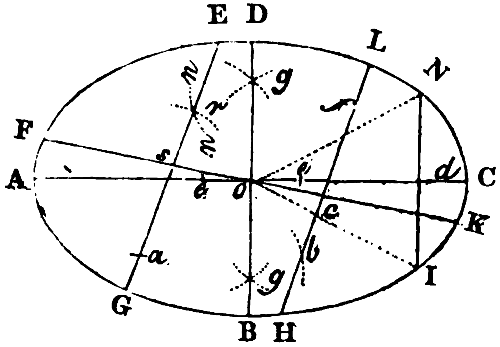
G

Make D m=8; and from the points D and m, with the distances 4 and 5, describe arcs intersecting each other in n. From D, through the point n, draw the line D C, and it will be the perpendicular required.

Note. This Problem may be performed by any other numbers in the same proportion; but 3, 4, and 5 are the least whole numbers that will form a right angled triangle.

PROBLEM XXIX.

To find the centre of a given ellipse, or oval figure, and draw the transverse and conjugate diameters.



Draw the line H L, in any part of the ellipse; then with one foot of your compasses on any assumed point a, at a convenient distance from H L, make the intersection b; and with the same extent, and f, as a centre, taken at a convenient distance from b, upon the line H L, describe the arc n n; from a as a centre, with the radius b f, describe another arc cutting n n in r; through a and r draw the line G E, and it will be parallel to H L.

Bisect G E and H L, in the points s and c; through these points draw the line F K; bisect this line in the point O, which will be the centre of the ellipse.

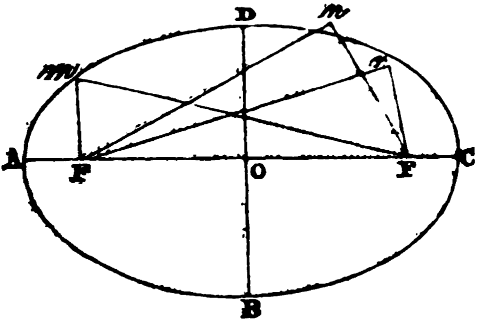
From O as a centre, and any radius of a convenient length, intersect the circumference in the two points N and I; join these points by the right line N I; and

through the point *d*, in the middle of this line, and the centre O, draw the line A C, which will be the transverse diameter of the ellipse.

From the points *e*, *e*, taken at equal distances from the centre O, describe arcs intersecting each other in *g*, *g*; through these points, and the centre O, draw the conjugate diameter B D, and it is done.

PROBLEM XXX.

To determine whether a given oval figure be greater or less than a true ellipse.



By the last Problem, draw the transverse and conjugate diameters A C and B D. With the radius A O in the compasses, and D as a centre, find the two focuses F, F. Take a string equal in length to the transverse diameter A C, and make its ends fast, with two pins, at the focuses. Draw the string tight, so as to form the triangle F *m* F; move the point *m* towards C, always keeping both parts of the string stretched, and if it every where coincide with the curve, the figure is a true ellipse; if it extend beyond the curve as at *n*, it is less than an ellipse; and if it fall short of the curve as at *r*, it is evidently greater than an ellipse. (*See Prob.* 15.)

Note. The last two Problems are frequently of considerable utility in Practical Gauging.

PROBLEMS

IN

PRACTICAL GEOMETRY,

FOR THE EXERCISE OF THE LEARNER.

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1. The length of a given line A B, is 15 inches; it is required to lay it down by a scale of equal parts, and bisect it geometrically.

2. It is required to bisect a given angle A B C?

3. Draw a line C D, parallel to a given line A B, at the distance of eight-tenths of an inch.

4. A given line measures 18 inches; it is required to erect a perpendicular to this line at the distance of 8 inches from one end.

5. The three sides of a triangle are 16, 20, and 24 inches respectively; it is required to lay it down by a scale of equal parts.

6. The base of a triangle measures 24, the distance of the perpendicular from one end of the base 14, and the perpendicular itself 12 inches; it is required to lay down the triangle.

7. Lay down a square, whose side is 16 inches.

8. The length of a rectangle is 24, and its breadth 12 inches; it is required to construct the figure.

9. Construct a regular rhombus whose side is 24 inches.

10. The base of an irregular rhombus measures 16, and the perpendicular breadth 12 inches; it is required to construct the figure.

11. Lay down a regular rhomboid whose sides measure 36 and 18 inches respectively.

12. The transverse diameter of an ellipse, measures

28, and the conjugate 16 inches; it is required to construct the figure.

13. The side of a regular hexagon measures 14 inches; it is required to construct the figure.

14. It is required to make an acute angle that shall contain 42° 30′.

15. It is required to make an obtuse angle that shall contain 136° 45′.

GEOMETRICAL THEOREMS,

THE

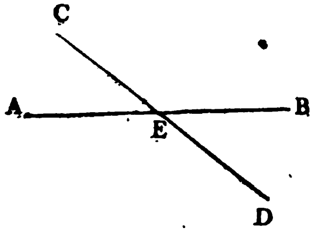
DEMONSTRATIONS

OF WHICH

May be seen in the Elements of Euclid, Simpson, and Emerson.

THEOREM I.

If two straight lines A B, C D, cut each other in the point E, the angle A E C will be equal to the angle D E B, and C E B to A E D. (*Euclid I.* 15. *Simp. I.* 3. *Em.* 2.)

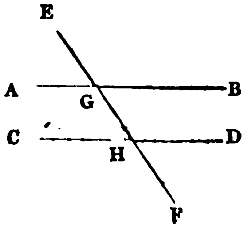


THEOREM II.

The greatest side of every triangle is opposite to the greatest angle. (*Euc. I.* 18. *Simp. I.* 13. *Em. II.* 4.)

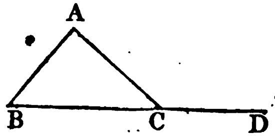
THEOREM III.

Let the right line E F fall upon the parallel right lines A B, C D; the alternate angles A G H, G H D, are equal to each other; and the exterior angle E G B, is equal to the interior and opposite, upon the same side, G H D; and the two interior angles B G H, G H D, upon the same side, are together equal to two right angles. (*Euc. I.* 29. *Simp. I.* 7. *Em. I.* 4.)



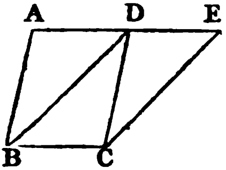
THEOREM IV.

Let A B C be a triangle, and let one of its sides B C be produced to D; the exterior angle A C D is equal to the two interior and opposite angles C A B, A B C; also the three interior angles of every triangle are together equal to two right angles. (*Euc. I.* 32. *Simp. I.* 9, 10. *Em. II.* 1, 2.)



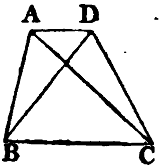
THEOREM V.

Let the parallelograms A B C D, D B C E be upon the same base B C, and between the same parallels A E, B C; the parallelogram A B C D is equal to the parallelogram D B C E. (*Euc. I.* 35. *Simp. II.* 2. *Em. III.* 6.)



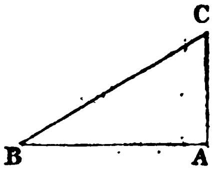
THEOREM VI.

Let the triangles A B C, D B C be upon the same base C, and between the same parallels A D, B C; the trian- A B C is equal to the triangle D B C. (*Euc. I.* 37. *'p. II.* 2. *Em. II.* 10.)



THEOREM VII.

.et A B C be a right angled triangle, having the right le B A C; the square of the side B C is equal to the of the squares of the sides A B, A C. (*Euc. I.* 47. *p. II.* 8. *Em. II.* 21.)

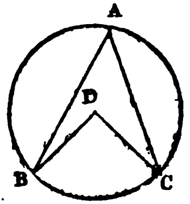


'te. Pythagoras, who was born about 2400 years ago, discovered elebrated and useful theorem; in consequence of which, it is said, 'ered a hecatomb to the gods.

THEOREM VIII.

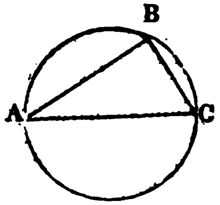
et A B C be a circle, and B D C an angle at the cen-

tre, and B A C an angle at the circumference, which have the same arc B C for their base; the angle B D C is double of the angle B A C. (*Euc. III.* 20. *Simp. III.* 10. *Em. IV.* 12.)



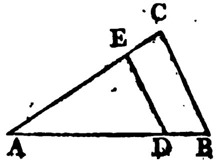
THEOREM IX.

Let A B C be a semicircle; then the angle A B C in that semicircle, is a right angle. (*Euc. III.* 31. *Simp. III.* 13. *Em. VI.* 14.)



THEOREM X.

Let D E be drawn parallel to B C, one of the sides of the triangle A B C; then B D is to D A, as C E to E A. (*Euc. VI.* 2. *Simp. IV.* 12. *Em. II.* 12.)

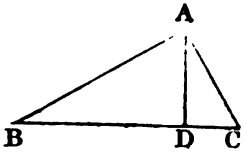


THEOREM XI.

n the preceding figure, D E being parallel to B C, the ngles A B C, A D E are equiangular or similar; efore A B is to B C, as A D to D E; and A B is to ', as A D to A E. (*Euc. VI.* 4. *Simp. IV.* 12. *Em.* 13.)

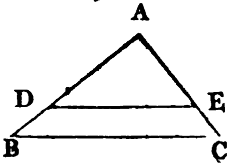
THEOREM XII.

et A B C be a right angled triangle, having the right e B A C; and from the point A let A D be drawn endicularly to the base B C; the triangles A B D, C are similar to the whole triangle A B C, and to other. Also the perpendicular A D is a mean pro- onal between the segments of the base; and each of ides is a mean proportional between the base and its ent adjacent to that side; therefore, B D is to D A, A to D C; B C is to B A, as B A to B D; and B C C A, as C A to C D. (*Euc. VI.* 8. *Simp. IV.* 19. *VI.* 17.)



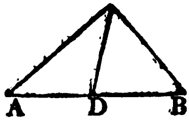
THEOREM XIII.

t A B C, A D E be similar triangles, having the A common to both; then the triangle A B C is to iangle A D E, as the square of B C to the square of That is, similar triangles are to one another in the cate ratio of their homologous sides. (*Euc. VI.* 19. *IV.* 24. *Em. II.* 18.)



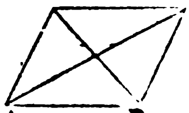
THEOREM XIV.

In any triangle A B C, double the square of a line C D, drawn from the vertex to the middle of the base A B, together with double the square of half the base A D or B D, is equal to the sum of the squares of the other sides A C, B C. *(Simp. II.* 11. *Em. II.* 28.*)*



THEOREM XV.

In any parallelogram A B C D, the sum of the squares of the two diagonals A C, B D, is equal to the sum of the squares of all the four sides of the parallelogram. *(Simp. II.* 12. *Em. III.* 9.*)*



THEOREM XVI.

All similar figures are in proportion to each other as the squares of their homologous sides. *(Simp. IV.* 26. *Em. III.* 20.*)*

THEOREM XVII.

The circumferences of circles, and the arcs and chords of similar segments, are in proportion to each other, as the radii or diameters of the circles. *(Em. IV.* 8, 9.*)*

THEOREM XVIII.

Circles are to each other as the squares of their radii, diameters, or circumferences. *(Em. IV. 35.)*

THEOREM XIX.

Similar polygons described in circles, are to each other, the circles in which they are inscribed; or as the uares of the diameters of those circles. *(Em. IV. 36.)*

THEOREM XX.

All similar solids are to each other as the cubes of their te dimensions. *(Em. VI. 24.)*

PART IV.

MENSURATION OF SUPERFICIES,

APPLIED TO

GAUGING.

PRELIMINARY OBSERVATIONS.

THE AREA of any plane figure is its superficial content, the measurement of its surface, without any regard to ckness. This area is computed from some space of a ermined form; namely, from a square whose side is e *inch,* one *foot,* one *yard,* &c. which is thence called *measuring unit.*

In the subject of Gauging, however, where the *measing unit* is not a *surface,* but a *solid,* namely one *alc lon,* one *wine gallon,* or one *malt bushel,* it is the univer custom, among Officers of the Excise, to consider every ne figure as a solid of *one inch* in depth or thick-

ness; for, by this method, if we divide the number which expresses the area of any plane figure, in square inches, by the number of cubic inches contained in the *ale gallon*, the *wine gallon*, and the *malt bushel*, we shall obtain the *area*, or rather the *solid content* of the figure, at one inch deep, in ale and wine gallons, and malt bushels.

Now, as the design of Gauging is to determine the number of gallons, bushels, &c. contained in any vessel, or the number of gallons, bushels, &c. that any vessel is capable of containing, it is evidently most agreeable to the *general practice* of Gauging, to consider every plane figure as the base of some vessel, the sides of which are perpendicular to the bottom; for having found the area of the base, we have only to multiply this area by the depth of the liquor in the vessel, or by the depth of the vessel, and we shall obtain the solid content, or capacity, in the same denomination in which the area is found.

By this method we enter directly on the *Practical Part* of Gauging; and the learner will perceive the *real* use of the Science even in reading the first Problem.

TABLES

OF

ALE, BEER, WINE, SPIRIT, AND DRY MEASURE.

TABLE I.

A Table of Ale and Beer Measure, 36 gallons to a barrel.

2 Pints, *pt.* .... make	1 Quart, *qt.*
4 Quarts	1 Gallon, *gal.*
9 Gallons	1 Firkin, *fir.*
2 Firkins, or 18 Gallons	1 Kilderkin, *kil.*
2 Kilderkins, or 36 Gallons	1 Barrel, *bar.*
1½ Barrels, or 54 Gallons	1 Hogshead, *hhd.*
2 Barrels, or 72 Gallons	1 Puncheon, *pun.*
3 Barrels, 2 Hogsheads, or 108 Gallons	1 Butt, *butt.*
2 Butts, 4 Hogsheads, or 216 Gallons	1 Tun, *tun.*

TABLE II.

A Table of the Cubic Inches in a Pint, Quart, Gallon, &c. &c. 36 Gallons to a Barrel.

ic Inches.

25= 1 Pint.

).5= 2= 1 Quart.

32= 8= 4= 1 Gallon.

38= 72= 36= 9= 1 Firkin.

76= 144= 72= 18= 2= 1 Kilderkin.

52= 288=144= 36= 4= 2=1 Barrel.

28= 432=216= 54= 6= 3=1½=1 Hogshead.

04= 576=288= 72= 8= 4=2 =1 Puncheon.

56= 864=432=108=12= 6=3 =2=1 Butt.

12=1728=864=216=24=12=6 =4=2=1 Tun.

TABLE III.

able of Ale and Beer Measure, 34 Gallons to a Barrel.

'ints .... make 1 Quart.

)uarts .... 1 Gallon.

;allons .... 1 Firkin.

'irkins, or 17 Gallons .... 1 Kilderkin.

ilderkins, or 34 Gallons .... 1 Barrel.

3arrel, or 51 Gallons .... 1 Hogshead.

3arrels, or 68 Gallons .... 1 Puncheon

3arrels, 2 Hogsheads, or 102 Gallons 1 Butt.

3utts, 4 Hogsheads, or 204 gallons ... 1 Tun.

TABLE IV.

A Table of the Cubic Inches in a Pint, Quart, Gallon, &c. &c. 34 Gallons to a Barrel.

ic Inches.

!5= 1 Pint.

.5= 2= 1 Quart.

32= 8= 4= 1 Gallon.

)7= 68= 34= 8½= 1 Firkin.

)4= 136= 68= 17 = 2= 1 Kilderkin.

38= 272=136= 34 = 4= 2=1 Barrel.

32= 408=204= 51 = 6= 3=1½=1 Hogshead.

76= 544=272= 68 = 8= 4=2 =1 Puncheon.

64= 816=408=102 =12= 6=3 =1 Butt.

28=1632=816=204 =24=12=6 =2=1 Tun.

TABLE V.

A Table of Ale and Beer Measure, 32 Gallons to a Barrel.

2	Pints	make	1 Quart.
4	Quarts		1 Gallon.
8	Gallons		1 Firkin.
2	Firkins, or 16 Gallons		1 Kilderkin.
2	Kilderkins, or 32 Gallons		1 Barrel.
$1\frac{1}{2}$	Barrel, or 48 Gallons		1 Hogshead.
2	Barrels, or 64 Gallons		1 Puncheon.
3	Barrels, 2 Hogsheads, or 96 Gallons		1 Butt.
2	Butts, 4 Hogsheads, or 192 Gallons ...		1 Tun.

TABLE VI.

A Table of the Cubic Inches in a Pint, Quart, Gallon, &c. &c. 32 Gallons to a Barrel.

Cubic Inches.

35.25 = 1 Pint.
70.5 = 2 = 1 Quart.
282 = 8 = 4 = 1 Gallon.
2256 = 64 = 32 = 8 = 1 Firkin.
4512 = 128 = 64 = 16 = 2 = 1 Kilderkin.
9024 = 256 = 128 = 32 = 4 = 2 = 1 Barrel.
13536 = 384 = 192 = 48 = 6 = 3 = $1\frac{1}{2}$ = 1 Hogshead.
18048 = 512 = 256 = 64 = 8 = 4 = 2 = 1 Puncheon.
27072 = 768 = 384 = 96 = 12 = 6 = 3 = 1 Butt.
54144 = 1536 = 768 = 192 = 24 = 12 = 6 = 2 = 1 Tun.

REMARKS.

By Chap. 23, Sec. 20, and Chap. 24, Sec. 34, of a Statute made in the 12th year of the reign of Charles II.; also by Chap. 24, Sec. 5, of another Statute made in the 1st year of the reign of William and Mary, "Every six and thirty gallons of beer taken by the gauge, according to the standard ale quart kept in the exchequer, four whereof shall make the gallon, shall be returned by the Gauger, within the limits of the bills of mortality, for a barrel of beer; and every two and thirty gallons of ale, taken by the gauge, according to the same standard, shall be returned by the Gauger, within the said limits, for a barrel of ale."

By Chap. 24, Sec. 5, of the last named Statute, "Every ur and thirty gallons of beer or ale, whether strong or nall, brewed out of the said limits, taken by the Gauger, cording to the fore-mentioned standard, shall be return- l by him for a barrel of beer or ale."

By Chap. 69, Sec. 12, of another Statute made in the rd year of the reign of George III., "Every thirty-six llons of beer or ale brewed by the common brewers in reat Britain, whether within or without the weekly bills mortality, taken according to the standard of the ale art, four whereof shall make the gallon, remaining in e exchequer, shall be returned by the Gauger, for a rrel of beer or ale."

By Sec. 14, of the same Statute, "No beer or ale ewed by the common Brewers in Great Britain, shall sold by them at any other rate or quantity for the rrel than thirty-six gallons; and that nothing herein ll extend to alter the quantity to be returned as and a barrel of beer or ale brewed by any victualler or re- ler, or any person other than the common brewer; but e same shall remain as declared by 1 W. and M. Chap. Sec. 5."

Vote 1. Here it may be observed, that the standard ale quart, kept he exchequer, contains 70½ cubic inches; and consequently the ale lon is 282 cubic inches.

2. By this measure, Ale, Beer, and Porter are measured; and the nputations in gauging these liquors, are made by it, in order to rge the Duty of Excise.

3. It may not be improper to observe, that if the area of any figure found in square inches, by the following Problems, and divided by cubic inches in a pint, a quart, a gallon, a firkin, &c. &c., the pective quotients will be the area of the figure in pints, quarts, gal- s, firkins, &c. &c.

4. The content of any vessel may be found in pints, quarts, gallons, . &c. by dividing the content in cubic inches by the respective divi- s. (*See the foregoing and following Tables, for the cubic inches in a t, quart, gallon, firkin, &c. &c.*)

TABLE VII.

A Table of Wine and Spirit Measure.

2	Pints	make	1 Quart, *qt.*
4	Quarts		1 Gallon, *gal.*
31½	Gallons		1 Barrel, *bar.*
42	Gallons		1 Tierce, *tier.*
63	Gallons		1 Hogshead, *hhd.*
84	Gallons		1 Puncheon, *pun.*
2	Hogsheads, or 126 Gallons		1 Pipe or butt, *pipe.*
2	Pipes, or 252 Gallons		1 Tun, *tun.*

TABLE VIII.

A Table of the Cubic Inches in a Pint, Quart, Gallon, &c. &c. Wine Measure.

Cubic Inches.

28.875=	1 Pint.								
57.750=	2=	1 Quart.							
231=	8=	4=	1 Gallon.						
7276.5=	252=	126=	31½=1 Barrel.						
9702=	336=	168=	42	=1 Tierce.					
14553=	504=	252=	63	=1½	=1 Hogshead.				
19404=	672=	336=	84	=2	=1⅓	=1 Puncheon.			
29106=	1008=	504=	126	=3	=2	=1½	=1 Pipe.		
58212=	2016=	1008=	252	=6	=4	=3	=2	=1 Tun.	

REMARKS.

By Chap. 27, Sec. 17, of a Statute made in the 5th year of the reign of Queen Anne, it is enacted that, "Any round vessel, (commonly called a cylinder) having an even bottom, and being seven inches in diameter throughout, and six inches deep from the top of the inside to the bottom, or any vessel containing 231 cubic inches, shall be deemed a lawful wine gallon."

By the same Statute it is also enacted, "that 63 such gallons, each consisting of 231 cubic inches, shall be deemed a hogshead; 126 such gallons a butt or pipe; and 252 such gallons a tun of wine."

Note 1. By this measure all sorts of wines, spirits, distilled waters, cyder, perry, mead, verjuce, and vinegar, are measured; and Excise Officers make their computations by it, in gauging these liquors, in order to charge the duty.

2. An anker is 10, and a rundlet 18 gallons.

TABLE IX.

A Table of Corn or Dry Measure.

Pints .... make 1 Quart, *qt.*
Quarts .... 1 Gallon, *gal.*
Gallons .... 1 Peck, *pk.*
Pecks .... 1 Bushel, *bu.*
Bushels .... 1 Quarter, *qr.*
Quarters, or 40 Bushels .... 1 Wey, *wey.*
Weys, or 10 Quarters .... 1 Last, *last.*

TABLE X.

A Table of the Cubic Inches in a Pint, Quart, Gallon, &c. &c. Dry Measure.

bic Inches.								
33.6=	1 Pint.							
67.2=	2=	1 Quart.						
268.8=	8=	4=	1 Gallon.					
537.6=	16=	8=	2=	1 Peck.				
50.42=	64=	32=	8=	4=	1 Bushel.			
03.36=	512=	256=	64=	32=	8=	1 Quarter.		
016.8=	2560=	1280=	320=	160=	40=	5=	1 Wey.	
033.6=	5120=	2560=	640=	320=	80=	10=	2=	1 Last.

REMARKS.

By Chap. 2, Sec. 7, of a Statute made in the 12th year the reign of Queen Anne, and by Chap. 34, Sec. 4, of ther Statute made in the 50th year of the reign of orge III., it is enacted that, "Malt shall be charged the Winchester bushel; and every round vessel with even bottom, being made eighteen inches and a half in neter throughout, and eight inches deep, shall be med a legal Winchester bushel."

By Chap. 2, Sec. 17, of the 12th of Anne, it is enacted t, "The Officers of the Excise shall take account of in making into malt, by gauge only, and not by ac- lly measuring it with a bushel."

Note 1. If we determine the content of a cylindrical vessel whose neter is 18½, and depth 8 inches, by Prob. 4, Part V., we shall

find it to be 2150.4252, the number of cubic inches in a Winche bushel.

2. It has been ascertained by experiments, that after the saccha substance has been extracted from malt, by the operation of *mash* its bulk is diminished a little more than one-sixth part of the wh in consequence of which the mash-tun gallon is always taken at cubic inches.

TABLE XI.

Avoirdupois Weight.

16 Drams	make	1 Ounce	*dr.* *oz.*
16 Ounces		1 Pound,	*lb.*
28 Pounds		1 Quarter,	*qr.*
4 Quarters, or 112 lb,		1 Hundred Weight,	*cwt*
20 Hundred Weight		1 Ton,	*ton,*

TABLE XII.

A Table of Drams in an Ounce, a Pound, a Quarter, &c. &c. Avoirdupois Weight.

Drams.

16= 1 Ounce.
256= 16= 1 Pound.
7168= 448= 28= 1 Quarter.
28672= 1792= 112= 4= 1 Hund. Wt.
573440=35840=2240=80=20=1 Ton.

Remark.

By various Statutes made in the reign of George III., it is enacted that, "Officers of the Excise and Customs shall take the weight of candles, coffee, tea, cocoa-nuts, chocolate, glass, hides, hops, paper, salt, soap, starch, stone-bottles, tobacco, and snuff, by Avoirdupois Weight, in order to charge the respective duties upon these articles."

THE METHOD OF FINDING THE

MULTIPLIERS, DIVISORS, AND GAUGE-POINTS CONTAINED IN THE SUBSEQUENT TABLE.

If the area of any vessel, in square inches, be divided by the square-divisors 282, 231, 227, 268.8, and 2150.42,

ective quotients will be the area of the vessel in ns, wine gallons, mash-tun gallons, malt gallons, lt bushels; but as dividing by these numbers is e as multiplying by $\frac{1}{282}$, $\frac{1}{231}$, $\frac{1}{227}$, &c., it is eviat if these fractions be reduced to decimals, we ain multipliers that will answer the same purpose ivisors, if we prefer multiplication, in making our ons.

ı, it has been found, by Mathematicians, that if meter of a circle be 1, the area will be .7854, and that the area of any circle may be obtained iplying the square of the diameter by this numıt as the area thus found, must be divided by the divisors, to reduce it to ale gallons, wine gallons, becomes desirable to find numbers that will pere work at one operation.

, as multiplying the square of the diameter by and then dividing the product by the square-divithe same thing as multiplying the square of the er by $\frac{.7854}{282}$, $\frac{.7854}{231}$, $\frac{.7854}{227}$, &c.; it is evident that if fractions be reduced to decimals, we shall obtain multipliers to be used in finding the areas of

we prefer division, it is the same thing to divide by $\frac{231}{.7854}$, &c. as to multiply by $\frac{.7854}{282}$, $\frac{.7854}{231}$, &c.; , if those fractions be reduced to mixed numbers, all have proper divisors for circles.

the same manner we can find multipliers and divior ascertaining the weights of bodies from their conand *vice versâ*, provided the specific gravities of the s be known. Thus, the solid content of a pound rd soap cold, is found to be 27.14 cubic inches; a d of hard soap hot, 28.0; a pound of green soap, , &c. &c.; hence, if the area of any vessel, in re inches, be divided by these numbers, the respecquotients will be the area of the vessel in pounds; if the area in pounds, be multiplied by the depth of vessel in inches, the product will be the content of the

vessel in pounds. Or, if the weight of any quantity of goods, in pounds, be multiplied by the cubic inches pound, the product will be the content in cubic inches

The multipliers for squares, in order to find their in pounds, are found by reducing $\frac{1}{27.14}$, $\frac{1}{28.0}$, $\frac{1}{25.67}$ to decimals; the multipliers for circles, by reducing $\frac{.7854}{27.14}$, $\frac{.7854}{28.0}$, $\frac{.7854}{25.67}$, &c. to decimals; and the divisors circles, by reducing $\frac{27.14}{.7854}$, $\frac{28.0}{.7854}$, $\frac{25.67}{.7854}$, &c. to numbers.

The gauge-points for squares are the square roots the square-divisors; being the sides of squares wh areas are one ale gallon, one wine gallon, one malt bush &c.; and the gauge-points for circles, are the squ roots of the circular-divisors; being the diameters circles whose areas are equal to one ale gallon, one wi gallon, one malt bushel, &c. &c.

Note. What has been said on this subject will be fully comprehe ed from the following Rules and Examples.

To find Multipliers for Squares.

RULE.

Divide unity by the square divisors, and the respecti quotients will be the factors for squares.

EXAMPLES.

Divisors for Squares.		Multipliers for Squares.	
282)	1.000000(	.003546	Ale gallons.
231)	1.000000(	.004329	Wine gallons.
268.8)	1.000000(	.003720	Malt gallons.
2217)	1.000000(	.000465	Malt bushels.
2150.42)	1.000000(	.004405	Mash-tun gallons.

To find Multipliers for Circles.

RULE.

Divide .7854 by the square-divisors, and the respective quotients will be factors for circles.

EXAMPLES.

Divisors for Squares.		Multipliers for Circles.	
282)	.7854(	.002785	Ale gallons.
231)	.7854(	.003399	Wine gallons.
268.8)	.7854(	.002922	Malt gallons.
227)	.7854(	.000365	Malt bushels.
2150.42)	.7854(	.00346	Mash-tun gallons.

To find Divisors for Circles.

RULE.

Divide the square-divisors by .7854, and the respective quotients will be divisors for circles.

EXAMPLES.

Area of a Circle whose Diameter is 1.	Divisors for Squares.	Divisors for Circles.	
.7854)	282.0000(	359.05	Ale gallons.
.7854)	231.0000(	294.12	Wine gallons.
.7854)	268.8000(	342.24	Malt gallons.
.7854)	2150.4200(	2738.00	Malt bushels.
.7854)	227.0000(	289.00	Mash-tun gallons.

To find Gauge-points for Squares.

RULE.

Extract the square roots of the square-divisors, you will obtain the gauge-points for squares.

EXAMPLES.

Divisors for Squares.		Gauge-points for Squares.
282.00	Their Square Roots are	16.79 Ale gallons.
231.00		15.19 Wine gallons.
268.80		16.39 Malt gallons.
2150.42		46.37 Malt bushels.
227.00		15.07 Mash-tun gallons.

To find Gauge-points for Circles.

RULE.

Extract the square roots of the circular divisors, a you will obtain the gauge-points for circles.

EXAMPLES.

Divisors for Circles.		Gauge-points, for Circles.
359.05	Their Square Roots are	18.95 Ale gallons.
294.12		17.15 Wine gallons.
342.24		18.5 Malt gallons.
2738.		52.32 Malt bushels.
289.		17. Mash-tun gallons.

In the same manner were found all the Factors, Divisors, and Gauge-points in the following Table.

of Factors, Divisors, and Gauge-points, for Squares and Circles.

	Squares.	Circles.	Squares.	Circles.	Squares.	Circles.
Inches the area of unity	1	.785398	1	1.27324	1	1.128
A superficial foot	.006944	.005454	144.	183.34	12.	13.54
A solid foot	.000578	.000454	1728.	2200.16	41.57	46.91
Ale gallon	.003546	.002785	282.	359.05	16.79	18.95
Wine gallon	.004329	.003399	231.	294.12	15.19	17.15
Malt or corn bushel	.000465	.000365	2150.42	2738.00	46.37	52.32
Malt gallon	.003720	.002922	268.8	342.24	16.39	18.5
Mash-tun gallon	.004405	.00346	227.	289.	15.1	17.07
A pound of hard soap cold	.036845	.028939	27.14	34.56	5.21	5.88
A pound of hot soap	.035714	.028050	28.0	35.65	5.29	5.97
A pound of green soap	.038956	.0306	25.67	32.68	5.06	5.72
A pound of white soft soap	.039123	.030721	25.56	32.54	5.05	5.7
A pound of tallow net	.031844	.025101	31.4	39.98	5.6	6.32
A pound of green starch	.028736	.028563	34.8	44.32	5.9	6.66
A pound of dry starch	.024813	.019491	40.3	51.3	6.35	7.16
A pound of flint glass	.094697	.074403	10.56	13.44	3.25	3.69
A pound of white glass	.071123	.05586	14.06	17.9	3.74	4.32
A pound of green glass	.082102	.064516	12.18	15.5	3.48	3.94

The use of the Multipliers, Divisors, and Gauge-points, is
he following Problems.
he figures, in the fourth column, of the above Table, are
es, except those in the first and second lines, which are
hes.

PROBLEMS

IN

PRACTICAL GAUGING.

PROBLEM I.

The side of a square being given, to find the area in ale gallons, wine gallons, mash-tun gallons, and malt bushels.

RULE.

By the Pen.

Multiply the length of one of the sides by itself, and the product will be the area in square inches; which being multiplied by .003546, .004329, .004405, and .000465; or divided by 282, 231, 227, and 2150.42, the respective products or quotients will be the area of the square in ale gallons, wine gallons, mash-tun gallons, and malt bushels.

Note 1. Excise Officers take their dimensions with measuring-tapes, divided into inches, or with gauging-rods, divided into inches and tenths.

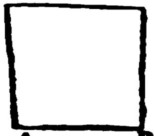
2. If the area of the base of any vessel whose sides are perpendicular, be multiplied by the depth of the vessel, or by the depth of the liquor in the vessel, the product will be the solid content, or capacity, in the same denomination in which the area is found.

3. When the words *square guile-tun*, *rectangular cooler*, &c. &c. occur in this or the following Problems, it is to be understood that the bases of those vessels are squares, rectangles, &c. &c.; and it must always be remembered, that when we assign depths or thicknesses to squares or rectangles, they become *prisms*, *parallelopipedons*, &c. &c. (See Definitions Part V.)

EXAMPLES.

1. If the side of the square A B C D measure 105 inches; what is the area in ale, wine, and mash-tun gallons, and malt bushels?

D C



A B

By Multiplication.

Inches.

```
    105 side.
    105 ditto.
    ---
    525
   105
  -----
  11025 area in square inches.
 .003546 multiplier.
  -----
  66150
  44100
 55125
33075
---------
39.094650 area in ale gallons.
```

Square inches.

```
    11025 area.
   .004329 multiplier.
    -----
    99225
   22050
  33075
 44100
---------
47.727225 wine gallons.
```

Square inches:

```
   11025 area.
  .004405 multiplier.
   -----
   55125
  44100
 44100
---------
48.565125 mash-tun gallons.
```

Square inches.
11025 *area.*
.000465 *multiplier.*
55125
66150
44100
5.126625 *malt bushels.*

By Division.

Sq. inches.
Square divisor 282)11025(39.0957 *ale gallons.*
846
2565
2538
2700
2538
1620
1410
2100
1974
.126

Sq. inches.
Square divisor 231)11025(47.7272 *wine gallons.*
924
1785
1617
1680
1617
630
462
1680
1617
630
462
168

```
                    Sq. inches.
are divisor  227)11025(48.5682 mash-tun gallons.
                 908
                 ----
                 1945
                 1816
                 ----
                  1290
                  1135
                  ----
                   1550
                   1362
                   ----
                    1880
                    1816
                    ----
                      640
                      454
                      ---
                      186
                      ===
```

```
                      Sq. inches.
isor  2150.42)11025.00(5.1269 malt bushels.
              .1075210
              --------
                272900
                215042
                ------
                 578580
                 430084
                 -------
                 1484960
                 1290252
                 -------
                  1947080
                  1935378
                  -------
                    11702
                    =====
```

By the Sliding Rule.

RULE I.

:his operation the square divisors 282, 231, 227, 150.42, upon the line A, must be used; and the tion will be, As the square divisor on A, is to the a B; so is the side on A, to the area on B.

	On A.	On B.	On A.	On B.
As	282	: 105 ::	105 :	39.10 *ale gallons.*
	231			47.73 *wine gallons.*
	227			48.57 *mash-tun gall*
	2150.42			5.13 *malt bushels.*

RULE II.

Set unity on C, to the square gauge-point on D; th against the side of the square on D, is the area on C.

	On D.	On C.	On D.	On C.
As	16.79	: 1 ::	105 :	39.10 *ale gallons.*
	15.19			47.73 *wine gallons.*
	15.1			48.57 *mash-tun gallons.*
	46.37			5.13 *malt bushels.*

2. Admit the foregoing figure to represent the base a square guile-tun; what quantity of ale does the vess contain, when the depth of the liquor is 23.6 inches?

Ale gallons.
39.1 *area.*
23.6 *depth.*
2346
1173
782
922.76 *Ans.*

By the Sliding Rule.

RULE I.

Multiply the area by the depth, and the product wil be the content in ale gallons.

On A. On B. On A. On B.
As 1 : 39.1 :: 23.6 : 922.76 *ale gallons.*

RULE II.

Set the depth on C, to the square gauge-point on D;

then against the side of the guile-tun on D, is the content on C.

	On D.		On C.		On D.	On C.
As	16.79	:	23.6	::	105	: 922.76 *ale gallons.*

3. The side of a square measures 153.4 inches; required the area in ale and wine gallons, and malt bushels.

Ans. 83.445 *ale gallons,* 101.868 *wine gallons, and* 10.942 *malt bushels.*

4. If the side of a square cooler be 82.2 inches; how many ale gallons does it contain, when the depth of the liquor is 5.4 inches? *Ans.* 129.384 *ale gallons.*

5. The side of a square wine-vat is 42.2 inches; what is the content in wine gallons, when the depth of the liquor is 52.7 inches? *Ans.* 425.71 *wine gallons.*

6. The side of a square cistern measures 75 inches; required the area and content in malt bushels, when the depth of the grain is 16.4 inches.

Ans. The area is 2.615, *and the content* 42.886 *malt bushels.*

7. The side of a square vessel measures 84 inches; required the area and content in ale, wine, and mash-tun gallons, and malt bushels; the depth being 14 inches.

Answer.

Areas.	*Contents.*
25.021	350.294 *ale gallons.*
30.545	427.630 *wine gallons.*
31.083	435.162 *mash-tun gallons.*
3.281	45.934 *malt bushels.*

REMARKS.

If the area of any figure, in square inches, be multiplied by .006944, the square factor for a superficial foot, taken from the second column of the foregoing Table, the product will be the area of the figure in square feet; and if the area in square inches, be divided by 144, the square divisor, for a superficial foot, taken from the fourth column of the same Table, the quotient will also be the area in square feet.

The area of the first Example in this Problem, is 11025 square inches; then, 11025 × .006944 = 76.5576,

the area in square feet; and $11025 \div 144 = 76.562$, which is also the area in square feet.

AGAIN, If the square of the diameter of a circle, or the product of the two diameters of an ellipse, be multiplied by .005454, the circular factor, for a superficial foot, taken from the third column of the foregoing Table, the product will be the area of the figure in square feet; and if the same number be divided by 183.34, the circular divisor, for a superficial foot, taken from the fifth column of the same Table, the quotient will also be the area of the figure in square feet.

The square of the diameter of the circle in the first Example, Problem 13, is 1156; then, $1156 \times .005454 = 6.304824$, the area of the figure in square feet; and $1156 \div 183.34 = 6.305$, which is likewise the area in square feet, as before.

It may also be observed that, if the area of any figure in square feet, be divided by 9, the number of square feet in a square yard, the quotient will be the area of the figure in square yards. *(For the Methods of measuring all kinds of Superficies and Solids, see Nesbit's Practical Mensuration.)*

PROBLEM II.

To find the area of a rectangle.

RULE.

By the Pen.

Multiply the length by the breadth, and the product will be the area in square inches; which being divided by 282, 231, 227, and 2150.42, the respective quotients will be the area in ale gallons, wine gallons, mash-tun gallons, and malt bushels.

EXAMPLES.

1. Required the area of the rectangle A B C D, in ale, wine, and mash-tun gallons, and malt bushels; its length A B, being 102.5, and its breadth B C, 54.4 inches.

Inches.
102.5 *length.*
54.4 *breadth.*
4100
4100
5125
Divisor 282)5576.00(19.773 *ale gallons.*
282
2756
2538
2180
1974
2060
1974
860
846
14

Sq. Inches.
Divisor 231)5576.00(24.138 *wine gallons.*
462
956
924
320
231
890
693
1970
1848
122

Sq. inches.

Divisor 227)5576.00(24.563 *mash-tun gallons.*

```
454
---
1036
 908
----
 1280
 1135
 ----
  1450
  1362
  ----
    880
    681
    ---
    199
```

Sq. inches.

Divisor 2150.42)5576.00(2.592 *malt bushels.*

```
430084
-------
1275160
1075210
-------
 1999500
 1935378
 -------
   641220
   430084
   ------
   211136
```

By the Sliding Rule.

As the square divisor on A, is to the length on B; so is the breadth on A, to the area on B.

	On A.	On B.	On A.	On B.
As	282 / 231 / 227 / 2150.42	: 102.5	:: 54.4 :	19.77 *ale gallons.* / 24.14 *wine gallons.* / 24.56 *mash-tun gallons.* / 2.59 *malt bushels.*

2. If the foregoing figure represent the base of a rectangular vessel; what is its content in ale and wine gallons, and malt bushels, when its depth is 5.2 inches?

Here $19.77 \times 5.2 = 102.804$ *ale gallons;* $24.14 \times 5.2 = 125.528$ *wine gallons; and* $2.59 \times 5.2 = 13.468$ *malt bushels.*

By the Sliding Rule.

one on A, is to the area on B; so is the depth on A, content on B.

h A.	On B.		On A.		On B.
1 :	19.77 / 24.14 / 2.59	::	5.2	:	102.80 *ale gallons.* / 125.53 *wine gallons.* / 13.47 *malt bushels.*

f the length of a rectangular cooler be 162.7, and adth 86.3 inches; how many gallons of ale does it 1, when the depth of the liquor is 6.8 inches?

Ans. 338.572 *gallons.*

f the sides of a rectangular wine-vat measure 74.6 .7 inches; required the area and content, in wine , supposing the depth of the liquor to be 24.7 in.

The area is 21.217, *and the content* 524.0599 *wine*

brewer has a rectangular mash-tun which mea- 4 inches in length, and 52 in breadth; how many s of malt had been brewed, when the Excise Offi- ind the depth of the grains to be 32 inches; ad- that each gallon of malt uniformly produces one of grains? (See Note 2, Page 78.)

Ans. 58.64 *bushels.*

maltster has a rectangular cistern whose length is inches, and breadth 98.2; required the area and t, in malt bushels, when the depth is 14.8 inches.

The area is 5.685, *and the content* 84.138 *malt*

floor of malt measures 245 inches in length, 152 dth, and 5.9 in depth; required the area and con- malt bushels.

The area is 17.317, *and the content* 102.1703 *malt*

PROBLEM III.

To find the area of a rhombus or rhomboides.

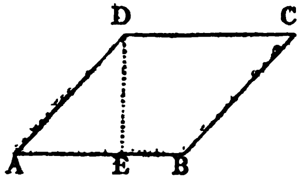
RULE.

By the Pen.

Multiply the length of the base by the perpendicul breadth, and the product will be the area in square i inches; which being divided by 282, 231, 227, a 2150.42, the respective quotients will be the area in al wine, and mash-tun gallons, and malt bushels.

EXAMPLES.

1. Let ABCD be a rhombus, whose base AB is inches, and perpendicular DE 40 inches; what is t area in ale, wine, and mash-tun gallons, and ma bushels?



Inches.

50 *base.*

40 *per. breadth.*

Divisor 282)2000(7.092 *ale gallons.*

```
           1974
            2600
            2538
              620
              564
               56
```

Sq. inches.

Divisor 231)2000(8.658 *wine gallons.*

```
1848
----
 1520
 1386
 ----
  1340
  1155
  ----
   1850
   1848
   ----
      2
```

Sq. inches.

Divisor 227)2000(8.810 *mash-tun gallons.*

```
1816
----
 1840
 1816
 ----
   240
   227
   ---
    130
```

Sq. inches.

ivisor 2150.42)2000.000(.9300 *malt bushels.*

```
1935378
-------
 646220
 645126
 ------
  109400
```

By the Sliding Rule.

le square divisor on A, is to the base on B; so is pendicular breadth on A, to the area on B.

On A.	On B.	On A.	On B.
282	: 50 ::	40 :	7.09 *ale gallons.*
231			8.66 *wine gallons.*
227			8.81 *mash-tun gallons.*
150.42			0.93 *malt bushels.*

2. Let the foregoing figure denote the base of a ve whose sides are perpendicular; required the conten ale, wine, mash-tun gallons, and malt bushels, at 17 ches from the bottom?

Here $7.092 \times 17 = 120.564$ *ale gallons;* 8.658×1 147.186 *wine gallons;* $8.81 \times 17 = 149.77$ *mash-tun* *lons; and* $.93 \times 17 = 15.81$ *malt bushels.*

By the Sliding Rule.

As one on A, is to the area on B; so is the depth A, to the content on B.

$$\begin{array}{cccc} \text{On A.} & \text{On B.} & \text{On A.} & \text{On B.} \\ \textit{As } 1 : & \left\{\begin{array}{l} 7.09 \\ 8.66 \\ 8.81 \\ 0.93 \end{array}\right\} & :: 17 : & \left\{\begin{array}{l} 120.56 \textit{ ale gallons.} \\ 147.19 \textit{ wine gallons.} \\ 149.77 \textit{ mash-tun gallons.} \\ 15.81 \textit{ malt bushels.} \end{array}\right. \end{array}$$

3. There is a cooler in the form of a rhomboid, wh base is 73.6, and perpendicular breadth 56.2 inch required the area and content, in ale gallons; the depth the liquor being 2.7 inches?

Ans. The area is 14.667, *and the content* 39.6009 *gallons.*

4. Required the area and content of a rhomboi vessel, in ale gallons, the length of the base being 10 inches, the perpendicular breadth 70.6, and the depth the liquor eight-tenths of an inch.

Ans. The area is 27.138, *and the content* 21.7104 *gallons.*

PROBLEM IV.

To find the area of a triangle, when the base and perpe dicular are given.

RULE.

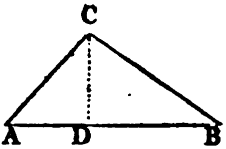
By the Pen.

Multiply the base by the perpendicular, and half th the product will be the area in square inches; whic being divided by 282, 231, and 2150.42, the respectiv quotients will be the area in ale and wine gallons, an malt-bushels.

Note. If the base be multiplied by half the perpendicular, or the perpendicular by half the base, the product will be the area in square inches.

EXAMPLES.

1. Required the area of the triangle A B C, in ale and wine gallons, and malt bushels; the base A B, being 216.8, and the perpendicular D C, 94.6 inches.



Note. The perpendicular C D, is the shortest distance from the angle C, to the base A B; and may be found by one person holding the end of the measuring-tape at C, while another, by trials, determines the point D.

```
            Inches.
            216.8 base.
             94.6 perpendicular.
            13008
            8672
         19512
      2)20509.28
Divisor 282)10254.64(36.363 ale gallons.
            846
            1794
            1692
             1026
              846
              1804
              1692
               1120
                846
                274
```

```
                Sq. inches.
Divisor 231)10254.64(44.392 wine gallons.
            924
            ----
            1014
             924
            ----
              906
              693
             ----
             2134
             2079
             ----
               550
               462
              ----
                88
```

```
                 Sq. inches.
Divisor 2150.42)10254.64(4.768 malt bushels.
                860168
               -------
               1652960
               1505294
               -------
                1476660
                1290252
                -------
                 1864080
                 1720336
                 -------
                  143744
```

By the Sliding Rule.

As the square divisor on A, is to the base on B; so is half the perpendicular on A, to the area on B.

	On A.		On B.		On A.		On B.
As	282	:	216.8	::	47.3	:	36.36 *ale gallons.*
	231						44.39 *wine gallons.*
	2150.42						4.77 *malt bushels.*

2. If the foregoing figure represent the base of a triangular vessel; how many ale gallons, wine gallons, and malt bushels will it contain, if its depth be 21.6 inches?

Here 36.368 × 21.6 = 785.4408 *ale gallons*; 44.392 ×
)=958.8672 *wine gallons*; *and* 4.768 × 21.6 = 102.9888
t bushels.

By the Sliding Rule.

s one on A, is to the area on B; so is the depth on A, e content on B.

$$\begin{array}{llll} \text{n A.} & \text{On B.} & \text{On A.} & \text{On B.} \\ 1 : & \left\{\begin{array}{r} 36.36 \\ 44.39 \\ 4.77 \end{array}\right\} & :: 21.6 : & \left\{\begin{array}{l} 785.44 \textit{ ale gallons.} \\ 958.87 \textit{ wine gallons.} \\ 102.98 \textit{ malt bushels.} \end{array}\right. \end{array}$$

The base of a triangular cooler measures 124, and perpendicular 94.7 inches; required the area and nt in ale gallons, when the depth of the liquor is iches.

s. The area is 20.82, *and the content* 85.362 *ale s.*

A maltster has a triangular cistern the base of which ires 136.7, and the perpendicular 86.8 inches; re- d its area and content, in malt bushels, when the of the grain is 42.3 inches.

s. The area is 2.758, *and the content* 116.6634 *malt s.*

PROBLEM V.

find the area of a triangle when the three sides are given.

RULE.

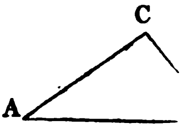
By the Pen.

m half the sum of the three sides subtract each verally; multiply the half sum and the three re- ers continually together; and the square root of ast product will be the area in square inches. e this area by 282, 231, and 2150.42, and you will the area in ale and wine gallons, and malt bushels.

EXAMPLES.

What is the area of the triangle A B C, in ale and

wine gallons, and malt bushels; t
50, A C 40, and B C 30 inches?



Here $\frac{50+40+30}{2}=\frac{120}{2}=60,$
then 60−50=10, *the first rema*
second remainder; and 60−30=
whence $\sqrt{60\times10\times20\times30}=\sqrt{3}$
inches; then 600÷282=2.127,
÷231=2.597, *area in wine g*
=.279, *area in malt bushels.*

By the Slidi

Multiply the half sum and
tinually together; extract the
duct, and you will obtain the
divide this area by the square
be the area in ale and win
(See the use of the Sliding I

Multi

On A.		On B.
As 1	:	60
As 1	:	600
As 1	:	1200

Extr

Set 1 *on* C, *to* 1 *on*
on D.

On A.
288
231

$2.127 \times 8.6 = 18.2922$ *ale gallons;* $2.597 \times 8.6 =$ *wine gallons; and* $.279 \times 8.6 = 2.3994$ *malt bushels.*

By the Sliding Rule.

e on A, is to the area on B; so is the depth on A, ontent on B.

n A. On B. On A. On B.

$$1 : \begin{Bmatrix} 2.13 \\ 2.60 \\ .28 \end{Bmatrix} :: 8.6 : \begin{Bmatrix} 18.29 \text{ ale gallons.} \\ 22.33 \text{ wine gallons} \\ 2.40 \text{ malt bushels.} \end{Bmatrix}$$

e three sides of a triangle measure 114, 112, and es respectively; what is its area in ale and wine and malt bushels?

The area is 17.689 *ale gallons;* 21.594 *wine gal- d* 2.319 *malt bushels.*

e base of a guile-tun is an equilateral triangle de is 74 inches; required the area and content gallons, when the depth of the liquor is 28.6

The area is 8.408, *and the content* 240.4688 *ale*

The first question in this Problem being a right-angled nay be more easily solved by the Rule given in the last and the fourth question being an equilateral triangle, may solved by Problem the tenth; the Rule given in this s, however, particularly useful in finding the areas of oblique angled isosceles and scalene triangles, when their sides only and also in determining the area of a trapezium from the and one of the diagonals.

PROBLEM VI.

To find the area of a trapezium.

RULE.

By the Pen.

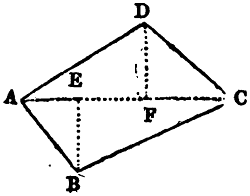
ply the sum of the two perpendiculars by the upon which they fall, and half the product will rea in square inches. Divide the area thus found, 231, and 2150.42, and the respective quotients the area of the figure in ale and wine gallons, t bushels.

Note 1. The area of a trapezium may also be found by dividing into two triangles, and computing the area of each triangle either of the last Problems.

2. Sometimes a trapezium may be very properly divided into t right-angled triangles and a trapezoid.

EXAMPLES.

1. What is the area of the trapezium A B C D, in a and wine gallons, and malt bushels; the diagonal A measuring 121 inches, and the perpendiculars B E, an D F, 38.3 and 43.1 inches respectively?



Inches.
38.3 } *perpendiculars.*
43.1 }
81.4 *sum.*
121 *diagonal.*
814
1628
814
2)9849.4
Divisor 282)4924.7(17.463 *ale gallons.*
282
2104
1974
1307
1128
1790
1692
980
846
134

, $4924.7 \div 231 = 21.319$ *wine gallons; and* $4924.7 \div$
$2 = 2.290$ *malt bushels.*

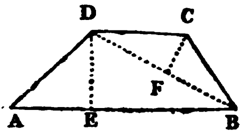
By the Sliding Rule.

he square divisor on A, is to the diagonal on B; so the sum of the perpendiculars on A, to the area

On A.	On B.	On A.	On B.
282			17.46 *ale gallons.*
231	: 121 ::	40.7 :	21.32 *wine gallons.*
150.42			2.29 *malt bushels.*

That is the area of the following trapezium, in ale ; AB measuring 130.5, DE 46.4, BD 97, and 6 inches?

This figure is divided into two triangles, because two perpen. cannot be taken upon either of the diagonals.



$\overline{130.5 \times 46.4} \div 2 = 6055.20 \div 2 = 3027.60$, *the area triangle* ABD*; and* $\overline{97 \times 24.6} \div 2 = 2386.2 \div 2 =$, *the area of the triangle* BCD*; then* $3027.60 +$ $= 4220.7$ *square inches, the area of the trapezium* D*; whence* $4220.7 \div 282 = 14.967$*, the area in ale*

By the Sliding Rule.

he square divisor on A, is to the base on B; so the perpendicular on A, to the area on B.

A.	On B.	On A.	On B.
2 :	130.5	:: 23.2 :	10.73 *triangle ABD.*
	97.0	12.3	4.23 *triangle BCD.*
			14.96 *trapezium ABCD.*

3. Let the last figure represent the base of a cooler; what is the content in ale gallons, when the depth of the liquor is 3.7 inches? *Ans.* 55.3779 *ale gallons.*

4. What is the area of a trapezium in wine gallons; the diagonal measuring 94.4, and the two perpendiculars 28.2 and 47 inches respectively?

Ans. 15.365 *wine gallons.*

5. How many bushels of barley can be steeped in a in a cistern at one time; when the diagonal of the bottom measures 126.6, the perpendiculars 58.4 and 46.8 respectively, and the depth 39.3 inches; allowing one-fifth of the capacity of the vessel for the *swell* of the grain? *Ans.* 97.3383 *bushels.*

PROBLEM VII.

To find the area of a trapezoid.

RULE.

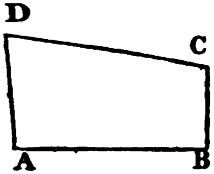
By the Pen.

Multiply the sum of the two parallel sides by the perpendicular distance between them, and half the product will be the area in square inches. Divide this area by 282, 231, and 2150.42, and the respective quotients will be the area in ale and wine gallons, and malt bushels.

Note. Half the sum of the parallel sides of a trapezoid, multiplied by the perpendicular distance between them, will give the area.

EXAMPLES.

1. What is the area of the trapezoid A B C D, in ale and wine gallons, and malt bushels; the parallel sides A D and B C of which measure 98.6 and 71.8 respectively; and A B, the perpendicular distance between them, 151.5 inches?




```
        Inches.
         98.6 } sides.
         71.8 }
        ------
        170.4 sum.
        151.5 base.
        ------
         8520
        1704
       8520
      1704
      --------
    2)25815.60
      --------
Divisor 282)12907.80(45.772 ale gallons.
            1128
            ----
             1627
             1410
             ----
              2178
              1974
              ----
               2040
               1974
               ----
                660
                564
                ---
                 96
```

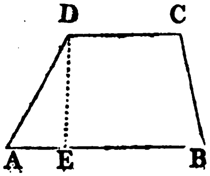
12907.80÷231=55.877 *wine gallons; and* 12907.80 .42=6.002 *malt bushels.*

By the Sliding Rule.

he square divisor on A, is to the base or perpendi- listance of the sides on B; so is half the sum of es on A, to the area on B.

	On A.		On B.		On A.		On B.
As	282	:	151.5	::	85.2	:	45.77 *ale gallo*
	231						55.88 *wine gal*
	2150.42						6.00 *malt bus*

2. Let the following figure represent a floor of m how many bushels does it contain, when the parallel si AB and DC measure 186 and 112 respectively; D the perpendicular distance between them, 106; and depth of the grain 5.7 inches?



Here $\overline{186+112} \times 106 = 298 \times 106 = 31588$; *and* 315 $\div 2 = 15794$, *the area in square inches; also* 15794 2150.42 = 7.344, *the area in malt bushels; then* $7.344 \times$ = 41.8608, *the number of bushels required.*

By the Sliding Rule.

Find the area as in the last example; then as one A, is to the area on B; so is the depth on A, to the content on B.

	On A.		On B.		On A.		On B.
As	2150.42	:	106	::	149	:	7.34, *the area; then,*
As	1	:	7.34	::	5.7	:	41.86, *the content.*

Or,

Without knowing the area, the content may be foun by the line marked M D, thus; As the perpendicular dis tance of the sides on B, is to the depth on M D; so i half the sum of the sides on A, to the content on B.

	On B.		On M D.		On A.		On B.
As	106	:	5.7	::	149	:	41.86, *the content.*

3. The base of a cooler is a trapezoid whose parallel sides measure 214 and 178 respectively; and the perpendicular distance between them 146 inches; what is the area in ale gallons? *Ans.* 101.475 *ale gallons.*

4. If the depth of the liquor, in the foregoing cooler, be one inch and seven-tenths; how many ale gallons does it contain? *Ans.* 172.5075 *ale gallons.*

PROBLEM VIII.

To find the area of an irregular polygon of any number of sides.

RULE.

By the Pen.

Divide the given figure into triangles and trapeziums, in the most convenient manner; and find the area of each separately by the foregoing Problems; then the sum of these areas will be the area of the polygon in square inches. Divide the area thus found by 282, 231, and 2150.42, and you will obtain the area of the figure in ale and wine gallons, and malt bushels.

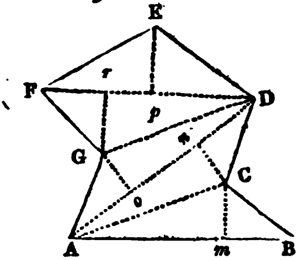
Note 1. It is sometimes more eligible to find the double area of each figure into which the polygon is divided; and half the sum of these double areas will be the area of the polygon.

2. Sometimes the area of an irregular figure may be very easily found by measuring a base line in a convenient position; and upon it erecting perpendiculars to the opposite angles, on each side, thus dividing the whole figure into right-angled triangles and trapezoids. (See Example 4.)

EXAMPLES.

1. Required the area of the irregular polygon A B C D E F G, in ale and wine gallons, and malt bushels, from the following dimensions.

Diagonals. Inches.		*Perpendiculars.* Inches.	
A m =	75.0	C m =	24.0
A B =	106.5	G o =	22.5
A o =	36.0	C n =	25.8
A n =	74.4	E p =	31.2
A D =	108.0	G r =	27.6
D p =	48.0		
D r =	70.5		
D F =	102.0		



CONSTRUCTION. Draw the line A B, which ma = 106.5; and lay off 75.0 from A to m, at which poi erect the perpendicular C m = 24.0; and join A C and B and you will have the triangle A B C.

With C as a centre, and the radius C n = 25.8, descri an arc; and with A as a centre, and the radius A n 74.4, describe another arc cutting the former in Through the point n draw the diagonal A D = 108. upon which lay off A o = 36.0. At o erect the pe pendicular G o = 22.5; join C D, D G, and G A, and t trapezium A C D G will be completed.

The trapezium D E F G may be constructed in a sim lar manner.

CALCULATION. *Here* 106.5 × 24 = 2556, *double the ar of the triangle A B C.*

Again, $\overline{22.5+25.8} \times 108 = 48.3 \times 108 = 5216.4$ *double th area of the trapezium A C D G.*

Also, $\overline{31.2+27.6} \times 102 = 58.8 \times 102 = 5997.6$, *double th area of the trapezium D E F G.*

$\frac{2556+5216.4+5997.6}{2}=\frac{13770}{2}=6885$ *square in-*
area of the irregular polygon ABCDEFG.
. $6885\div282=24.414$, *the area in ale gallons;*
$31=29.805$, *the area in wine gallons; and* $6885\div$
$=3.201$, *the area in malt bushels.*

By the Sliding Rule.

is operation, recourse must be had to the Rules
Problems IV. and VI.

Ale Measure.

$$\begin{array}{ccccc} \text{On B.} & & \text{On A.} & & \text{On B.} \\ \left\{\begin{array}{l}106.5\\108.0\\102.0\end{array}\right\} & :: & \left\{\begin{array}{l}12.0\\24.15\\29.4\end{array}\right\} & : & \left\{\begin{array}{ll}4.53 & \textit{triangle } ABC.\\9.25 & \textit{trapezium } ACDG.\\10.63 & \textit{trapezium } DEFG.\\\hline 24.41 & \textit{whole polygon.}\end{array}\right. \end{array}$$

Wine Measure.

$$\begin{array}{ccccc} \text{On B.} & & \text{On A.} & & \text{On B.} \\ \left\{\begin{array}{l}106.5\\108.0\\102.0\end{array}\right\} & :: & \left\{\begin{array}{l}12.0\\24.15\\29.4\end{array}\right\} & : & \left\{\begin{array}{ll}5.53 & \textit{triangle } ABC.\\11.29 & \textit{trapezium } ACDG.\\12.98 & \textit{trapezium } DEFG.\\\hline 29.80 & \textit{whole polygon.}\end{array}\right. \end{array}$$

Malt Measure.

$$\begin{array}{ccccc} \text{On B.} & & \text{On A.} & & \text{On B.} \\ 2:\left\{\begin{array}{l}106.5\\108.0\\102.0\end{array}\right\} & :: & \left\{\begin{array}{l}12.0\\24.15\\29.4\end{array}\right\} & : & \left\{\begin{array}{ll}0.59 & \textit{triangle } ABC.\\1.21 & \textit{trapezium } ACDG.\\1.39 & \textit{trapezium } DEFG.\\\hline 3.19 & \textit{whole polygon.}\end{array}\right. \end{array}$$

the foregoing figure represent the base of a
what quantity of ale does the vessel contain,
e depth of the liquor is 6.7 inches?

Ans. 163.5738 *ale gallons.*

L

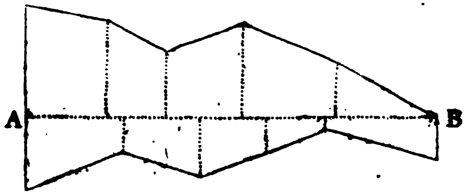
3. Required the area of an irregular polygon in ale and wine gallons, and malt bushels; the first side measuring 94, the second 102, the third 64, the fourth 140, and the fifth 100 inches; and the diagonal from the first to the third angle 160, and that from the first to the fourth 130 inches.

Note. This figure is divided into three triangles; the areas of which may be found by Problem 5.

Ans. The area is 52.336 ale gallons, 63.890 wine gallons, and 6.862 malt bushels.

4. Required the area of the following irregular figure, in ale and wine gallons, and malt bushels; all the dimensions being taken in inches.

	AB	
0	1314	126
234	1005	
	980	52
	785	125
312	700	
	555	152
215	460	
	335	100
336	260	
360	000	232
Begin at A,	and	measure to B.



Note 1. The dimensions of the foregoing figure, are taken as directed in Note 2; and evidently divide the figure into one right-angled triangle, and nine trapezoids. The base line is measured from A to B; the dimensions are entered from the bottom towards the top; the point on the base line, where each perpendicular rises to the opposite

angle, is entered in the middle column; and the perpendiculars themselves in the right and left-hand columns respectively. The base of each trapezoid may be found by subtracting the distances on the base line from each other; thus, the base of the first trapezoid on the left, is 260; the base of the first on the right, 335; the base of the second on the left, 460 — 260 = 200; the base of the second on the right, 555 — 335 = 220, &c. &c.

2. When the dimensions are numerous, the foregoing method of entering them in the Note Book, will be found very convenient; if, however, a rough sketch of the figure be preferred, it may be drawn, and the dimensions entered upon the respective parts of the figure.

ANSWER.

Double Areas.

324337 *trapezoids on the right.*
656476 *ditto on the left.*
2)980813 *sum.*
490406 *area in square inches.*

Then, 490406÷282=1739.028, *the area in ale gallons;* 490406÷231=2122.969, *the area in wine gallons; and* 490406÷2150.42=228.051, *the area in malt bushels.*

REMARK.

When any side of an irregular figure is curved, draw a chord-line so as to join the extremities of the curve; and erect perpendiculars from the chord-line to the curve, in such a manner as to divide the space contained between the chord and the curve into a number of small right-angled triangles and trapezoids, the areas of which must be found as in the last Example. Or the area of the space contained between the chord and the curve may be found by the method of Equi-distant Ordinates, described in Problem XX.

PROBLEM IX.

To find the area of a regular polygon.

RULE.

By the Pen.

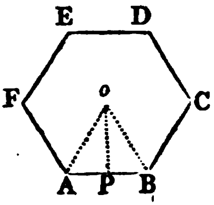
Multiply the sum of the sides, or perimeter of the

polygon, by the perpendicular falling from its ce
upon one of the sides; and half the product will be
area in square inches. Divide this area by 282, 231,
2150.42, and the respective quotients will be the are
ale and wine gallons, and malt bushels.

Note. The sum of the sides may be obtained by multiplyin
length of one side by the number of sides.

EXAMPLES.

1. Required the area of the regular hexagon A B
E F, in ale and wine gallons, and malt bushels; the
A B being 41.2, and the perpendicular P *o* 35.6 inches



Inches.

41.2 *side.*
6 *number of sides.*
247.2 *perimeter.*
35.6 *perpendicular.*
14832
12360
7416
2)8800.32
Divisor 282)4400.16(15.603 *ale gallons.*
282
1580
1410
1701
1692
960
846
114

4400.16 ÷ 231 = 19.048 *wine gallons*; and 4400.16
).42 = 2.046 *malt bushels*.

By the Sliding Rule.

ie square divisor on A, is to half the sum of the
n B; so is the perpendicular on A, to the area

On A.		On B.		On A.		On B.
?						15.60 *ale gallons.*
l	:	123.6	::	35.6	:	19.05 *wine gallons.*
50.42						2.05 *malt bushels.*

lmit the foregoing figure to represent the base of
in; what number of ale gallons does the vessel
when the depth of the liquor is 38.9 inches?

Ans. 606.9567 *ale gallons.*

hat is the area of a regular pentagon, in ale gal-
he side measuring 64.8, and the perpendicular
hes? *Ans.* 25.621 *ale gallons.*

he side of the base of a heptagonal wine vat
s 46.3, and the perpendicular 48.1 inches; re-
the area and content in wine gallons, when the
f the liquor is 48.6 inches?

The area is 33.742, *and the content* 1639.8612
lons.

PROBLEM X.

the area of a regular polygon, when the side only is given.

RULE.

By the Pen.

ply the square of the given side by the number
standing opposite the name of the polygon, in the
ig Table, and the product will be the area in
inches, ale gallons, wine gallons, or malt bushels
ively.

A TABLE OF POLYGONS, &c.

No. of Sides.	Names of the Polygons.	Area in Sq. Inches.	Area in Ale Gall.	Area in WineGall.	Are in MaltB
3	Trigon	0.433013	.001536	.001875	.0002
4	Tetragon	1.000000	.003546	.004329	.0004
5	Pentagon	1.720477	.006101	.007448	.0008
6	Hexagon	2.598076	.009213	.011247	.0012
7	Heptagon	3.633912	.012886	.015731	.0016
8	Octagon	4.828427	.017122	.020902	.0022
9	Nonagon	6.181824	.021921	.026761	.00287
10	Decagon	7.694209	.027284	.033308	.00357
11	Undecagon	9.365640	.033211	.040544	.00435
12	Duodecagon	11.196152	.039702	.048468	.00520

EXAMPLES.

1. The side of a regular pentagon is 38 inches, wh is the area in ale and wine gallons, and malt bushels?

Inches.
38 *side.*
38 *side.*
―――
304
114
―――
1444 *square of the side.*
.006101 *multiplier.*
―――
1444
1444
8664
―――
8.809844 *ale gallons.*

), 1444 × .007448 = 10.754912 *wine gallons*; *and* (.0008 = 1.1552 *malt bushels.*

By the Sliding Rule.

l on D, is to the multiplier on C; so is the side of lygon on D, to the area on C.

On D.		On C.		On D.		On C.
1	:	.006101 .007448 .0008	::	38	:	8.81 *ale gallons.* 10.75 *wine gallons.* 1.16 *malt bushels.*

Required the area and content of a hexagonal guile ale gallons; the side of the vessel measuring 21.7 depth 41.6 inches.

The area is 4.3383, *and the content* 180.47328 *ale*

What is the area, in wine gallons, of a heptagonal at whose side measures 24.3 inches?

Ans. 9.2889 *wine gallons.*

f the side of an octagonal cistern measures 31.2, what is the area in malt bushels?

Ans. 2.1853 *malt bushels.*

PROBLEM XI.

n the diameter of a circle to find the circumference; or, the circumference to find the diameter.

RULE I.

By the Pen.

7 is to 22, so is the diameter to the circumference: 22 is to 7, so is the circumference to the diameter.

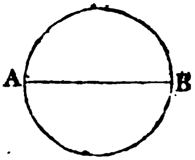
RULE II.

ltiply the diameter by 3.1416, and the product will e circumference: or, divide the circumference by 3, and the quotient will be the diameter.

. The second Rule is more correct than the first, and is there- nerally preferred.

EXAMPLES.

1. If the diameter A B of a circle be 84.5 inches, w is the circumference?



By Rule I.

As 7 : 22 :: 84.5 : 265.571 *inches, the circumference quired.*

By Rule II.

Here 3.1416 × 84.5 = 265.4652 *inches, the circumfere required.*

By the Sliding Rule.

RULE I.

As 7 on A, is to 22 on B; so is the diameter on A, the circumference on B: and as 22 on B, is to 7 on A *so is the circumference on* B, *to the diameter on* A.

On A. On B. On A. On B.
As 7 : 22 :: 84.5 : 265.57 *inches, the circumferen required.*

RULE II.

As 1 on A, is to 3.1416 on B; so is the diameter on A to the *circumference on* B; and *vice versâ.*

On A. On B. On A. On B.
As 1 : 3.1416 :: 84.5 : 265.46 *inches, the cir cumference required.*

2. The circumference of a circle is 265.4652 inches what is the diameter? *Ans.* 84.5 *inches*

3. The diameter of a cylindrical vessel is 36.9 inches, what is its circumference? *Ans.* 115.925 *inches.*

'he circumference of a globe or sphere is 86.7 in- what is its diameter? *Ans.* 27.5974 *inches.*

. *There is no figure that affords a greater variety of useful pro-* than the circle; nor is there any that contains so large an area the same perimeter.

atio of the diameter of a circle to its circumference has never exactly determined; although this celebrated Problem, called ring *of the circle*, has engaged the attention and exercised the of the ablest Mathematicians, both ancient and modern. But he relation between the diameter and circumference cannot be efined in known numbers; yet approximating ratios have been ed sufficiently correct for practical purposes.

nedes, a native of Syracuse, who flourished about 200 years e Christian æra, after attempting in vain to determine the true the diameter to the circumference, found it to be nearly as 7

roportion given by Vieta, a Frenchman, and Metius, a Dutch- out the end of the 16th century, is as 113 to 355, which is ore accurate than the former; and is a very commodious ratio, reduced into decimals, it agrees with the truth to the sixth clusively.

rst, however, who ascertained this ratio to any great degree ness, was Ludolph Van Ceulen, a Dutchman. He found the diameter of a circle be 1, the circumference will be 26535897932384626433832795028884 nearly, which is true to s of decimals. This was thought so extraordinary a perform- at the numbers were cut on his *tomb-stone*, in St. Peter's yard, at Leyden.

the invention of fluxions, by the illustrious Sir Isaac Newton, ring of the circle has become more easy; and the late ingeni- Abraham Sharp, of Little Horton, near Bradford, in York- as not only confirmed Ceulen's ratio, but extended it to 72 f decimals.

John Machin, Professor of Astronomy in Gresham College, has also given us a quadrature of the circle, which is true to ces of figures; and even this has been extended, by the French aticians, to 128.

PROBLEM XII.

To find the length of the arc of a circle.

RULE I.

By the Pen.

om 8 times the chord of half the arc subtract the

Also, 1156 × .003399 = 3.929244, *the area in wine gallons;* 1156 × .00346 = 3.99976, *the area in mash-tun gallons;* *and* 1156 × .000365 = .421940, *the area in malt bushels.*

By Division.

```
                  Sq. of diameter.
Divisor 359.05)1156.00(3.2196 ale gallons.
               107715
               ------
                 78850
                 71810
                 -----
                  70400
                  35905
                  ------
                  344950
                  323145
                  ------
                   218050
                   215430
                   ------
                     2620
```

Also, 1156 ÷ 294.12 = 3.9303, *the area in wine gallons;* 1156 ÷ 289, = 4.0, *the area in mash-tun gallons; and* 1156 ÷ 2738 = .4222, *the area in malt bushels.*

By the Table of Ale Areas, in Part VII.

Find the diameter 34, in the first or left-hand column; and opposite to it, in the second column, you will have 3.2196, *the area in ale gallons.*

By the Table of Wine Areas, in Part VII.

Having found the diameter 34, in the first column; opposite to it, in the second column, we have 3.9304, *the area in wine gallons.*

By the Sliding Rule.

RULE I.

As the circular divisor on A, is to the diameter on B; so is the diameter on A, to the area on B.

n A. On B. On A. On B.

59.05, 94.12, 39.0, 38.0 : 34 :: 34 : 3.22 *ale gallons.*, 3.93 *wine gallons.*, 4.00 *mash-tun gallons.*, .422 *malt bushels.*

On A.	On B.	On A.	On B.	
59.05	: 34 ::	34 :	3.22	*ale gallons.*
94.12			3.93	*wine gallons.*
39.0			4.00	*mash-tun gallons.*
38.0			.422	*malt bushels.*

RULE II.

ircular gauge-point on D, is to unity on C; so
ieter on D, to the area on C.

On D.	On C.	On D.	On C.	
18.95	: 1 ::	34 :	3.22	*ale gallons.*
17.15			3.93	*wine gallons.*
17.07			4.00	*mash-tun gallons.*
52.32			.422	*malt bushels.*

diameter of a circle is 215.8 inches, what is the
: and wine gallons?

By Multiplication.

215.8 × 215.8 × .002785 = 46569.64 × .002785 =
174, *the area in ale gallons; and* 46569.64 ×
= 158.29020636, *the area in wine gallons.*

By Division.

6569.64 ÷ 359.05 = 129.7023, *the area in ale gal-*
l 46569.64 ÷ 294.12 = 158.3355, *the area in wine*

ie Tables of Ale and Wine Areas, in Part VII.

ig found 215 in the first column of the Table of
s, and .8 at the top of the page; then in the
angle of meeting, we have 129.7011, *the area in*
ns.
roceeding in the same manner with the Table of
eas, we obtain 158.3367, *the area in wine gallons.*

M

By the Sliding Rule.

RULE I.

	On A.		On B.		On A.		On B.
As	359.05 / 294.12	:	215.8	::	215.8	:	129.70 *ale gallon* / 158.34 *wine gallo*

RULE II.

	On D.		On C,		On D.		On C.
As	18.95 / 17.15	:	1	::	215.8	:	129.70 *ale gallons.* / 158.34 *wine gallons.*

3. If the circle, in the last question, be the base of cylindrical guile-tun; how many gallons of ale does t vessel contain, when the depth of the liquor is 48.6 i ches?

Ale Gallons.

```
 129.7011 area.
     48.6 depth.
 --------
  7782066
 10376088
 5188044
 ----------
 6303.47346 Ans.
```

By the Sliding Rule.

As 1 on A, is to the area on B; so is the depth on A to the content on B.

	On A.		On B.		On A.		On B.
As	1	:	129.7	::	48.6	:	6303.47, *ale gallons.*

4. The diameter of a circle is 38.6 inches, what is i area in ale, wine, and mash-tun gallons, and ma bushels?

Ans. By Multiplication, the area is 4.1280 *ale gallons* 5.0381 *wine gallons;* 5.1285 *mash-tun gallons; and* .541 *malt bushels.*

5. If the diameter of a cylindrical wine-vat be 78.6 inches, what is the area of its base, in wine gallons?

Ans. By Division, the area is 21.0048; and by the table of wine areas, 21.005 wine gallons.

6. The diameter of a cylindrical vessel is 53.2 inches, required its area and content in ale gallons, when the depth of the liquor is 35.7 inches.

Ans. By Multiplication, the area is 7.8822, and the content 281.39454 ale gallons.

GENERAL RULES.

The following Rules will solve most of the useful Problems relating to the circle and its equal or inscribed square, &c.

RULE 1. The diameter of a circle multiplied by .8862269, will give the side of a square equal in area.

2. The circumference of a circle multiplied by .2820948, will give the side of a square equal in area.

3. The diameter of a circle multiplied by .7071068, will give the side of the inscribed square.

4. The circumference of a circle multiplied by .2250791, will give the side of the inscribed square.

5. The area of a circle multiplied by .6366197, and the square root of the product extracted, will give the side of the inscribed square.

6. The side of a square multiplied by 1.414214, will give the diameter of its circumscribing circle.

7. The side of a square multiplied by 4.442883, will give the circumference of its circumscribing circle.

8. The side of a square multiplied by 1.128379, will give the diameter of a circle equal in area.

9. The side of a square multiplied by 3.544908, will give the circumference of a circle equal in area.

PROBLEM XIV.

To find the area of the sector of a circle.

RULE.

By the Pen.

Multiply the length of the arc by the radius of the

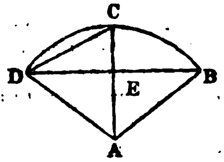
sector, and half the product will be the area in square inches; which being divided by 282, 231, and 2150.42, the respective quotients will be the area in ale and wine gallons, and malt bushels.

Note 1. The length of the arc may be found by Problem XII.

2. The area of a quadrant may be obtained by taking $\frac{1}{4}$, and the area of a semi-circle by taking $\frac{1}{2}$ of the area of the whole circle.

EXAMPLES.

1. What is the area of the sector A B C D, in ale gallons, when the radius A B or A D measures 45, the chord B D of the whole arc 72, and the versed sine C E 18 inches?



Here $\sqrt{36^2+18^2}=\sqrt{1296+324}=\sqrt{1620}=40.2492$ $=$ *D C, the chord of half the arc; and* $\frac{40.2492\times 8-72}{3}=\frac{321.9936-72}{3}=\frac{249.9936}{3}=83.3312$, *the length of the arc D C B; then* $\frac{83.3312\times 45}{2}=\frac{3749.904}{2}=1874.952$, *the area in square inches; whence* $\frac{1874.952}{282}=6.648$, *the area in ale gallons.*

By the Sliding Rule.

As the square divisor on A, is to the radius of the sector on B; so is half the length of the arc on A, to the area on B.

	On A.		On B.		On A.		On B.
As	282	:	45	::	41.66	:	6.65 *ale gallons.*

2. The radius of a sector is 31.5, and the length of the arc 42.6 inches; what is its area in wine gallons?

Ans. 2.9045 *wine gallons.*

3. The radius of a sector is 60, and the chord of the whole arc 72 inches; what is its area in malt bushels?

Ans. 1.0768 *malt bushels.*

4. The radius of a cooler, placed in an oblique angle of a brew-house, measures 80, the chord of the whole arc 124.8, and the chord of half the arc 69.3 inches; required the area and content in ale gallons, when the depth of the liquor is 8.7 inches.

Ans. The area is 20.312, *and the content* 176.7144 *ale gallons.*

PROBLEM XV.

To find the area of the segment of a circle.

RULE I.

By the Pen.

To two thirds of the product of the chord and height of the segment, add the cube of the height divided by twice the chord, and the sum will be the area in square inches. Divide this area by 282, 231, and 2150.42, and the respective quotients will be the area in ale and wine gallons, and malt bushels.

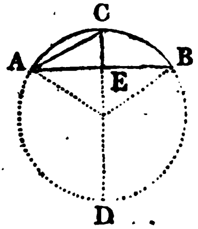
Note 1. If the segment be greater than a semi-circle, find the area of the remaining segment, which subtract from the area of the whole circle; and the remainder will be the area of the segment required.

2. When the chord and versed sine are given, divide the square of half the chord by the versed sine; to the quotient add the versed sine, and the sum will be the diameter of the circle; hence its area may be found by Prob. 13. Or, multiply the square of the diameter by .7854, and the product will be the area in square inches; from this take the area of the remaining segment, and the difference will be the area of the segment required, in square inches.

EXAMPLES.

1. The chord A B measures 120, and the versed sine

C E 30 inches; what is the area of the segment A C B, ale and wine gallons, and malt bushels?



Here $120 \times 30 \times \frac{2}{3} = 3600 \times \frac{2}{3} = \frac{7200}{3} = 2400$, *two-thi of the product of the chord and versed sine, or height of* 1 *segment; and* $\frac{30^3}{120 \times 2} = \frac{27000}{240} = 112.5$, *the cube of* 1 *height divided by twice the chord; then* $2400 + 112.5$ 2512.5, *the area in square inches; whence,* $2512.5 \div 282$ 8.9095, *the area in ale gallons;* $2512.5 \div 231 = 10.876$ *the area in wine gallons; and* $2512.5 \div 2150.42 = 1.168$ *the area in malt bushels.*

2. The chord A B is 128, and the versed sine C E : inches; what is the area of the segment in ale and wi gallons?

Ans. The area is 10.137 *ale, and* 12.375 *wine gallons.*

3. Required the area of the segment A D B, great than a semi-circle, in ale and wine gallons, and ma bushels; the chord A B measuring 60, and the verse sine E D 40 inches.

Ans. 7.351 *ale gallons,* 8.974 *wine gallons, and* .964 *ma bushels.*

RULE II.

Divide the versed sine or height of the segment by th diameter of the circle of which the segment is a part and find the quotient in the column of heights or verse sines, in the Table at the end of Part IV.

Take out the corresponding *Area Seg.* which multipl

by the square of the diameter; and the product will be the area of the segment in square inches. Divide the area thus obtained by 282, 231, and 2150.42, and the respective quotients will be the area in ale and wine gallons, and malt bushels.

Note 1. If the quotient of the height by the diameter do not terminate in three places of figures, without a fractional remainder, find the *Area Seg.* answering to the first three decimals of the quotient; subtract it from the next greater *Area Seg.*; multiply the remainder by the fractional part of the quotient, and the product will be the corresponding proportional part to be added to the first *Area Seg.*

EXAMPLE. If the versed sine be 25, and the diameter 55, we have $\frac{25}{55}=.454\frac{6}{11}$, the tabular height. The *Area Seg.* answering to .454, is .346764; the next greater *Area Seg.* is .347759; their difference is .000995; then $.000995\times\frac{6}{11}=.000542$, which being added to .346764, gives .347306, the *Area Seg.* corresponding to $.454\frac{6}{11}$.

Note 2. When the area of a segment greater than a semi-circle is required, subtract the quotient of the height by the diameter, from 1; find the *Area Seg.* corresponding to the remainder, which take from .785398, and the difference will be the *Area Seg.* answering to the quotient.

EXAMPLE. If the versed sine be 66, and the diameter 80, we have $\frac{66}{80}=.825$, the tabular height; then $1-.825=.175$; and the *Area Seg.* answering to .175, is .092313, which being taken from .785398, leaves .693085, the *Area Seg.* corresponding to .825.

EXAMPLES.

1. The chord A B measures 120, and the versed sine C E 30 inches; what is the area of the segment A C B, in ale and wine gallons, and malt bushels?

By Note 2, under Rule 1, we have $\frac{60^2}{30}+30=\frac{3600}{30}+30$ $=120+30=150$, *the diameter* C D; *then* $\frac{30.0}{150}=.2$, *the tabular height, or quotient of the versed sine divided by the diameter. The Area Seg. corresponding to this quotient, is* .111823; *then* $.111823\times150^2=.111823\times22500=2516.0175$, *the area in square inches; and* $2516.0175\div282=8.922$, *the area in ale gallons;* $2516.0175\div231=10.8918$, *the area in wine gallons; and* $2516.0175\div2150.42=1.17$, *the area in malt bushels.*

2. The versed sine C E is 32, and the diameter C D 1 inches; what is the area of the segment A C B, in and wine gallons?

Ans. 10.151 *ale gallons, and* 12.392 *wine gallo*

3. What is the area of the segment A D B, great than a semi-circle, in ale and wine gallons, and ma bushels; the versed sine E D being 40, and the diamet C D 62.5 inches?

Ans. 7.353 *ale gallons,* 8.976 *wine gallons, and* .96 *malt bushels.*

PROBLEM XVI.

To find the area of an ellipse.

RULE.

By the Pen.

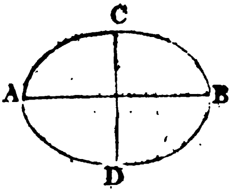
Divide the product of the two diameters by 359.05 294.12, and 2738; and the respective quotients will be the area of the ellipse, in ale and wine gallons, and malt bushels.

Note 1. If the two diameters of an ellipse and the number .7854 be multipled continually together, the last product will be the area of the ellipse in square inches.

2. If the sum of the two diameters of an ellipse be multiplied by 1.5708, the product will be the circumference of the ellipse, *nearly.*

EXAMPLES.

1. Required the area of the ellipse A B C D, in ale and wine gallons, and malt bushels; the transverse diameter A B measuring 102.5, and the conjugate C D 75.2 inches.



```
                         Inches.
                          102.5
                           75.2
                         ------
                           2050
                          5125
                         7175
Divisor 359.05)7708.00(21.467 ale gallons.
               71810
               -----
                52700
                35905
                ------
                167950
                143620
                ------
                 243300
                 215430
                 ------
                  278700
                  251835
                  ------
                   27365
```

Also, 7708.00 ÷ 294.12 = 26.206 *wine gallons; and* 7708.00 ÷ 2738 = 2.815 *malt bushels.*

By the Sliding Rule.

As the circular divisor on A, is to the transverse diameter on B; so is the conjugate diameter on A, to the area on B.

$$\begin{array}{ccccc} & \text{On A.} & \text{On B.} & \text{On A.} & \text{On B.} \\ \textit{As} & \left\{\begin{array}{r} 359.05 \\ 294.12 \\ 2738.00 \end{array}\right\} : 102.5 :: 75.2 : & & & \left\{\begin{array}{l} 21.47 \textit{ ale gallons.} \\ 26.21 \textit{ wine gallons.} \\ \ \ 2.82 \textit{ malt bushels.} \end{array}\right. \end{array}$$

2. Suppose the foregoing figure to represent the base of an elliptical cooler; what is the content in ale gallons, when the depth of the liquor is 10.8 inches?

Ans. 231.8436 *ale gallons.*

3. The transverse diameter of an elliptical wine-vat measures 85.9, and the conjugate 63.8 inches; what is its area in wine gallons? *Ans.* 18.633 *wine gallons.*

4. A maltster has an elliptical cistern whose transv diameter measures 96.8, and conjugate 73.2 inches; quired the area and content in malt bushels, when depth of the grain is 34.7 inches.

Ans. The area is 2.587, *and the content* 89.7689 *n bushels.*

5. The transverse diameter of an elliptical guile-t measures 96.4, and the conjugate 82.3 inches; requi the area and content, in ale gallons, when the depth the liquor is 53.8 inches.

Ans. The area is 22.096, *and the content* 1188.7648 *gallons.*

PROBLEM XVII.

To find the area of an elliptical segment, the base which is parallel to either of the diameters of t ellipse.

RULE.

By the Pen.

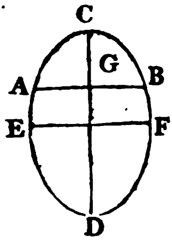
Divide the height of the segment by that diameter the ellipse of which it is a part, and find the *Area Se* answering to the quotient, in the Table at the end Part IV. Multiply the two diameters of the ellipse an the *Area Seg.* thus found, continually together, and th last product will be the area in square inches.

Divide this area by 282, 231, and 2150.42, and th respective quotients will be the area in ale and wine gal lons, and malt bushels.

Note. If the segment be greater than a semi-ellipse, find the are of the remaining segment, which subtract from the area of the whol ellipse; and the remainder will be the area of the segment required Or, proceed as directed in Problem XV., Rule II., Note II.

EXAMPLES.

1. Required the area, in ale gallons, of the elliptical segment A C B, cut off by the double ordinate A B; the height G C being 36.4, the diameter C D of the whole ellipse 120.8, and the diameter E F 75.4 inches.



Here $\frac{36.4}{120.8}$=.301, *the tabular height; and the corresponding Area Seg. is* .199085; *then* .199085 × 120.8 × 75.4=1813.3298, *the area of the segment in square inches;* and $\frac{1813.3298}{282}$= 6.4302, *the area in ale gallons.*

2. What is the area of the segment A D B, in wine gallons; the dimensions being the same as in the last example? *Ans.* 23.1181 *wine gallons.*

3. Required the area in malt bushels, of the greater segment of an ellipse made by a double ordinate parallel to the conjugate diameter, at the distance of 27 inches from the centre of the ellipse; the diameters being 180 and 120 inches. *Ans.* 5.4282 *malt bushels.*

PROBLEM XVIII.

To find the area of a parabola, its base and height being given.

RULE.

By the Pen.

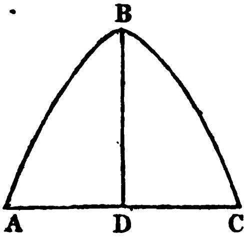
Multiply the base by the height, and ⅔ of the product will be the area in square inches, which being divided by 282, 231, and 2150.42, will give the area in ale and wine gallons, and malt bushels.

Note. A parabola is a figure or section formed by cutting a cone by a plane parallel to one of its slant sides; and several parabolas may be

cut from the same cone, which will all vary in their bases and altitu according to the distance of the cutting plane from the parallel sid the cone. (See the definitions in Part V.)

EXAMPLES.

1. What is the area, in ale gallons, of the parab A B C, whose height D B is 54.5, and the base or doul ordinate A C 72.8 inches?



Here $72.8 \times 54.5 \times \frac{2}{3} = \frac{72.8 \times 54.5 \times 2}{3} = \frac{3967.6 \times 2}{3} =$ $\frac{7935.2}{3} = 2645.0666$, *the area in square inches; an* $2645.0666 \div 282 = 9.3796$, *the area in ale gallons.*

By the Sliding Rule.

As the square divisor on A, is to the perpendicular on B; so is $\frac{2}{3}$ of the base on A, to the area on B.

On A.		On B.		On A.		On B.
As 282	:	54.5	::	48.5	:	9.38 *ale gallons.*

2. The absciss or height B D is 36.2, and the double ordinate A C 108.6; what is the area of the parabola A B C, in wine gallons? *Ans.* 11.3458 *wine gallons.*

PROBLEM XIX.

To find the area of compound figures.

RULE.

By the Pen.

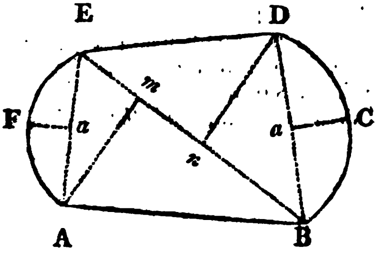
By the foregoing Problems, find the areas of the several figures of which the whole compound figure is composed; and the sum of these areas will be the area required.

Note 1. Mixed or compound figures are such as are composed of rectilineal and curvilineal figures united.

2. If the curvilineal part of the figure be irregular, its area must be found by the method described in the next Problem; and as it is sometimes difficult to determine the nature of curves, it would be advisable always to adopt it in Practice, when the true nature of the curve cannot be discovered.

EXAMPLES.

1. What is the area, in ale and wine gallons, of the compound figure A B C D E F, composed of a trapezium and two circular segments; the chord A E measuring 75.6, and the versed sine F *a* 9.6; the diagonal E B 136, the perpendicular A *m* 64, and D *n* 64.8; the chord B D 92, and the versed sine C *a* 14 inches?



N

By Prob. VI., we have $\frac{64+64.8\times136}{2} = \frac{128.8\times136}{2}$ $= \frac{17516.8}{2} = 8758.4$; *the area of the trapezium A B D E, in square inches.*

By Prob. XV., we have $75.6\times9.6\times\frac{2}{3}=725.76\times\frac{2}{3}=$ $\frac{1451.52}{3}=483.84$, *two-thirds of the product of the chord and versed sine; and* $\frac{9.6^3}{75.6\times2}=\frac{884.736}{151.2}=5.851$, *the cube of the height divided by twice the chord; then* $483.84+5.851$ $=489.691$, *the area of the segment A F E, in square inches.*

Again, by the same Prob. we have $92\times14\times\frac{2}{3}=1288\times\frac{2}{3}$ $=\frac{2576}{3}=858.666$, *two-thirds of the product of the chord and versed sine; and* $\frac{14^3}{92\times2}=\frac{2744}{184}=14.913$, *the cube of the height divided by twice the chord; then* $858.666+14.913=$ 873.579, *the area of the segment B C D, in square inches.*

Now, $8758.4+489.691+873.579=10121.67$, *the area of the whole compound figure in square inches; hence,* $10121.67\div282=35.892$, *the area in ale gallons; and* $10121.67\div231=43.816$, *the area in wine gallons.*

2. Required the area and content in ale gallons, of a cooler composed of a triangle, and a circular segment, whose chord is the longest side of the triangle; the versed sine of the segment being 26, the sides of the triangle 75,100, and 125 inches respectively, and the depth of the liquor 8.6 inches?

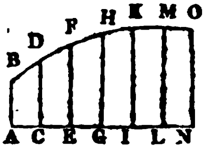
Ans. The area is 21.23, *and the content* 182.578 *ale gallons.*

PROBLEM XX.

To find the area of any curvilineal figure by means of equi-distant ordinates.

RULE.

By the Pen.



If a right line A N be divided to any even number of equal parts C, C E, E G, &c.; and at the oints of division be erected perendicular ordinates A B, C D, E F, c., terminated by any curve B D F, c., and if A be put for the sum of e extreme or first and last ordites A B, N O; B for the sum of the even ordinates D, G H, L M, &c.; viz. the second, fourth, sixth, &c., d C for the sum of all the rest E F, I K, &c.; viz. the ird, fifth, &c., or the odd ordinates, wanting the first d last: then the common distance A C, or C E, &c. of e ordinates, being multiplied by the sum arising from e addition of A, four times B, and two times C: oneird of the product will be the area A B O N, in square ches, very nearly; that is, $\frac{A+4B+2C}{3} \times D =$ the area, itting D=A C, the common distance of the ordinates.

Note. The foregoing Rule being expressed in Algebraic terms, is dom perfectly comprehended by learners; but when it is simplified the following manner, it may be easily understood and committed to emory.

RULE.

To the sum of the extreme, or first and last ordinates, d four times the sum of all the even ordinates, and ice the sum of all the odd ordinates, not including the st and last; multiply this sum by ⅓ of the common distance the ordinates, and the product will be the area in square ches. Divide this area by 282, 231, and 2150.42, and

the respective quotients will be the area in ale and wi[ne] gallons, and malt bushels.

Note 1. If the common distance of the ordinates will not divide [by] 3, multiply by that distance, divide the product by 3, and the quoti[ent] will be the area in square inches.

2. The length of the base line must be ascertained before y[ou] begin to measure the ordinates, in order that you may divide it into [an] even number of equal parts; and it is evident that you will determi[ne] the area most correctly when you use the greatest number of or[di]nates.

3. If the figure whose area is required be elliptical, the ordinat[es] must be taken perpendicularly to the transverse diameter; and th[e] areas of the two segments cut off by the extreme ordinates may [be] found by Problem XVII. If, however, the segments be small, it w[ill] be more easy, and correct enough for *general practice*, to consider the[m] as parabolas; consequently their areas must be found by Pro[b.] XVIII.

Or, if the heights of the segments be equal to each other, multip[ly] the sum of the extreme ordinates by the sum of the heights of th[e] segments, and ⅓ of the product will be the area of both segments *nearly*. To the area thus found, add the area of that part of th[e] figure measured by equi-distant ordinates; and the sum will be the whole area in square inches, which must be divided by the proper divisors as before directed.

4. If the area of any figure resembling a parabola, the segment of a circle, or the segment of an ellipse be required, and it cannot be determined to which of these denominations it belongs, divide the base or chord-line into an even number of equal parts; and measure ordinates from the base to the curve. In this case, the first and last ordinates will be nothing; but notwithstanding this, the Rule for equi-distant ordinates will give the area of the figure extremely near the truth; and if there be an ordinate at one end of the base, and none at the other, the Rule may still be applied with success. (See Ex. 3.)

SCHOLIUM.

The method of finding the areas of curvilineal figures, by means of equi-distant ordinates, was first demonstrated by the illustrious Sir Isaac Newton.

Mr. Robert Shirtcliffe, in his Theory and Practice of Gauging, appears to have been the first who applied it to finding the areas of curvilineal vessels used by brewers, distillers, &c.; and after him Mr. Samuel Farrer, in the Appendix to Overley's Gauging. Their Rules, however, were extremely tedious; and, though true to demonstra-

tion, were not general, but particular, according to the number of ordinates used.

To obviate this inconvenience, the *general rule*, given in this Problem, was deduced from Simpson's Dissertations, page 109, by Mr. Thomas Moss; and demonstrated in his valuable Treatise of Gauging, page 235.

Dr. Hutton, in his Mensuration, Proposition I., Section II., Part IV., has also given an elegant demonstration of the same Rule; and adds, in a Corollary, that it will obtain for the contents of all solids, by using the areas of the sections perpendicular to the axe, instead of the ordinates.

The Doctor particularly recommends it to the notice of Practical *Gaugers*, as being the best approximation that has yet been, or perhaps ever can be given; for by taking an indefinite number of ordinates, or sections, the areas, or contents of irregular figures may be obtained to any degree of accuracy.

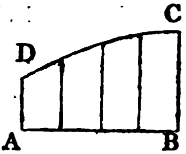
Now as the bases of Distillers' wash-backs, Brewers' coolers, &c. &c. are sometimes greater and sometimes less than true ellipses, it becomes absolutely necessary to find the areas of all such vessels by the method of Equidistant Ordinates, otherwise the Trader or the Revenue may be materially injured. Of this it appears that the Honourable Commissioners of the Board of Excise were fully convinced; as they, in a General Letter, dated July 1775, issued an order to that effect.

This order was again repeated, on the 3rd of February, 1818, in another General Letter, which also contains some observations relating to the qualifications and mathematical knowledge of Officers of the Excise.

EXAMPLES.

EXAM. 1.

What is the area of the curved figure A B C D, in ale and wine gallons, and malt bushels; the lengths of five equidistant ordinates being as follow; viz. the first or A D = 40, the second = 50, the third = 60, the fourth = 70, and the fifth or last B C = 75; and the length of the base A B = 120 inches?



Here 40+75=115, *the sum of the first and last ordinates;* 50+70×4=120×4=480, *four times the sum of the even ordinates;* 60×2=120, *twice the sum of the odd ordinates; and* 120÷4=30, *the common distance of the ordinates; then* 115+480+120×10 *(one-third of the common distance)* =715×10=7150, *the area in square inches; hence* 7150÷282=25.354, *the area in ale gallons* 7150÷231=30.952, *the area in wine gallons; and* 7150÷2150.42=3.324, *the area in malt bushels.*

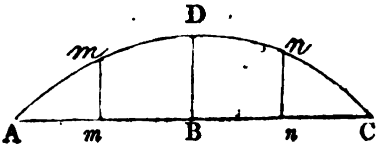
EXAM. 2.

If the foregoing figure represent the base of a cooler; what is its content in ale gallons, when the depth of the liquor is 12.6 inches?

Here 25.354×12.6=319.4604 *ale gallons, the answer required.*

EXAM. 3.

Required the area of the curvilineal figure A B C D A, in ale and wine gallons; the base A C being 95.2, the ordinate *m m* 15.9, B D 22.3, and *n n* 17.4 inches.



(See Note 4.)

Here the sum of the extreme ordinates =0; *and* 15.9+17.4×4=33.3×4=133.2, *four times the sum of the even ordinates; also* 22.3×2=44.6, *twice the sum of the odd ordi-*

nates; and $95.2 \div 4 = 23.8$, *the common distance of the ordinates, which will not divide by 3, (See Note 1.;) then*

$$\frac{133.2+44.6\times 23.8}{3}=\frac{177.8\times 23.8}{3}=\frac{4231.64}{3}=1410.546,$$

the area in square inches; and $1410.546 \div 282 = 5.001$, *the area in ale gallons; and* $1410.546 \div 231 = 6.106$, *the area in wine gallons.*

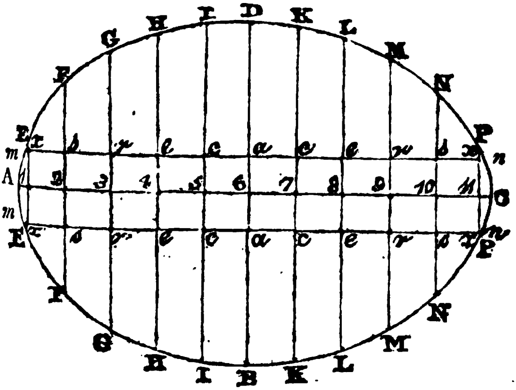
Note. It is evident that the figures A *m m*, and C *n n*, are greater than triangles; and that the figures B *m m* D, and B *n n* D, are greater than trapezoids; but if we consider them as triangles and trapezoids, and find their areas by Problems IV. and VII., we hence obtain the area of the whole figure A B C D A = 1323.28 square inches, which is 87.266 square inches less than the area found by equi-distant ordinates.

Again, it is evident, from the dimensions, that the figure A B C D A cannot be a parabola; but by inspection, it appears that the nature of the curve approaches very nearly to that figure. Now, if we consider it as a parabola, and find its area by Problem XVIII., we obtain 1415.306 square inches, which area is only 4.86 square inches more than that obtained by equi-distant ordinates.

These observations tend to prove that the Rule for equi-distant ordinates will always give *nearly* the true area, whatever be the shape of the figure.

EXAM. 4.

Let the following figure represent the base of an elliptical vessel; it is required to take the dimensions, and find the area in ale and wine gallons, and malt bushels.



TO TAKE THE DIMENSIONS.

By Prob. XXIX., Part III., find the centre of the vessel, and also the transverse and conjugate diameters A C, and B D, which must be struck with a chalk-line.

At a convenient distance from A C, by Prob. III., make the two parallel lines *m n*, *m n*.

Take the common distance in the compasses, and set it from 6 to 7, from 7 to 8, &c.; also, from 6 to 5, from 5 to 4, &c.; and likewise both ways from *a* to *c*, from *c* to *e*, &c.

Through the three points *x* 1 *x*, *s* 2 *s*, *r* 3 *r*, &c. with a chalk-line, strike the ordinates, or rather double ordinates, E 1 E, F 2 F, G 3 G, &c. &c. until you have finished the whole; the lengths of which may then be taken, in inches and tenths, with the dimension-cane, and other proper instruments.

Or, the common distance may be set off in the following manner, without the compasses: let your assistant hold the end of the tape at 6, (the centre of the vessel,) and suppose the common distance of the double ordinates to be 12 inches; then with a thin piece of chalk, make marks upon the bottom of the cooler, both ways, from the centre, at the distance of 12, at 24, at 36 inches, &c. from the end of the tape; and you will thus determine the points, 7, 8, 9, &c. and also the points 5, 4, 3, &c. Do the same upon the two parallel lines *m n*; and then strike the double ordinates as before directed.

Note 1. As the operation of finding the centre of an elliptical vessel, and drawing the diameters by Prob. XXIX., Part III., is rather tedious, it is seldom followed in Practical Gauging. The general method is to let an assistant hold one end of the tape at A, and by repeated trials to find the longest diameter A C, which must then be struck with a chalk-line. By the same method the conjugate diameter B D, is determined, which will always be the longest that can be found within the vessel, perpendicular to A C.

2. In the practice of Gauging, the method described in the last Note, has a decided advantage over that given in Prob. XXIX., Part III.; for it is frequently found in taking the dimensions of an oval utensil, that the conjugate diameter does not intersect the transverse in the centre of the vessel. This will always be the case when the vessel is made in an irregular manner.

3. When the transverse and conjugate diameters of the base of an oval vessel are drawn, the vessel is said, by Gaugers, to be quartered.

Dimensions of the foregoing Figure.

		Inches.
Transverse diameter A C	=	128
Conjugate diameter B D	=	85.5
Common distance of the ordinates	=	12
Height of each segment cut off by the extreme ordinates	=	4

Ordinates.

			Inches.
1.	E E	=	29.75
2.	F F	=	56.55
3.	G G	=	70.69
4.	H H	=	79.26
5.	I I	=	83.98
6.	B D	=	85.50
7.	K K	=	83.98
8.	L L	=	79.26
9.	M M	=	70.69
10.	N N	=	56.55
11.	P P	=	29.75

To find the Area of the Figure E B P P D E.

(See the Rule.)

	Inches.
First ordinate	29.75
Last ditto	29.75
Sum	59.50

Even Ordinates.

	Inches.
Second	56.55
Fourth	79.26
Sixth	85.50
Eighth	79.26
Tenth	56.55
Sum	357.12
Multiply by	4
Four times the sum	1428.48

Odd Ordinates.

	Inches.
Third	70.69
Fifth	83.98
Seventh........................	83.98
Ninth	70.69
Sum	309.34
Multiply by	2
Twice the sum	618.68

	Inches.
Sum of the first and last ordinates	59.50
Four times the sum of the even ordinates	1428.48
Twice the sum of the odd ordinates	618.68
Sum total	2106.66
Multiply by ⅓ *of* 12 *(the common distance)*	4
Area of the figure E B P P D E	8426.64

To find the Area of the two Segments E A E and P C P.

(See Note 3.)	Inches.
Sum of the two extreme ordinates	59.5
Multiply by the sum of their distance from the curve	8
Divide the product by	3)476.0
Area of the segments E A E and P C P	158.66
Area of the figure E B P P D E	8426.64
Area of the whole ellipse A B C D, in square inches	8585.30

Sq. inches.

Divisor 282)8585.300(30.444 *ale gallons.*

```
846
---
 1253
 1128
 ----
  1250
  1128
  ----
  1220
  1128
  ----
    92
```

Sq. Inches.

Divisor 231)8585.300(37.165 *wine gallons.*

```
           698
           ----
           1655
           1617
           ----
             383
             231
             ----
             1520
             1386
             ----
              1340
              1155
              ----
               185
               ====
```

Sq. inches.

Divisor 2150.42)8585.300(3.992 *malt bushels.*

```
           645126
           -------
           2134040
           1935378
           -------
            1986620
            1935378
            -------
              512420
              430084
              ------
               82336
               =====
```

Note. The figure belonging to the last example is a true ellipse; the lengths of the ordinates having been found by Prob. I., Part VII., of Nesbit's Mensuration.

Its true area obtained by Prob. XVI., Part IV., of this Work, is 30.480 ale gallons. This exceeds the area found by equi-distant ordinates no more than .036 of a gallon, which proves the approximation, by equi-distant ordinates, to be *very near* the truth; and if the ordinates had been taken at the distance of six inches from each other, instead of twelve, the difference would have been still less.

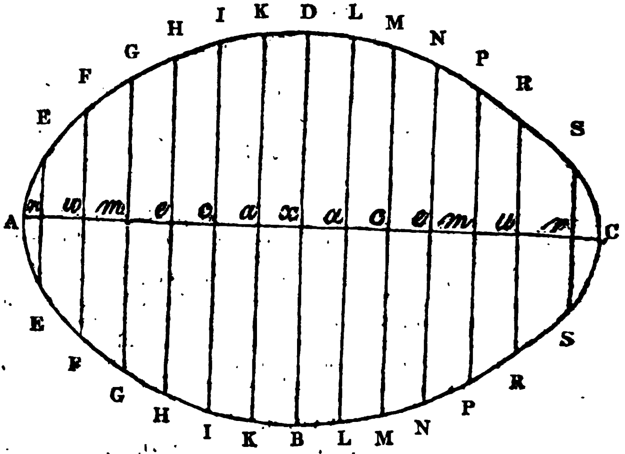
EXAM. 5.

Required the area in ale gallons, of the curvilineal figure A B C D, from the following dimensions: namely, the transverse diameter A C measures 156 inches; the conjugate diameter B D, which falls 4 inches from the middle of the transverse, 112 inches; the height A *n*, of

the segment E A E, 4 inches; the height C r, of the segment S C S, 8 inches; the common distance of the ordinates 12 inches; and the ordinates themselves as below:

Ordinates.

			Inches.
1.	E E	=	34.8
2.	F F	=	66.8
3.	G G	=	84.0
4.	H H	=	95.2
5.	I I	=	104.8
6.	K K	=	110.0
7.	B D	=	112.0
8.	L L	=	110.4
9.	M M	=	106.2
10.	N N	=	97.6
11.	P P	=	84.6
12.	R R	=	69.8
13.	S S	=	48.8



To construct the figure from the foregoing dimensions.

By any convenient scale of equal parts, draw the conjugate diameter B D = 112, and bisect it perpendicularly with the transverse diameter A C, drawn at pleasure.

'ake the common distance 12, in your compasses, and
't off upon the diameter A C, from x to a, from a to c,
six times, both ways, to r and n; and you will have
places of all the ordinates.
't off 4 from n to A, and 8 from r to C; and you
have the transverse diameter A C, which will mea-
156 inches, if the various operations have been accu-
y performed.
rallel to the conjugate diameter B D, and through
oints a, c, e, &c. with a parallel-ruler, draw all the
ates at pleasure.
om the point n, set off both ways to E, 17.4, half of
irst ordinate; also, from u, set off both ways to
3.4, half of the second ordinate; and thus proceed
you have determined the extreme points of all the
ates, through which, with a steady hand, describe the
mference of the figure.

To find the Area.

re $34.8+48.8=83.6$, *the sum of the extreme ordi-*
; $(66.8+95.2+110.+110.4+97.6+69.8)\times 4=$
$\times 4=2199.2$, *four times the sum of all the even ordi-*
and $(84.0+104.8+112.0+106.2+84.6)\times 2=491.6$
983.2, *twice the sum of all the odd ordinates; then*
$+2199.2+983.2)\times 4=3266\times 4=13064$, *the area of*
ure E B S S D E.
Prob. XVIII., we have $34.8\times 4=139.2$, *the product*
base and height of the segment E A E; and 48.8×8
.4, *the product of the base and height of the segment*
; *then* $139.2+390.4\times\frac{2}{3}=529.6\times\frac{2}{3}=\frac{529.6\times 2}{3}=$
$=353.066$, *the area of both the segments.*

v, $13064.0+353.066=13417.066$ *square inches, the*
f the curvilineal figure A B C D; and $13417.066\div$
47.578 *ale gallons, the area required.*

Remark.

: area of the foregoing figure, found by the Rule
le ellipse, Prob. XVI., is 48.661 ale gallons. This

is 1.088 gallons more than the area found by equi-distant ordinates, which shows the figure to be less than a true ellipse.

Now, if we suppose the figure to represent the base of a guile-tun whose sides are perpendicular, and the depth of the liquor 36 inches, and the area found as an ellipse, we have 1.088 × 36 = 38.988 ale gallons, the quantity of liquor for which the trader will be unduly charged duty in consequence of not finding the area of the vessel by equi-distant ordinates.

EXAM. 6.

Required the area of a curvilineal vessel, in ale and wine gallons; the transverse diameter being 174.6 inches, the conjugate diameter 106.4 inches, the versed sine of each segment 3.3 inches, the common distance of 15 perpendicular ordinates 12 inches, and the ordinates themselves as below:

Ordinates.		Inches.
1.	=	35.2
2.	=	72.4
3.	=	88.6
4.	=	99.0
5.	=	102.8
6.	=	104.2
7.	=	105.2
8.	=	106.4
9.	=	105.3
10.	=	104.5
11.	=	102.7
12.	=	99.2
13.	=	88.7
14.	=	72.5
15.	=	35.3

Ans. *The area is 55.725 ale gallons, and 68.029 wine gallons.*

REMARK.

The area found by the Rule for the ellipse, Prob. XVI. is 51.740 ale gallons, and 63.162 wine gallons, which is less than the true area by 3.985 ale gallons, and 4.867 wine gallons.

ow, if we suppose the figure to represent the base of ler, and let the depth of the liquor be 8 inches, we 3.985 × 8 = 31.88 ale gallons, the quantity of liquor which no duty will be charged, if the area of the be found by the Rule for the ellipse, and the duty imated from the gauge thus obtained. And this will ppen once only, but every brewing, if the cooler be by this method, in the Officer's Dimension Book.
ain, if we suppose the figure to represent the base of tiller's Wash Back, whose sides are perpendicular, e depth of the liquor 36 inches, we have 4.867 × 75.212 wine gallons, the quantity of liquor for no duty will be charged, if the area of the vessel nd by the Rule for the ellipse; hence it appears e revenue, or the trader, may be greatly injured, if s do not gauge and fix all oval vessels which are e ellipses, by the method of equi-distant ordinates. e last remark; and also Problem XXX., Part III., a method is given to determine whether an oval be greater or less than a *true* ellipse.)

EXAM. 7.

ired the area of a curvilineal vessel, in ale and allons, and malt bushels; the transverse diameter 22 inches, the conjugate diameter 71 inches, the sine of each segment 5 inches, the common dis- f 9 perpendicular ordinates 14 inches, and the or- themselves as below:

Ordinates.

		Inches.
1.	=	31.2
2.	=	50.8
3.	=	62.0
4.	=	68.2
5.	=	71.0
6.	=	68.8
7.	=	63.0
8.	=	52.0
9.	=	32.0

The area is 24.153 *ale gallons*, 29.485 *wine gal- l* 3.167 *malt bushels.*

The area found by the Rule for the ellipse, Prob. XVI., is

24.121 ale gallons. This differs from the area found by equidistant ordinates, only .032 of a gallon, which shows the figure to be nearly a *true* ellipse.

A

TABLE

OF THE

Areas of the Segments of a Circle,

Whose Diameter is Unity, and supposed to be divided into 1000 equal Parts.

Height.	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.
.001	.000042	.023	.004618	.045	.012554
.002	.000119	.024	.004921	.046	.012971
.003	.000219	.025	.005230	.047	.013392
.004	.000337	.026	.005546	.048	.013818
.005	.000470	.027	.005867	.049	.014247
.006	.000618	.028	.006194	.050	.014681
.007	.000779	.029	.006527	.051	.015119
.008	.000951	.030	.006865	.052	.015561
.009	.001135	.031	.007209	.053	.016007
.010	.001329	.032	.007558	.054	.016457
.011	.001533	.033	.007913	.055	.016911
.012	.001746	.034	.008273	.056	.017369
.013	.001968	.035	.008638	.057	.017831
.014	.002199	.036	.009008	.058	.018296
.015	.002438	.037	.009383	.059	.018766
.016	.002685	.038	.009763	.060	.019239
.017	.002940	.039	.010148	.061	.019716
.018	.003202	.040	.010537	.062	.020196
.019	.003471	.041	.010931	.063	.020680
.020	.003748	.042	.011330	.064	.021168
.021	.004031	.043	.011734	.065	.021659
.022	.004322	.044	.012142	.066	.022154

The Areas of the Segments of a Circle.

Height.	Area Segment.	Height.	Area Segment.	Height.	Area Segment.
67	.022652	.100	.040875	.133	.062026
68	.023154	.101	.041476	.134	.062707
69	.023659	.102	.042080	.135	.063389
70	.024168	.103	.042687	.136	.064074
71	.024680	.104	.043296	.137	.064760
72	.025195	.105	.043908	.138	.065449
73	.025714	.106	.044522	.139	.066140
74	.026236	.107	.045139	.140	.066833
75	.026761	.108	.045759	.141	.067528
76	.027289	.109	.046381	.142	.068225
77	.027821	.110	.047005	.143	.068924
78	.028356	.111	.047632	.144	.069625
79	.028894	.112	.048262	.145	.070328
80	.029435	.113	.048894	.146	.071033
81	.029979	.114	.049528	.147	.071741
82	.030526	.115	.050165	.148	.072450
83	.031076	.116	.050804	.149	.073161
4	.031629	.117	.051446	.150	.073874
5	.032186	.118	.052090	.151	.074589
6	.032745	.119	.052736	.152	.075306
7	.033307	.120	.053385	.153	.076026
8	.033872	.121	.054036	.154	.076747
9	.034441	.122	.054689	.155	.077469
0	.035011	.123	.055345	.156	.078194
1	.035585	.124	.056003	.157	.078921
2	.036162	.125	.056663	.158	.079649
3	.036741	.126	.057326	.159	.080380
4	.037323	.127	.057991	.160	.081112
5	.037909	.128	.058658	.161	.081846
6	.038496	.129	.059327	.162	.082582
7	.039087	.130	.059999	.163	.083320
8	.039680	.131	.060672	.164	.084059
9	.040276	.132	.061348	.165	.084801

The Areas of the Segments of a Circle.

Height.	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.
.166	.085544	.198	.110226	.230	.136465
.167	.086289	.199	.111024	.231	.137307
.168	.087036	.200	.111823	.232	.138150
.169	.087785	.201	.112624	.233	.138995
.170	.088535	.202	.113426	.234	.139841
.171	.089287	.203	.114230	.235	.140688
.172	.090041	.204	.115035	.236	.141537
.173	.090797	.205	.115842	.237	.142387
.174	.091554	.206	.116650	.238	.143238
.175	.092313	.207	.117460	.239	.144091
.176	.093074	.208	.118271	.240	.144944
.177	.093836	.209	.119083	.241	.145799
.178	.094601	.210	.119897	.242	.146655
.179	.095366	.211	.120712	.243	.147512
.180	.096134	.212	.121529	.244	.148371
.181	.096903	.213	.122347	.245	.149230
.182	.097674	.214	.123167	.246	.150091
.183	.098447	.215	.123988	.247	.150953
.184	.099221	.216	.124810	.248	.151816
.185	.099997	.217	.125634	.249	.152680
.186	.100774	.218	.126459	.250	.153546
.187	.101553	.219	.127285	.251	.154412
.188	.102334	.220	.128113	.252	.155280
.189	.103116	.221	.128942	.253	.156149
.190	.103900	.222	.129773	.254	.157019
.191	.104685	.223	.130605	.255	.157890
.192	.105472	.224	.131438	.256	.158762
.193	.106261	.225	.132272	.257	.159636
.194	.107051	.226	.133108	.258	.160510
.195	.107842	.227	.133945	.259	.161386
.196	.108636	.228	.134784	.260	.162263
.197	.109430	.229	.135624	.261	.163140

The Areas of the Segments of a Circle.

Height.	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.
.262	.164019	.294	.192684	.326	.222277
.263	.164899	.295	.193596	.327	.223215
.264	.165780	.296	.194509	.328	.224154
.265	.166663	.297	.195422	.329	.225093
.266	.167546	.298	.196337	.330	.226033
.267	.168430	.299	.197252	.331	.226974
.268	.169315	.300	.198168	.332	.227915
.269	.170202	.301	.199085	.333	.228858
.270	.171089	.302	.200003	.334	.229801
.271	.171978	.303	.200922	.335	.230745
.272	.172867	.304	.201841	.336	.231689
.273	.173758	.305	.202761	.337	.232634
.274	.174649	.306	.203683	.338	.233580
.275	.175542	.307	.204605	.339	.234526
.276	.176435	.308	.205527	.340	.235473
.277	.177330	.309	.206451	.341	.236421
.278	.178225	.310	.207376	.342	.237369
.279	.179122	.311	.208301	.343	.238318
.280	.180019	.312	.209227	.344	.239268
.281	.180918	.313	.210154	.345	.240218
.282	.181817	.314	.211082	.346	.241169
.283	.182718	.315	.212011	.347	.242121
.284	.183619	.316	.212940	.348	.243074
.285	.184521	.317	.213871	.349	.244026
.286	.185425	.318	.214802	.350	.244980
.287	.186329	.319	.215733	.351	.245934
.288	.187234	.320	.216666	.352	.246889
.289	.188140	.321	.217599	.353	.247845
.290	.189047	.322	.218533	.354	.248801
.291	.189955	.323	.219468	.355	.249757
.292	.190864	.324	.220404	.356	.250715
.293	.191775	.325	.221340	.357	.251673

The Areas of the Segments of a Circle.

Height.	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.
.358	.252631	.390	.283592	.422	.315016
.359	.253590	.391	.284568	.423	.316004
.360	.254550	.392	.285544	.424	.316992
.361	.255510	.393	.286521	.425	.317981
.362	.256471	.394	.287498	.426	.318970
.363	.257433	.395	.288476	.427	.319959
.364	.258395	.396	.289453	.428	.320948
.365	.259357	.397	.290432	.429	.321938
.366	.260320	.398	.291411	.430	.322928
.367	.261284	.399	.292390	.431	.323918
.368	.262248	.400	.293369	.432	.324909
.369	.263213	.401	.294349	.433	.325900
.370	.264178	.402	.295330	.434	.326892
.371	.265144	.403	.296311	.435	.327882
.372	.266111	.404	.297292	.436	.328874
.373	.267078	.405	.298273	.437	.329866
.374	.268045	.406	.299255	.438	.330858
.375	.269013	.407	.300238	.439	.331850
.376	.269982	.408	.301220	.440	.332843
.377	.270951	.409	.302203	.441	.333836
.378	.271920	.410	.303187	.442	.334829
.379	.272890	.411	.304171	.443	.335822
.380	.273861	.412	.305155	.444	.336816
.381	.274832	.413	.306140	.445	.337810
.382	.275803	.414	.307125	.446	.338804
.383	.276775	.415	.308110	.447	.339798
.384	.277748	.416	.309095	.448	.340793
.385	.278721	.417	.310081	.449	.341787
.386	.279694	.418	.311068	.450	.342782
.387	.280668	.419	.312054	.451	.343777
.388	.281642	.420	.313041	.452	.344772
.389	.282617	.421	.314029	.453	.345768

The Areas of the Segments of a Circle.

Height.	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.	*Height.*	*Area* SEGMENT.
.454	.346764	.470	.362717	.486	.378701
.455	.347759	.471	.363715	.487	.379700
.456	.348755	.472	.364713	.488	.380700
.457	.349752	.473	.365712	.489	.381699
.458	.350748	.474	.366710	.490	.382699
.459	.351745	.475	.367709	.491	.383699
.460	.352742	.476	.368708	.492	.384699
.461	.353739	.477	.369707	.493	.385699
.462	.354736	.478	.370706	.494	.386699
.463	.355732	.479	.371705	.495	.387699
.464	.356730	.480	.372704	.496	.388699
.465	.357727	.481	.373703	.497	.389699
.466	.358725	.482	.374702	.498	.390699
.467	.359723	.483	.375702	.499	.391699
.468	.360721	.484	.376702	.500	.392699
.469	.361719	.485	.377701		

te. The use of the foregoing Table is given in Problem XV., II.; and the method of constructing it may be seen in Moss's ing, page 92.

PART V.

MENSURATION OF SOLIDS

APPLIED TO

GAUGING.

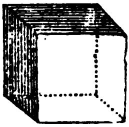
DEFINITIONS OF SOLIDS.

1. A SOLID is a figure which generally consists of three dimensions; viz. length, breadth, and thickness.

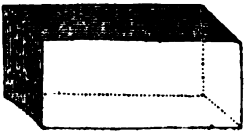
2. The *measurement* of a solid is called its solidity, capacity, or content.

3. The *contents* of solids are estimated by a cube whose side is one inch, one foot, one yard, &c. called the *measuring-unit*; hence the solidity of a body is said to be so many cubic inches, feet, yards, &c. as are contained in that body. In Gauging, however, the contents of all vessels are reduced to *ale gallons, wine gallons, malt bushels,* &c. &c.

4. A *cube* is a solid having six equal square sides.

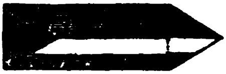


5. A *parallelopipedon* is a solid having six rectangular sides, every opposite two of which are equal and parallel.



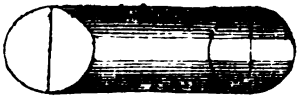
6. A *prism* is a solid whose ends are two equal, parallel, and similar plane figures; and its sides rectangles.

It is called a *triangular prism* when its ends are triangles; a *square prism*, when its ends are squares; a *pentagonal prism*, when its ends are pentagons, &c.

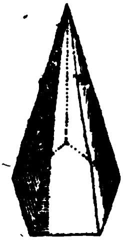


7. A *cylinder* is a solid conceived to be described by the revolution of a right-angled parallelogram about one of its sides, which remains fixed, and is called the axis of the cylinder; or it is a solid whose ends are parallel circles, and its sides right-lines.

Note. When the parallel ends of a solid are bounded by dissimilar curves; that is, when one end is bounded by an ellipse and the other by a circle, the figure is called a *cylindroid.*



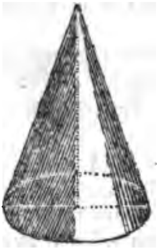
8. A *pyramid* is a solid the base of which is any plane figure whatever, and its sides are triangles, meeting in a point, called the *vertex* of the pyramid



9. A *cone* is a solid conceived to be described by the revolution of a right-angled triangle about one of its legs,

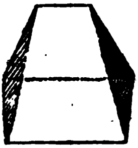
which remains fixed, and is called the *axis* of the cone; or it is a pyramid of an infinite number of sides, having a circle for its base.

Note. When the base of a cone is an ellipse, the solid is called an elliptical cone.

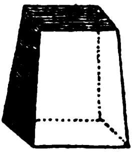


10. The *frustum* of a pyramid or cone is that part which remains when the top is cut off by a plane parallel to the base. The part cut off is called a *segment*.

11. A *wedge* is a solid whose base is a rectangle, its two ends plane triangles, and its two opposite sides terminate in an edge.



12. A *prismoid* is a solid whose bases or ends are two right-angled parallelograms, being parallel but not similar to each other; and its sides four plane trapezoids.



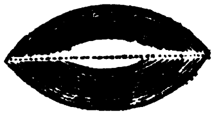
13. A *sphere* or *globe* is a solid conceived to be forme

e rotation of a semi-circle about its diameter, which ins fixed, and is called the *axis* or diameter; or it is d bounded by one continued convex surface, every of which is equally distant from a point within, calle e centre.

The *segment* of a sphere is any part of it cut off plane. If the plane pass through the centre, it livide the sphere into two equal parts called *hemis-*

The *zone of a sphere* is a part intercepted between arallel planes, and if these planes be equally dis- rom the centre, it is called the *middle zone of the*

A circular spindle is a solid conceived to be formed revolution of a circular segment about its chord, remains fixed.



Cylindrical hoofs or *ungulas* are solids formed by a cylinder in different directions, and may be l into six varieties; viz. 1st, when the cutting is parallel to the axis of the cylinder, and passes h both ends; 2nd, when the plane is oblique to the d passes through both ends; 3rd, when the plane obliquely through the sides; 4th, when the plane the side, passes through the base, and makes the segment of the base less than a semi-circle; 5th, he plane enters the side, passes through the base, akes the segment of the base a semi-circle; and en the plane enters the side, passes through the

base, and makes the segment of the base greater than a semi-circle.

18. In the two first cases, the sections will be *rectangles;* in the third case, the section will be an *ellipse;* in the fourth case, it will be the less *segment* of an *ellipse;* in the fifth case, it will be a *semi-ellipse;* and in the sixth case, it will be the greater *segment* of an *ellipse.*

19. A cylinder may also be cut by a plane passing obliquely through the opposite extremities of the two ends. In this case the cylinder will be divided into two *ungulas,* each of which will be half a cylinder; and the section will be an *ellipse.*

20. *Conic hoofs* or *ungulas* are solids formed by cutting the frustum of a cone in different directions; and may be divided into eight varieties; viz. 1st, when the cutting plane is parallel to the axis of the frustum, and passes through both ends; 2nd, when the plane is oblique to the axis, and passes through both ends; 3rd, when the plane passes obliquely through the opposite extremities of the two ends; 4th, when the plane passes obliquely through the sides; 5th, when the plane enters the side, and passes through the less end; 6th, when the plane enters the side, passes through the greater base or end, and makes a less angle with the base than that made by the base and the side of the frustum; 7th, when the plane cuts the side, passes through the greater base, and is parallel to one of the sides; and 8th, when the plane enters the side, passes through the greater base, and is parallel to the axis of the frustum.

21. In the two first cases, the sections will be *trapezoids;* in the third and fourth cases, the sections will be *ellipses;* in the fifth and sixth cases, the sections will be *elliptical segments;* in the seventh case, the section will be a *parabola;* and in the eighth case, the section will be a *hyperbola.* (See the definitions of the Conic Sections.)

22. *Pyramidical* and *prismoidal ungulas* are solids formed by cutting the frustum of a square pyramid, or a prismoid in different directions; and are generally either *wedges* or *prismoids.*

Note. All the *ungulas* mentioned in the preceding definitions, may be formed by *liquor* in cylindrical, conical, pyramidical, and prismoidal vessels, placed in different positions.

DEFINITIONS

OF THE

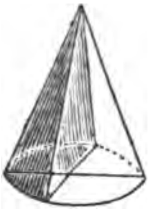
Conic Sections and their Solids.

---

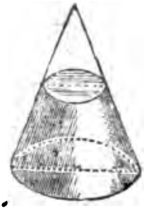
1. Conic Sections are plane figures formed by cutting a cone.

According to the different positions of the cutting plane, there will arise five different figures or sections.

2. If the cutting plane pass through the vertex of the cone, and any part of the base, the section will be a *triangle*.



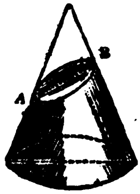
3. If a cone be cut into two parts, by a plane parallel to the base, the section will be a *circle*.



4. If a cone be cut by a plane passing through its two slant sides in an oblique direction, the section will be an

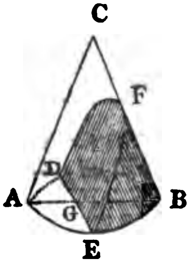
ellipse or ellipsis; A and B being the vertices, and A the transverse diameter.

Note. The method of constructing an ellipse is given in Prob XV., Part III.



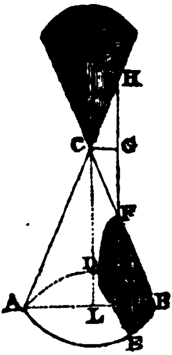
5. If a cone be cut by a plane parallel to either of i slant sides, the section will be a *parabola;* thus, if t cone A B C be cut by a plane parallel to the slant sic A C, the section D E F will be a *parabola;* F being i vertex, G F its axis, and D E its base. The soli D E B F D, cut off by the plane, is called a parabol hoof or ungula.

Note. The method of describing a parabola may be seen in Nesbit Mensuration, page 319.



6. If a cone be cut by a plane parallel to its axis, or i such a manner that the plane, if continued, would mee the opposite cone, the section will be a *hyperbola;* thu if the cone A B C be cut by a plane parallel to its axi C L, or in such a manner that the plane, if continued would meet the opposite side of a similar cone, as at H the section D E F will be a *hyperbola;* D E being its base F and H its vertices, G its centre, F H its transverse

diameter, and C G its semi-conjugate. The solid DEBFD, cut off by the plane, is called a hyperbolic hoof or ungula.



Note 1. The method of constructing a hyperbola, may be seen in Nesbit's Mensuration, page 323.

2. All the figures that can possibly be formed by cutting a cone, are mentioned in the preceding definitions; and are five in number; viz. a *triangle*, a *circle*, an *ellipse*, a *parabola*, and a *hyperbola*; but the three last only, are usually denominated CONIC SECTIONS.

7. The *vertex* of a conic section is that point where the cutting plane meets the opposite sides of the cone, as at A and B, in the ellipse; at F, in the parabola; and at F and H, in the hyperbola; hence the ellipse and hyperbola have each two vertices, but the parabola has only one vertex.

8. The *axis* of a parabola or a hyperbola, is a right-line drawn from the vertex to the middle of the base.

9. An *ordinate* is a right line drawn perpendicularly from any point in the axis, and terminating in the curve; consequently, the double ordinate is a right-line passing perpendicularly through the axis, and terminating at both ends, in the curve.

10. An *absciss* is that part of the axis which is contained between the vertex and the ordinate.

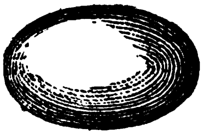
11. The *parameter* of any diameter is a third proportional to that diameter and its conjugate.

The parameter is sometimes called the *latus rectum*.

12. The *focus* is a point in the axis where the ordina is equal to half the parameter.

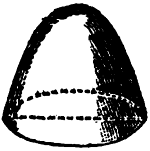
13. The ellipse and hyperbola have each two *foci*: b the parabola has only one *focus*.

14. A *spheroid* or *ellipsoid* is a solid generated by th revolution of an ellipse about one of its diameters. the revolution be made about the transverse diameter, th solid is called a *prolate spheroid*; but if about the con jugate diameter, an *oblate* spheroid.



15. A *conoid* is a solid formed by the revolution of a parabola, or hyperbola, about its axis; and is accordingly called *parabolic*, or *hyperbolic*.

The parabolic conoid is also called a *paraboloid*; and the hyperbolic conoid, a *hyperboloid*.



16. An *elliptic*, a *parabolic*, or a *hyperbolic spindle*, is a solid formed by the revolution of a segment of an ellipse, a parabola, or a hyperbola, about its double ordinate, which remains fixed. (See the circular spindle.)

GENERAL PROPERTIES

OF THE

Conic Sections.

PROPERTIES OF THE ELLIPSE.

1. The square of the distance of the focus from the centre, is equal to the difference of the squares of the semi-axes.

2. The sum of two lines drawn from the foci, to meet at any point in the curve, is equal to the transverse axis.

3. The parameter, or double ordinate at the focus, is equal to the square of the conjugate diameter divided by the transverse.

4. The rectangles of the segments of any diameter, are as the squares of their ordinates.

5. All the parallelograms circumscribed about an ellipse are equal to one another, and each equal to the rectangle of the two axes.

6. An ellipse is to the rectangle of the two axes, as any circle is to the square of its diameter.

7. The areas of ellipses are to one another, as the rectangles of their transverse and conjugate axes.

8. As the transverse axis of an ellipse is to the conjugate, so is the area of a circle whose diameter is the transverse, to the area of the ellipse.

PROPERTIES OF THE PARABOLA.

1. The distance of the focus from the vertex is equal to the square of any ordinate divided by 4 times its absciss.

2. The parameter, or double ordinate at the focus, is equal to the square of any ordinate divided by its absciss.

3. The distance between the focus and the vertex is equal to ¼ of the parameter, or ½ of the ordinate at the focus.

4. As the parameter is to the sum of any two ordinates, so is the difference of those ordinates to the difference of their abscisses.

5. Abscisses are to each other as the squares of their ordinates.

6. All parabolas are similar to each other.

7. The area of a parabola is equal to ⅔ of the area of the circumscribing parallelogram.

PROPERTIES OF THE HYPERBOLA.

1. The square root of the sum of the squares of the semi-axes, is equal to the distance of the focus from the middle of the transverse axis.

2. If half the transverse axis be subtracted from the distance between the focus and the middle of the said axis, the remainder will be the distance of the focus from the vertex.

3. The difference of two lines drawn from the two foci to any point in the curve, is equal to the transverse axis.

4. The parameter, latus rectum, or double ordinate at the focus, is equal to the square of the conjugate divided by the transverse.

Note. Those who desire to obtain a complete knowledge of the properties of Conic Sections, are referred to the Works of Emerson, Hutton, and Simson on that subject.

PROBLEMS

IN

PRACTICAL GAUGING.

PROBLEM I.

The side of a vessel in the form of a cube being given, to find its content in ale and wine gallons, and malt bushels.

RULE.

By the Pen.

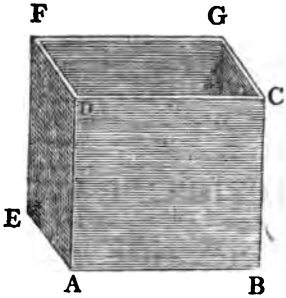
Multiply the side of the cube by itself, and that product again by the side; and the last product will be the content in cubic inches. Multiply the content thus found by .003546, .004329, and .000465; or divide it by 282, 231, and 2150.42; and the respective products, or quotients, will be the content in ale and wine gallons, and malt bushels.

Note 1. If the content of a cubical vessel be given in gallons or bushels, its side may be found by extracting the cube root of the given content, reduced to cubic inches.

2. It is scarcely necessary to observe to the young Gauger, that the internal dimensions of all vessels must be taken; hence, when we say the side of a vessel measures so many inches, we mean the internal, not the external side.

EXAMPLES.

1. The side A B or B C, of the cubical vessel A B C D E F G, measures 43.7 inches; what is the content in ale and wine gallons, and malt bushels?



By Multiplication.

Inches.	
43.7	*side.*
43.7	*side.*
3059	
1311	
1748	
1909.69	*product.*
43.7	*side.*
1336783	
572907	
763876	
83453.453	*content in cubic inches.*
.003546	*multiplier.*
500720718	
333813812	
417267265	
250360359	
295.926044338	*content in ale gallons.*

Cubic inches.	
83453.453	*content.*
.004329	*multiplier.*
751081077	
166906906	
250360359	
333813812	
361.269998037	*wine gallons.*

```
      Cubic inches.
        83453.453 content.
           .000465 multiplier.
       ──────────
       417267265
      500720718
     333813812
    ─────────────
    38.805855645 malt bushels.
```

By Division.

```
           Cubic inches.
Divisor 282)83453.453(295.934 ale gallons.
            564
            ────
            2705
            2538
            ────
             1673
             1410
             ────
              2634
              2538
              ────
                965
                846
               ────
                1193
                1128
                ────
                  65
```

```
           Cubic inches.
Divisor 231)83453.453(361.270 wine gallons.
            693
            ────
            1415
            1386
            ────
              293
              231
              ───
               624
               462
               ────
               1625
               1617
               ────
                  83
```

Cubic inches.

Divisor 2150.42)83453.453(38.807 *malt bushels.*

```
 645126
 ------
 1894085
 1720336
 -------
  1737493
  1720336
  -------
   1715700
   1505294
   -------
    210406
    ======
```

By the Sliding Rule.

RULE.

As the square gauge-point on D, is to the side on C so is the side on D, to the content on C.

	On D.	On C.	On D.	On C.
As	16.79	: 43.7 ::	43.7 :	295.93 *ale gallons.*
	15.19			361.27 *wine gallons.*
	46.36			38.80 *malt bushels.*

2. If the side of a cubical vessel measures 58.7 inches; what is its content in ale and wine gallons, and malt bushels?

Ans. 717.241 *ale gallons*, 875.593 *wine gallons, and* 94.056 *malt bushels.*

3. The side of a cubical wine-vat measures 86.5 inches; what is its content in wine gallons?

Ans. 2801.794 *wine gallons.*

4. A cubical vessel contains 862 ale gallons; required the length of its side, in inches? *Ans.* 62.4 *inches.*

REMARK.

If the content of any figure, in cubic inches, be multiplied by .000578, the square factor for a solid foot, taken from the second column of the Table of Factors, in Part IV., the product will be the content in cubic feet;

and if the content in cubic inches be divided by 1728, the square divisor, for a solid foot, taken from the fourth column of the same Table, the quotient will be the content in cubic feet.

The content of the first Example in this Problem, is 83453.453 cubic inches; then 83453.453 × .000578 = 48.236, the content in cubic feet; and 83453.453 ÷ 1728 = 48.294, which is also the content in cubic feet.

AGAIN, if the square of the diameter of a cylinder be multiplied by the height, and the product thence arising by .000454, the circular factor, for a solid foot, taken from the third column of the same Table, the product will be the content in cubic feet; or if the product arising from the square of the diameter by the height, be divided by 2200.16, the circular divisor for a solid foot, taken from the fifth column of the same Table, the quotient will be the content in cubic feet.

The number produced by multiplying the square of the diameter by the height, in the first Example, Problem V., is 277781.2; then 277781.2 × .000454 = 126.112, the content in cubic feet; and 277781.2 ÷ 2200.16 = 126.254, which is also the content in cubic feet.

PROBLEM II.

To find the content of a vessel in the form of a parallelopipedon.

RULE.

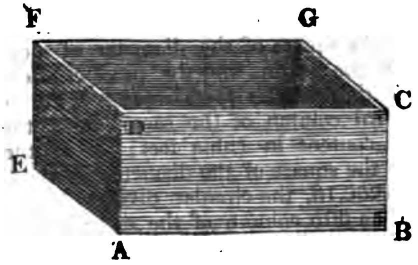
By the Pen.

Multiply the length by the breadth, and that product by the depth or altitude, and it will give the content in cubic inches. Divide the content thus obtained by 282, 231, and 2150.42; and the respective quotients will be the content in ale and wine gallons, and malt bushels.

Note. If the content of a parallelopipedon be divided by the product of any two of its dimensions, the quotient will be the other dimension.

EXAMPLES.

1. The length A B, of a vessel in the form of a parallelopipedon, measures 82 inches, its breadth A E 54 inches, and its depth or altitude B C 38.5 inches; what is its content in ale and wine gallons, and malt bushels?



Inches.
82 *length.*
54 *breadth.*
328
410
4428 *product.*
38.5 *depth.*
22140
35424
13284

Divisor 282)170478.0(604.531 *content in ale gallons.*

Also, 170478÷231=738.00, *the content in wine gallons;* *and* 170478÷2150.42=79.27, *the content in malt bushels.*

By the Sliding Rule.

RULE I.

As the length on C, is to the same on D; so is the breadth on C, to the mean proportional between the length and breadth, on D. (See Prob. X., Part II.)

Then, as the square gauge-point on D, is to the depth

on C; so is the mean proportional on D, to the content on C.

	On C.		On D.		On C.		On D.	
As	82	:	82	::	54	:	66.54,	the mean proportional.

	On D.		On C.		On D.		On C.
As	16.79 15.19 46.36	:	38.5	::	66.54	:	604.53 *ale gallons.* 738.00 *wine gallons.* 79.27 *malt bushels.*

RULE II.

As the square divisor on A, is to the length on B; so is the breadth on A, to the area of the base on B.

And, as one on A, is to the area of the base on B; so is the depth on A, to the content on B.

Areas.

	On A.		On B.		On A.		On B.
As	282 231 2150.42	:	82	::	54	:	15.70 *ale gallons.* 19.17 *wine gallons.* 2.06 *malt bushels.*

Contents.

	On A.		On B.		On A.		On B.
As	1	:	15.70 19.17 2.06	::	38.5	:	604.53 *ale gallons.* 738.00 *wine gallons.* 79.27 *malt bushels.*

2. The length of a vessel is 92.8 inches, its breadth ..5 inches, and its depth 46.2 inches; what is its content in ale and wine gallons, and malt bushels?

Ans. 980.619 *ale gallons,* 1197.12 *wine gallons, and* 28.59 *malt bushels.*

3. A wine-vat measures 104 inches in length, 92.4 inches in breadth, and 38.6 inches in depth; how many gallons of wine will it contain?

Ans. 1605.76 *wine gallons.*

4. Admit the foregoing figure A B C D E F G to represent a couch, the length of which measures 146 in-

ches, its breadth 92 inches, and its depth 35.6 inches how many bushels of malt will it contain?

Ans. 222.365 *bushels*

5. A floor of malt measures 246 inches in length, 12 in breadth, and 6.8 inches in depth; required its content in malt bushels? *Ans.* 97.236 *malt bushels*

6. A stone trough or cistern measures 83.8 inches in length, 45.3 inches in breadth, and holds 436.5 ale gallons; required its depth? *Ans.* 32.646 *inches*

PROBLEM III.

To find the content of a vessel in the form of a prism.

RULE.

By the Pen.

Find the area of the base in ale and wine gallons, and malt bushels; multiply these areas by the perpendicular depth; and the respective products will be the content of the vessel, in ale and wine gallons, and malt bushels.

Note 1. If the base be a square, a rectangle, a rhombus, a triangle, a trapezium, a trapezoid, or an irregular polygon, its area may be found by Prob. I., II., III., IV., V., VI., VII., or VIII.; but if the base be a regular polygon, its area may be obtained by Prob. IX. or X., Part IV.

2. A regular prism has all the sides of its base equal to each other; when they are unequal, the prism is irregular.

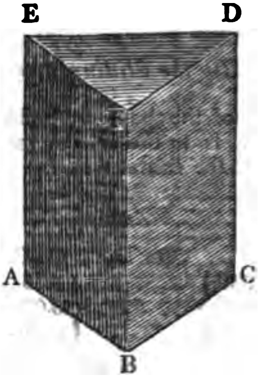
3. The perpendicular depth of a prismatic vessel may be found by dividing the content by the area of the base.

4. In order to find the content of any prism in cubic feet, find the area of the base in inches, as directed in Note 1; multiply this area by the perpendicular height, and the product will be the content of the prism in cubic inches. Divide this content by 1728, and you will obtain the content in cubic feet. (See Nesbit's Mensuration, Prob. III., Part IV.)

EXAMPLES.

1. Let A B C D E F represent a vessel in the form of a triangular prism; required its content in ale and wine gallons, and malt bushels; the perpendicular depth or al-

titude A E, B F, or C D measuring 65 inches, and each of the equal sides of the base A B C 52 inches?



(*See Prob. X., Part IV.*)

```
   Inches.
     52 side of the base.
     52 side of the base.
    104
   260
   2704 square of the side.
.001536 multiplier.
  16224
  8112
 13520
 2704
4.153344 area in ale gallons.
     65 depth.
20766720
24920064
269.967360 content in ale gallons.
```

Also, 2704×.001875=5.07, *the area of the base, in wine gallons; and* 5.07×65=329.55, *the content in wine gallons.*

Likewise, $2704 \times .000201 = .543504$, *the area of the base, in malt bushels; and* $.543504 \times 65 = 35.32776$, *the content in malt bushels.*

By the Sliding Rule.

Find the area of the base by Problem X., Part IV.; then, as one on A, is to the area of the base on B; so is the depth on A, to the content on B.

Areas.

$$\begin{array}{lllll} \text{On D.} & \text{On C.} & & \text{On D.} & \text{On C.} \\ \textit{As } 1 : & \left\{\begin{array}{l} .001536 \\ .001876 \\ .000201 \end{array}\right\} & :: & 52 : & \left\{\begin{array}{l} 4.15 \textit{ ale gallons.} \\ 5.07 \textit{ wine gallons.} \\ 0.543 \textit{ malt bushels.} \end{array}\right. \end{array}$$

Note. If it be thought inconvenient to find the area of the base by the Sliding Rule, it may be found by the Pen.

Contents.

$$\begin{array}{lllll} \text{On A.} & \text{On B.} & & \text{On A.} & \text{On B.} \\ \textit{As } 1 : & \left\{\begin{array}{l} 4.15 \\ 5.07 \\ 0.543 \end{array}\right\} & :: & 65 : & \left\{\begin{array}{l} 269.96 \textit{ ale gallons.} \\ 329.55 \textit{ wine gallons.} \\ 35.33 \textit{ malt bushels.} \end{array}\right. \end{array}$$

2. The side of a pentagonal vessel measures 24.5 inches, and its perpendicular depth 57.4 inches; what is the content in ale and wine gallons and malt bushels?

Ans. 210.20598935 *ale gallons,* 256.6159988 *wine gallons, and* 27.56348 *malt bushels.*

3. The side of a hexagonal wine-vat measures 32.6 inches, and its perpendicular depth 49.7 inches; what is its content in wine gallons?

Ans. 594.057 *wine gallons.*

4. The three sides of the base of an irregular, prismatic vessel, measure 96, 125, and 159 inches respectively; and its perpendicular depth 38.4 inches; what is its content in ale gallons? *Ans.* 710.5182 *ale gallons.*

5. The base of a maltster's cistern is a trapezium, the longer diagonal of which measures 274 inches, one of the perpendiculars 112 inches, and the other 93 inches;

required its content in malt bushels, its depth being 39.4 inches? *Ans.* 514.564 *malt bushels.*

6. An octagonal, prismatic vessel contains 579.3 wine gallons; and the side of its base measures 24.6 inches; required its perpendicular depth? *Ans.* 45.797 *inches.*

PROBLEM IV.

To find the content of a vessel in the form of a cylinder.

RULE I.

By the Pen.

Multiply the square of the diameter by the perpendicular depth; divide the product by 359.05, 294.12, and 2738, and the respective quotients will be the content in ale and wine gallons, and malt bushels.

RULE II.

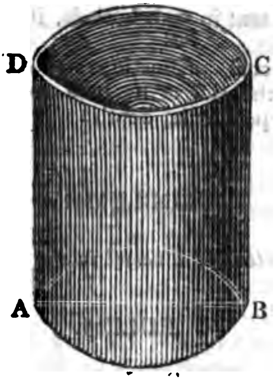
Find the area of the base in ale and wine gallons, by the Tables of ale and wine areas, Part VII.; then multiply these areas by the depth, and the respective products will be the content in ale and wine gallons. (See Prob. XIII. Part IV.)

Note 1. If the square of the diameter of a cylinder be multiplied by .7854, the product will be the area of the base in square inches; and if this area be multiplied by the depth or altitude, the product will be the content in cubic inches; which being divided by 1728, will give the content in cubic feet.

2. The depth or altitude of a cylinder may be found by dividing the content by the area of the base.

EXAMPLES.

1. Required the content of the cylindrical vessel A B C D, in ale and wine gallons, and malt bushels; the diameter of the base A B being 54.8 inches, and the depth A D or B C 62.5 inches?



By Rule I.

```
     Inches.
       54.8 diameter.
       54.8 ditto.
      -----
       4384
     2192
    2740
   --------
    3003.04 square of the diameter.
       62.5 depth.
   --------
    1501520
    600608
   1801824
Divisor 359.05)187690.000(522.740 content in ale gallons.
```

Also, 18760.000÷294.12=638.140, *the content in wine gallons; and* 18760.000÷2738=68.550, *the content in malt bushels.*

By Rule II.

By the Table of ale areas, Part VII., we find the area of the base to be 8.3638 *ale gallons; then* 8.3638 × 62.5= 522.7375, *the content in ale gallons.*

Again, by the Table of wine areas, Part VII., we find

the area of the base to be 10.2103 *wine gallons; then* $10.2103 \times 62.5 = 638.14375$, *the content in wine gallons.*

By the Sliding Rule.

As the circular gauge-point on D, is to the depth or altitude on C; so is the diameter on D, to the content on C.

$$\begin{array}{cccccccc} & \text{On D.} & & \text{On C.} & & \text{On D.} & & \text{On C.} \\ As & \begin{cases} 18.95 \\ 17.15 \\ 52.32 \end{cases} & : & 62.5 & :: & 54.8 & : & \begin{cases} 522.74 \textit{ ale gallons.} \\ 638.14 \textit{ wine gallons.} \\ 68.55 \textit{ malt bushels.} \end{cases} \end{array}$$

2. What is the content of a cylindrical guile-tun, in ale gallons; the diameter of the base being 68.6 inches, and the perpendicular depth 74 inches?

Ans. 969.895 *ale gallons.*

3. The depth of a cylindrical vessel is 57.4 inches, and its diameter 43.1 inches; what is the content in wine gallons? *Ans.* 362.527 *wine gallons.*

4. A cylindrical mash-tun measures 96.8 inches in diameter, and 84.6 inches in depth; how many bushels of malt will it contain? *Ans.* 206.094 *bushels.*

5. At Heidelberg, in Germany, is a cylindrical wine-cask, the perpendicular height of which measures 27 feet or 324 inches, and the diameter 21 feet or 252 inches; how many wine gallons will it hold?

Ans. 69955.446 *wine gallons.*

Note. This convivial monument of ancient hospitality, was formerly kept full of the best Rhenish wine, and the electors have given many entertainments on its platform; but it now only serves as a melancholy instance of the extinction of that hospitality; for it is suffered to moulder in a damp vault, quite empty.

Although this vessel is of an extraordinary magnitude; yet it is much inferior, in capacity, to many of the London porter-vats.

PROBLEM V.

To find the content of a vessel in the form of a pyramid.

RULE.

By the Pen.

Multiply the area of the top of the vessel, in square

inches, by the perpendicular depth, and $\frac{1}{3}$ of the product will be the content in cubic inches. Divide this content by 282, 231, and 2150.42; and the respective quotients will be the content in ale and wine gallons, and malt bushels.

Note 1. If the perpendicular depth will divide by 3, the content may be found by multiplying the area of the top by $\frac{1}{3}$ of the perpendicular depth.

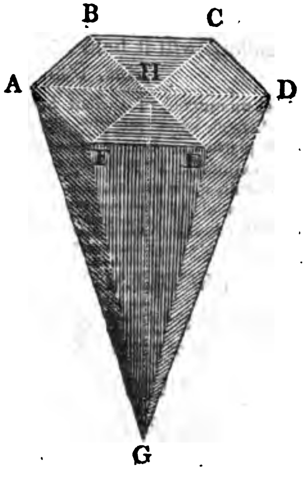
2. The area of the top may be found by Prob. IX. or X., Part IV. (See Note I., Prob. III.)

3. Vessels in the form of pyramids and cones are seldom met with in the Practice of Gauging; and it is evident, from the nature of the figures, that they must always be placed with their bases upwards, and their vertices downwards, like wine-glasses, silver-cups, &c. &c.; consequently what is called the base of a pyramid or cone, in the Mensuration of Solids, will become the top of a pyramidical or conical vessel, in Gauging.

4. If the area of the top, in ale and wine gallons, and malt bushels be multiplied by $\frac{1}{3}$ of the perpendicular depth in inches, the respective products will be the content in ale and wine gallons, and malt bushels.

EXAMPLES.

1. Required the content of the hexagonal pyramidical vessel A B C D E F G, in ale and wine gallons, and malt bushels; each side of the top measuring 46 inches, and the perpendicular depth H G, 90.3 inches?



(See Prob. X., Part IV.)

By the Rule.

Inches.

```
            46 side of the top.
            46 ditto.
           ----
           276
          184
          ----
          2116 square of the side.
      2.598076 multiplier.
      --------
         12696
        14812
      16928
     19044
    10580
    4232
   -------------
   5497.528816 area of the top in inches.
          30.1 one-third of the per. depth.
    -----------
    5497528816
  164925864480
  ---------------
Divisor 282)165475.6173616(586.792 content in ale gallons.
```

Also, 165475.6173616 ÷ 231 = 716.344, *the content in wine gallons; and* 165475.6173616 ÷ 2150.42 = 76.950, *the content in malt bushels.*

By Note 4.

```
      2116 square of the side.
   .009213 multiplier.
   -------
      6348
     2116
    4232
  19044
  ----------
  19.494708 area of the top in ale gallons.
       30.1 one-third of the per. depth.
  ----------
  19494708
 58484124
 -----------
 586.7907108 content in ale gallons.
```

Also, $2116 \times .011247 \times 30.1 = 23.798652 \times 30.1 = 716.339$, *the content in wine gallons; and* $2116 \times .001208 \times 30.1 = 2.556128 \times 30.1 = 76.939$, *the content in malt bushels.*

By the Sliding Rule.

Find the area of the top by Problem X., Part IV.: then, As one on A, is to the area of the top on B; so is one-third of the perpendicular depth on A, to the content on B.

Areas.

On D.	On C.	On D.	On C.	
As 1 :	.009213	:: 46 :	19.49	*ale gallons.*
	.011247		23.79	*wine gallons.*
	.001208		2.556	*malt bushels.*

Note. If it be thought more convenient, and accurate, in Practice, the area of the top may be found by the Pen.

Contents.

On A.	On B.	On A.	On C.	
As 1 :	19.49	:: 30.1 :	586.79	*ale gallons.*
	23.79		716.34	*wine gallons.*
	2.556		76.939	*malt bushels.*

Note. If the top of a pyramidical vessel be a square, the proportion by the Sliding Rule will be, As the square gauge-point on D, is to ⅓ of the perpendicular depth on C; so is the side of the top on D, to the content on C.

2. The perpendicular depth of a square, pyramidical vessel is 126 inches, and the side of its top 52.5 inches; what is the content in wine gallons?

Ans. 501.13 *wine gallons.*

3. The three sides of the top of a triangular, pyramidical vessel are 46, 48, and 54 inches, and its perpendicular depth 75 inches; what is the content in malt bushels?

Ans. 12.066 *malt bushels.*

PROBLEM VI.

To find the content of a vessel in the form of the frustum of a pyramid.

RULE I.

General Rule by the Pen.

To the sum of the areas of the two ends, add the square root of their product; multiply this sum by $\frac{1}{3}$ of the perpendicular depth, and the product will be the content in cubic inches. Divide this content by 282, 231, and 2150.42, respectively; and the quotients will be the content in ale and wine gallons, and malt bushels.

Note 1. The areas of the ends must be found as directed in Note 1, Prob. III.

2. The above Rule is general, whatever may be the form of the two similar ends of the frustum; that is, whether they be polygons, circles, or ellipses.

RULE II.

When the ends are regular polygons.

Add together the square of a side of each end, and the product of those sides; multiply the sum by $\frac{1}{3}$ of the depth, and this product by the tabular areas belonging to the polygon, (Prob. X., Part IV.;) and the respective products will be the content in ale and wine gallons, and malt bushels.

EXAMPLES.

1. What is the content in ale and wine gallons, and malt bushels, of the vessel A B C D E F G, in the form of the frustum of a square pyramid; the side A B of the greater end being 56 inches, the side D E of the lesser end 32 inches, and the perpendicular depth H *n*, 98 inches?

By Rule I.

Here $56 \times 56 = 3136$, *the area of the greater end*; $32 \times 32 = 1024$, *the area of the less end*; *and* $\sqrt{3136 \times 1024} = \sqrt{3211264} = 1792$, *the square root of their product*; *then* $(3136 + 1024 + 1792) \times 98 \div 3 = 5952 \times 98 \div 3 = 583296 \div 3 = 194432$, *the content in cubic inches. Hence* $194432 \div 282 = 689.476$, *the content in ale gallons*; $194432 \div 231 = 841.696$, *the content in wine gallons*; *and* $194432 \div 2150.42 = 90.415$, *the content in malt bushels.*

By Rule II.

Here $56 \times 56 = 3136$, *the square of a side of the greater end*; $32 \times 32 = 1024$, *the square of a side of the less end*; *and* $56 \times 32 = 1792$, *the product of the sides*; *then* $(3136 + 1024 + 1792) \times 98 \div 3 = 5952 \times 98 \div 3 = 583296 \div 3 = 194432$, *the content in cubic inches. Hence* $194432 \times .003546 = 689.455$, *the content in ale gallons*; $194432 \times .004329 = 841.696$, *the content in wine gallons*; *and* $194432 \times .000465 = 90.410$, *the content in malt bushels.*

By the Sliding Rule.

he areas of the two ends, and also a geometrical

mean between those areas; add these three numbers together; multiply the sum by one-third of the perpendicular depth; and the product will be the content.

Note. If the ends be squares, their areas may be found by Prob. I.; and if they be regular polygons, their areas may be obtained by Prob. IX. or X., Part IV. A geometrical mean proportional between the areas of the ends, may be found by Prob. I., Part I., or Prob. X., Part II.

Ale Gallons.

	On A.	On B.	On A.	On B.
As	282 :	56, 32 ::	56, 32 :	11.12 *bottom area.* / 3.63 *top area.*

As 11.12 *on* C : 11.12 *on* D :: 3.63 *on* C : 6.35 *on* D, *mean area.*

21.10 *sum.*

	On A.	On B.	On A.	On B.
As	1 :	21.1 ::	32.66 :	689.29 *content.*

Wine Gallons.

	On D.	On C.	On D.	On C.
As	15.19 :	1 :	56, 32 :	13.57 *bottom area.* / 4.43 *top area.*

As 13.57 *on* C : 13.57 *on* D :: 4.43 *on* C : 7.75 *mean area on* D.

25.75 *sum.*

	On A.	On B.	On A.	On B.
As	1 :	25.75 ::	32.66 :	841.0 *content.*

Note. In finding the areas of the ends, in ale gallons, the square divisor 282 is used; but in finding their areas in wine gallons, the square gauge-point 15.19 is used.

It may also be observed that the content in malt bushels may be obtained by either of the above methods.

2. What is the content of a hexagonal wine-vat; each side of the greater end being 92 inches, each side of the less end 54 inches, and the perpendicular depth 126 inches? *Ans.* 7722.370 *wine gallons.*

3. Required the content of an octagonal vessel, in ale and wine gallons, and malt bushels; each side of the

greater base being 35 inches, each side of the less be 28 inches, and the perpendicular depth 94 inches?

Ans. 1603.566 *ale gallons,* 1957.583 *wine gallons,* a 210.256 *malt bushels.*

PROBLEM VII.

To find the content of a vessel in the form of a cone.

RULE I.

By the Pen.

Multiply the square of the diameter of the top of the vessel, by $\frac{1}{3}$ of the perpendicular depth; divide the product by 359.05, 294.12, and 2738; and the respective quotients will be the content in ale and wine gallons, and malt bushels.

RULE II.

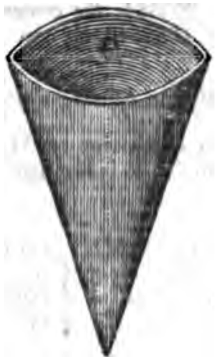
By Problem XIII., Part IV., find the area of the top of the vessel, in ale and wine gallons, and malt bushels; multiply these areas by $\frac{1}{3}$ of the perpendicular depth; and the respective products will be the content in ale and wine gallons, and malt bushels.

Note 1. If the square of the diameter of the base of a cone be multiplied by .7854, the product will be the area of the base in square inches. Multiply this area by $\frac{1}{3}$ of the cone's perpendicular height, and the product will be the content in cubic inches; which being divided by 1728, will give the content in cubic feet. (See Nesbit's Mensuration, Prob. VII., Part IV.)

2. The area of the top of a vessel in the form of an elliptical cone may be found by Prob. XVI., Part IV.; and hence the content may be obtained by Rule II.

EXAMPLES.

1. The diameter A B, of the top of a conical vessel, is 46 inches, and the perpendicular depth D C, 78 inches; what is the content in ale and wine gallons, and malt bushels?



By Rule I.

Inches.
46 *diameter of the top.*
46 *ditto.*
276
184
2116 *square of the diameter.*
26 *one-third of the depth.*
12696
4232
Divisor 359.05)55016(153.226 *ale gallons.*

ain, 55016÷294.12 = 187.052, *the content in wine* ns; *and* 55016÷2738 = 20.093, *the content in malt* els.

By Rule II.

ere 46 × 46 × .002785 = 2116 × 002785 = 5.893060, *the* of *the base in ale gallons*; 2116 × .003399 = 7.192284, rea *of the base in wine gallons*; *and* 2116 × .000365 72340, *the area of the base in malt bushels. Hence,* 3060 × 26 = 153.219560, *the content in ale gallons;*

$7.192284 \times 26 = 186.999384$, *the content in wine gallons*; and $.772340 \times 26 = 20.080840$, *the content in malt bushels.*

By the Sliding Rule.

As the circular gauge-point on D, is to ⅓ of the perpendicular depth on C; so is the diameter on D, to the content on C.

$$\begin{array}{lcccc} & \text{On D.} & \text{On C.} & \text{On D.} & \text{On C.} \\ As & \left\{\begin{array}{l} 18.95 \\ 17.15 \\ 52.32 \end{array}\right\} & : 26 :: & 46 : & \left\{\begin{array}{l} 153.21 \textit{ ale gallons.} \\ 186.99 \textit{ wine gallons.} \\ 20.08 \textit{ malt bushels.} \end{array}\right. \end{array}$$

2. The diameter of the top of a conical vessel measures 84 inches, and the length of its slant side 112 inches; what is the content in ale gallons?

Ans. 680.1115 *ale gallons.*

3. The perpendicular depth of a conical vessel is 9 inches, and the circumference of the top 216 inches; required the content in malt bushels?

Ans. 55.232 *malt bushels.*

PROBLEM VIII.

To find the content of a vessel in the form of the frustum of a cone.

RULE I.

By the Pen.

To three times the product of the top and bottom diameters, add the square of their difference; multiply the sum by the perpendicular depth; divide the product by 1077.15, 882.36, and 8214; and the respective quotients will be the content in ale and wine gallons, and malt bushels.

RULE II.

From the square of the sum of the top and bottom ımeters, subtract the product of those diameters; mul- ly the remainder by the perpendicular depth; divide : product by the foregoing divisors; and you will ob- n the content in ale and wine gallons, and malt shels.

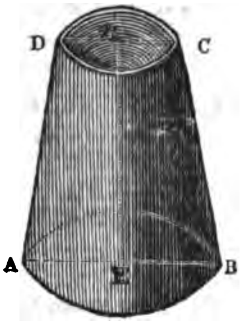
Vote 1. The General Rule given in Prob. VI., will also give the tent of a vessel in the form of the frustum of a cone, whether the s be circular or elliptical.

. When the ends are elliptical, their areas must be found by Prob. 'I., Part IV.

. The equal frustums of two similar cones, joined together at their ater ends, form a figure, which, by Gaugers, is called a cask of the rth variety. This variety, however, is seldom, or perhaps never met h in Practice.

EXAMPLES.

1. What is the content in ale and wine gallons, and ılt bushels, of the vessel A B C D, in the form of the ıstum of a cone; the diameter A B of the bottom being inches, the diameter C D of the top 40 inches, and the rpendicular depth E = 86 inches?



By Rule I.

Inches.
72 *the bottom diameter.*
40 *the top diameter.*
32 *difference.*
32 *ditto.*
64
96
1024 *square of the difference.*

Inches.
72 *bottom diameter.*
40 *top diameter.*
2880 *their product.*
3
8640 *three times their product.*
1024 *square of the difference.*
9664 *sum.*
86 *perpendicular depth.*
57984
77312
Divisor 1077.15)831104(771.576 *ale gallons.*

Also, 831104÷882.36=941.910, *the content in wine gallons; and* 831104÷8214=101.181, *the content in malt bushels.*

By Rule II.

Inches.
72 *bottom diameter.*
40 *top diameter.*
2880 *product.*

Inches.

```
    72 bottom diameter.
    40 top diameter.
   ---
   112 sum.
   112 sum.
   ---
   224
  112
 112
 -----
 12544 square of their sum.
  2880 product of the diameters.
 -----
  9664 difference.
    86 perpendicular depth.
 ------
  57984
 77312
 ------
 831104 the same as before.
```

Hence we obtain the contents by division, as in the last e.

By the Sliding Rule.

By Problem X., Part II., find a mean diameter between two given diameters; then find areas from these three meters, by Problem XIII., Part IV. Multiply the of these areas by ⅓ of the depth; and the product be the content.

Ale Gallons.

```
C.   On D.   On C.   On D.
72  :  72  ::  40  :  53.66 mean diameter.

  On D.   On C.   On D.      On C.
                ⎧ 72.00 ⎫  ⎧ 14.44 bottom area.
As 18.95 : 1 :: ⎨ 53.66 ⎬ :⎨  8.02 mean area.
                ⎩ 40.00 ⎭  ⎩  4.45 top area.
                             -----
                             26.91 sum.

n A.    On B.     On A.     On B.
 1   :  26.91  ::  28.66  :  771.45 content.
```

Wine Gallons.

$$\begin{array}{ccccccc} & \text{On D.} & & \text{On C.} & & \text{On D.} & & \text{On C.} \\ \textit{As} & 17.15 & : & 1 & :: & \left\{\begin{array}{c} 72.00 \\ 53.66 \\ 40.00 \end{array}\right\} & : & \left\{\begin{array}{l} 17.63 \textit{ bottom area.} \\ \ \ 9.79 \textit{ mean area.} \\ \ \ 5.44 \textit{ top area.} \end{array}\right. \\ & & & & & & & \overline{32.86} \textit{ sum.} \end{array}$$

$$\begin{array}{cccccccc} & \text{On A.} & & \text{On B.} & & \text{On A.} & & \text{On B.} \\ \textit{As} & 1 & : & 32.86 & :: & 28.66 & : & 941.98 \textit{ content.} \end{array}$$

Note. The content in malt bushels, may be found by a similar process.

2. The top diameter of a guile-tun is 60 inches, the bottom diameter 40 inches, and the perpendicular depth 74 inches; what is the content in ale gallons?

Ans. 522.118 *ale gallons.*

3. What is the content, in malt bushels, of a vessel whose top diameter is 40 inches, bottom diameter 36 inches, and depth 52 inches? *Ans.* 27.449 *malt bushels.*

PROBLEM IX.

To find the content of a vessel in the form of a prismoid.

RULE.

By the Pen.

To the sum of the areas of the two ends, add four times the area of a section parallel to, and equally distant from both ends; multiply this sum by the perpendicular depth, and $\frac{1}{6}$ of the product will be the content in cubic inches. Divide this content by 282 for ale gallons, 231 for wine gallons, and 2150.42 for malt bushels.

Note 1. The length of the middle section is equal to half the sum of the lengths of the two ends; and its breadth is equal to half the sum of their breadths.

2. If the ends be elliptical, the transverse diameter of the middle section will be equal to half the sum of the transverse diameters of the two ends; and the conjugate diameter equal to half the sum of the conjugate diameters of the two ends.

3. If one end be an ellipse and the other a circle, add the transverse diameter of the elliptical end to the diameter of the circular end; and

e half the sum for the transverse diameter of the middle section. e conjugate diameter of the middle section must be found in a simi- manner.

. If the square of the diameter of a circle be multiplied by .7854, product will be the area in square inches; and if the rectangle of two diameters of an ellipse be multiplied by .7854, the product be the area in square inches. (See Problems XIII. and XVI., t IV.)

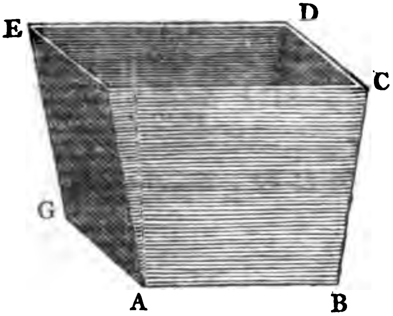
SCHOLIUM.

The Rule given in this Problem for a prismoid, is very irly demonstrated in Simpson's Fluxions, page 178, t edition; and also in a similar manner at page 302, of lliday's Fluxions.

There is likewise a very elegant demonstration given Proposition III., Section I., Part IV., of Dr. Hutton's nsuration, third edition, in which it is shewn to be e for all frustums whatever, and for all solids whose allel sections are similar figures; and Mr. Fletcher, he second edition of his Universal Measurer, Part III., e 54, and Mr. Moss, in his Gauging, page 175, third ion, say, that it is *nearly* true for any other solid, atever may be its form.

EXAMPLES.

. Required the content, in ale and wine gallons, and t bushels, of the prismoidal vessel A B C D E F G; length A B of its bottom being 40, and its breadth ; 21 inches; the length C F of its top 52, and its adth F E 25 inches, and the perpendicular depth A n inches?



Here $40\times21=840$, *the area of the bottom; and* $52\times25=1300$, *the area of the top of the vessel.*

Also $\frac{40+52}{2}=\frac{92}{2}=46$, *the length of the middle section; and* $\frac{21+25}{2}=\frac{46}{2}=23$, *the breadth of the middle section; then* $46\times23\times4=1058\times4=4232$, *four times the area of the middle section; whence* $(840+1300+4232)\times\frac{60}{6}=6372\times10=63720$, *the content in cubic inches; and* $63720\div282=225.957$, *the content in ale gallons;* $63720\div231=275.844$, *the content in wine gallons; and* $63720\div2150.42=29.631$, *the content in malt bushels.*

By the Sliding Rule.

By Problem II., Part IV., find the areas of the two ends; and also the area of the middle section. Add the areas of the two ends, and four times the area of the middle section, together; then say, as six on A, is to the sum of these areas on B; so is the perpendicular depth on A, to the content on B.

Ale Gallons.

	On A.	On B.		On A.		On B.	
As	282 :	46	::	23	:	3.75	*area of the middle section.*
						4	*multiply by four.*
						15.00	*four times the area of ditto.*
As	282 :	52	::	25	:	4.61	*area of the top.*
		40		21		2.98	*area of the bottom.*
						22.59	*sum.*

As 6 : 22.59 :: 60 : 225.90 *content.*

Wine Gallons.

On A. On B. On A. On B.
231 : 46 :: 23 : 4.58 *area of the middle section.*
4 *multiply by four.*

18.32 *four times the area of ditto.*

$$231 : \begin{Bmatrix} 52 \\ 40 \end{Bmatrix} :: \begin{Bmatrix} 25 \\ 21 \end{Bmatrix} : \begin{cases} 5.63 & \textit{area of the top.} \\ 3.64 & \textit{area of the bottom.} \end{cases}$$

27.59 *sum.*

ls 6 : 27.59 :: 60 : 275.90 *content.*

'ote. The content in malt bushels may be found by a similar ress.

!. The length and breadth of the bottom of a vessel he form of a prismoid, are 42.5 and 38.4; the length l breadth of the top 54.6 and 42.2; and the depth 54 hes; what is the content in ale gallons?

Ans. 375.395 *ale gallons.*

3. The length and breadth of the top of a cistern, in form of a prismoid, measure 70 and 56; the length l breadth of the bottom 66 and 53.8; and the depth inches; what is the content in malt bushels?

Ans. 83.345 *malt bushels.*

l. The perpendicular depth of a vessel, in the form of frustum of an elliptical cone, is 46.8 inches; the nsverse and conjugate diameters of the bottom meae 49.6 and 37.8 inches; the transverse and conjugate meters of the top, 67.2 and 50.4 inches; required the tent in ale and wine gallons?

lns. The content is 338.099 *ale gallons, and* 412.744 *e gallons.*

5. The perpendicular depth of a vessel with an ellipl base and a circular top, generally called a cylindroid, 52.6 inches; the transverse and conjugate diameters of bottom measure 61.6 and 46.2 inches; and the diater of the top measures 42.6 inches; required the cont in ale and wine gallons?

lns. The content is 339.716 *ale gallons, and* 414.719 *wine lons.*

Remarks.

It is evident that the middle section of the cylindroid, in the last question, cannot be a *true* ellipse, because the top of the vessel is a circle; consequently, the area of the middle section cannot be correctly found by the Rule for the ellipse.

Now, as we have not the *true* area of the middle section, the content of the vessel, found by the Rule given in this Problem, is not *mathematically* correct; although it is near enough the truth for all practical purposes.

If the true areas of all the horizontal sections of *any vessel* can be obtained, either by equi-distant ordinates, as directed in Problem XX., Part IV., or by any other method; then the Rule given in this Problem, will give *very nearly* the true content of the vessel. (See the foregoing Scholium.)

PROBLEM X.

To find the content of a vessel in the form of a sphere or globe.

RULE I.

By the Pen.

Multiply the cube of the diameter by .5236, and the product will be the content in cubic inches; which divide by 282, 231, and 2150.42, and the respective quotients will be the content in ale and wine gallons, and malt bushels.

RULE II.

Multiply the cube of the diameter by .0018567, .00227, and .000243, or divide it by 538.58, 441.18, and 4107; and the respective products or quotients will be the content in ale and wine gallons, and malt bushels.

Note 1. As every sphere is equal to two-thirds of its circumscribing

inder, it is evident that $\frac{2}{3}$ of the circular multipliers will be the ltipliers for a sphere, as given in the last Rule.

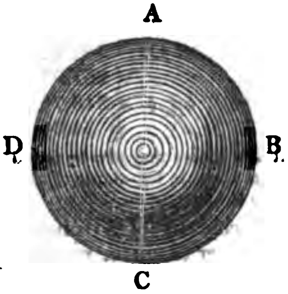
The divisors for a sphere must also be in the same ratio, but in-sely; hence $\frac{3}{2}$ of the circular divisors will be the divisors for a ere, as given in the same Rule.

?. The gauge-points for a sphere are obtained by extracting the are roots of the divisors; and are found to be 23.2 for ale, 21.0 for e gallons, and 64.1 for malt bushels.

. Vessels in the form of a sphere or globe are seldom met with in ctice; and when they are, it is evident they must have bung-holes casks.

EXAMPLES.

. What is the content of the spherical vessel A B C D, le and wine gallons, and malt bushels; if the inner meter A C, measure 52 inches?



By Rule I.

'ere $52^3 = 52 \times 52 \times 52 = 140608$, *the cube of the dia-r; and* $140608 \times .5236 = 73622.3488$, *the content in inches; then* $\frac{73622.3488}{282} = 261.072$, *the content in ale*

ns; $\frac{73622.3488}{231} = 318.711$, *the content in wine gal-*

; and $\frac{73622.344}{2150.42} = 34.236$, *the content in malt bushels.*

S 2

By Rule II.

By Multiplication.

Here 140608×.0018567=261.0668736, *the content* i *ale gallons;* 140608×.00227=319.18016, *the content* i *wine gallons; and* 140608×.000243=34.167744, *the con tent in malt bushels.*

By Division.

Here 140608÷538.58=261.071, *the content in ale gal lons;* 140608÷441.18=318.708, *the content in wine gal lons; and* 140608÷4107=34.236, *the content in mal bushels.*

By the Sliding Rule.

As the gauge-point on D, is to the diameter on C; s is the diameter on D, to the content on C. (See Note 2.)

$$\begin{array}{cccccccc} & \text{On D.} & & \text{On C.} & & \text{On D.} & & \text{On C.} \\ \textit{As} & \left\{\begin{array}{l} 23.2 \\ 21.0 \\ 64.1 \end{array}\right\} & : & 52 & :: & 52 & : & \left\{\begin{array}{l} 261.07 \textit{ ale gallons.} \\ 318.71 \textit{ wine gallons.} \\ 34.24 \textit{ malt bushels.} \end{array}\right. \end{array}$$

Note. The content of a sphere may also be found by saying, As th circular gauge-point on D, is to two-thirds of the diameter on C; s is the diameter on D, to the content on C.

2. Required the content of a spherical vessel, in al gallons; when the inner diameter measures 116 inches?

Ans. 2898.174 *ale gallons*

3. The inner diameter of a spherical vessel is 16.1 in ches; what is the content in malt bushels?

Ans. 1.014 *malt bushels*

PROBLEM XI.

To find the content of a vessel in the form of the seg ment of a sphere.

RULE I.

By the Pen.

To three times the square of the radius of the top of

the vessel, add the square of its depth; multiply this sum by the depth; and the product again by .5236, and you will obtain the content in cubic inches. Divide this content by 282, 231, and 2150.42; and the respective quotients will be the content in ale and wine gallons, and malt bushels.

Note 1. A vessel in the form of the segment of a sphere may be placed with the convex part either upwards or downwards. If the convex part be placed upwards, the vessel must evidently have a bung-hole at the top; but if the vessel be placed with the convex part downwards, it may be open at the top, as in the following figure. (See Problems V., VII., and XVII.)

2. When a vessel in the form of the segment of a sphere, is placed with its convex part upwards; it is evident that the part which is denominated the top of the vessel, in the above Rule, will become the base.

RULE II.

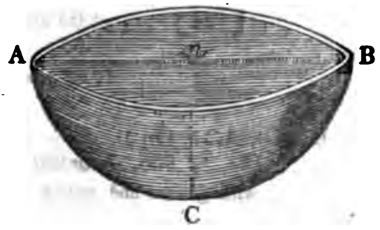
To the square of the diameter of the top of the vessel, add the square of the depth, and one-third of the same; this sum being multiplied by half the depth, and the product divided by 359.05, 294.12, and 2738, will give the content in ale and wine gallons, and malt bushels.

Note 1. Both the foregoing Rules are true when the segment is greater than a semi-sphere.

2. The altitude of the remaining segment may be found by dividing the square of A *n* by C *n*. (See the next figure; and also Theorems IX. and XII., Part III.)

EXAMPLES.

1. The diameter A B, of the top of a vessel, in the form of the segment of a sphere, measures 120 inches, and the perpendicular depth C *n*, 24 inches; what is the content in ale gallons?



By Rule I.

Here $60^2 \times 3 = 360 \times 3 = 10800$, *three times the square half the diameter of the top; also,* $24^2 = 576$, *the squa of the depth; then* $(10800 + 576) \times 24 \times .5236 = 11376$ $24 \times .5236 = 273024 \times .5236 = 142955.3664$, *the content cubic inches; hence* $142955.3664 \div 282 = 506.933$, *the co tent in ale gallons.*

By Rule II.

Here $120 \times 120 = 14400$, *the square of the diameter the top; and* $24 \times 24 = 576$, *the square of the depth; al* $\frac{576}{3} = 192$, *one-third of the square of the depth; the* $(14400 + 576 + 192) \times 12 = 15168 \times 12 = 182016$; *an* $182016 \div 359.05 = 506.937$, *the content in ale gallons.*

By the Sliding Rule.

By Problem XIII., Part IV., find the area of the segment's base or top; and also the area of a circle equal in diameter to the segment's height. To the sum of these two areas add one-third of the latter area; multiply the sum thus obtained by one-half of the height; and the product will be the content.

Note. This method of finding the content by the Sliding Rule, is founded on Rule II.

	On D.		On C.		On D.		On C.
As	18.95	:	1	::	120	:	40.10 *area of the top.*
					24		1.60 *ditto of a circle.*
							.53 *one-third of ditto.*
							42.23 *sum in ale gallons.*

	On A.		On B.		On A.		On B.
As	1	:	42.23	::	12	:	506.76 *content in ale gallons.*

Note. The content in wine gallons and malt bushels, may be obtained by a similar process.

2. The diameter of the top of a vessel, in the form of the segment of a sphere, measures 48 inches, and its depth 21 inches; what is the content in wine gallons?

Ans. 103.244 *wine gallons.*

3. The altitude of a segment of a sphere is 15 inches, and the diameter of the base 38 inches; what is the content in malt bushels? *Ans.* 4.777 *malt bushels.*

PROBLEM XII.

To find the content of a vessel in the form of the frustum or zone of a sphere.

RULE.

By the Pen.

To half the sum of the squares of the two diameters, add $\frac{2}{3}$ of the square of the depth of the vessel; multiply the sum by the depth; and divide the product by 359.05 for ale gallons, 294.12 for wine gallons, and 2738 for malt bushels.

EXAMPLES.

1. What is the content, in ale gallons, of the vessel ABCD, in the form of the frustum of a sphere; the bottom diameter AB being 42 inches, the top diameter CD 36 inches; and the perpendicular depth $m\,n$ 30 inches?

Here $\frac{42^2+36^2}{2}=\frac{1764+1296}{2}=\frac{3060}{2}=1530$, *half the sum of the squares of the two diameters; and* $30^2\times\frac{2}{3}=900\times\frac{2}{3}=\frac{1800}{3}=600$, *two-thirds of the square of the depth then* $1530+600\times30=2130\times30=63900$; *and* $63900\div359.05=177.969$, *the content in ale gallons.*

By the Sliding Rule.

By Problem VIII., Part II., find the squares of the diameters of the two ends, and also the square of the altitude. To half the sum of the squares of the diameters, add two-thirds of the square of the altitude; then say, As the circular divisor on A, is to this sum on B; so is the altitude of the zone on A, to the content on B.

Note. This method of obtaining the content by the Sliding Rule, is founded on the preceding Rule, given for the Pen.

On D.	On C.	On D.	On C.	
As 1 :	1 ::	42 :	1764	*the sq. of the greater diameter.*
		36	1296	*the sq. of the less ditto.*
			2)3060	*sum.*
			1530	*half the sum.*

On D.	On C.	On D.	On C.	
As 1 :	1 ::	30 :	900	*the square of the altitude.*
			2	
			3)1800	
			600	*two-thirds of ditto.*
Add.....................			1530	*brought down.*
			2130	*sum.*

On A.	On B.	On A.	On B.	
As 359.05 :	2130 ::	30 :	177.92	*the content in ale gallons.*

Note The content in wine gallons and malt bushels, may be found by a similar process.

2. What is the content, in wine gallons, of a vessel in the form of the frustum of a sphere; the greater diameter being 52 inches, the less diameter 30 inches, and the depth 18 inches? *Ans.* 123.500 *wine gallons.*

3. The top and bottom diameters of the frustum of a sphere are 24 and 25 inches, and the altitude 20 inches; what is the content in malt bushels?

Ans. 6.334 *malt bushels.*

PROBLEM XIII.

To find the content of a vessel in the form of a prolate spheroid.

RULE.

By the Pen.

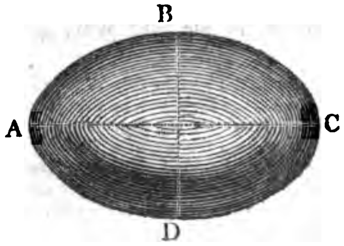
Multiply the square of the conjugate diameter by the transverse diameter; and divide the product by 538.58 for ale gallons, 441.18 for wine gallons, and 4107 for malt bushels.

Note 1. A spheroid is equal to $\frac{2}{3}$ of its circumscribing cylinder; hence the same multipliers, divisors, and gauge-points, are used for the spheroid that are used for the sphere. (See Problem X.)

2. Vessels in the form of an oblate spheroid are never met with in Gauging.

EXAMPLES.

1. What is the content of the spheroidal vessel A B C D, in ale gallons; the transverse diameter A C being 54 inches, and the conjugate diameter B D 33 inches?



Here $33 \times 33 \times 54 = 1089 \times 54 = 58806$; *then* $\frac{5880}{538.5}$ $= 109.187$, *the content in ale gallons.*

By the Sliding Rule.

As the gauge-point on D, is to the transverse diamet on C; so is the conjugate diameter on D, to the con tent on C.

On D. On C. On D. On C.
As 23.19 : 54 :: 33 : 109.19 *the content in ale gallons.*

Note 1. The content in wine gallons and malt bushels may b found in a similar manner.

2. The content of a spheroid may also be found by saying, As th circular gauge-point on D, is to two-thirds of the transverse diamete on C; so is the conjugate diameter on D, to the content on C.

2. Required the content, in wine gallons, of a spheroid whose transverse diameter is 72.5, and conjugate 56 in ches? *Ans.* 515.345 *wine gallons*

3. If the transverse diameter of a vessel, in the form o a spheroid, be 48 inches, and the conjugate 42 inches what is the content in malt bushels?

Ans. 20.616 *malt bushels*

REMARK.

The content of a vessel in the form of a prolate semi-spheroid, may be found by the following Rule: Multiply the square of the diameter of the base by the perpendicular height; divide the product by 538.58, 441.18, and 4107; and the respective quotients will be the content of the vessel, in ale and wine gallons, and malt bushels.

Note. Any diameter parallel to the base of a vessel, in the form of a prolate semi-spheroid, may be found by the following Rule: To the altitude of the given vessel, add the distance between its base and the required diameter; and from the altitude subtract the said distance; multiply the sum by the remainder, and extract the square root of the product. Then say, as twice the altitude of the given vessel, is to the diameter of its base; so is twice the above root, to the diameter required.

PROBLEM XIV.

nd the content of a vessel in the form of the middle frustum of a prolate spheroid.

RULE.

By the Pen.

twice the square of the middle diameter add the : of the diameter of the end; multiply the sum e length of the frustum; divide the product by .5, and 892.36; and the respective quotients will be ntent in ale and wine gallons.

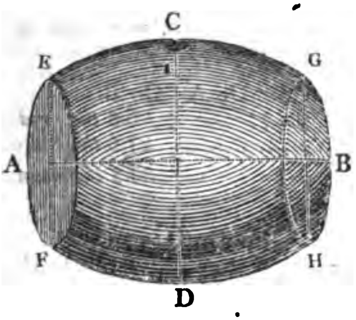
1. If the content be required in malt bushels, the divisor will

1e above divisors are obtained by multiplying 359.05, 294.12, 38, the circular divisors, for ale and wine gallons, and malt by three.

cask in the form of the middle frustum of a prolate spheroid augers, called a cask of the *first variety*.

EXAMPLES.

What is the content in ale and wine gallons, of the E G H F, in the form of the middle frustum of a : spheroid; the middle diameter C D being 36, imeter of the end E F or G H 27, and the length) inches?



Middle diameter.

$$\begin{array}{rl}
 & \textit{Inches.} \\
 & 36 \\
 & \underline{36} \\
 & 216 \\
 & \underline{108} \\
 & 1296 \textit{ square.} \\
 & \underline{2} \\
 & 2592 \textit{ twice the sq. of the middle diam.} \\
27 \times 27 = & \underline{729} \textit{ the square of the end diameter.} \\
 & 3321 \textit{ sum.} \\
 & \underline{40} \textit{ the length.} \\
\end{array}$$

Divisor 1077.15)132840(123.325, *the content in ale gall.*

Also, 132840 ÷ 882.36 = 150.550, *the content in wine gallons.*

By the Sliding Rule.

Multiply the difference between the middle and end diameters by .68, when that difference is less than 6 inches, but by .7, when it exceeds 6 inches; add the product to the less diameter, and the sum will be the mean diameter; then say, as the circular gauge-point on D, is to the length on C; so is the mean diameter on D, to the content on C.

Note. The multipliers given in this Problem, and some of the following Problems, are the same that are used in Cask Gauging, in order to reduce casks of different varieties, to cylinders of equal capacities, by finding mean diameters.

	On A.		On B.		On A.		On B.
As	1	:	9	::	.7	:	6.3 *the product.*
Add......................							27.0 *the end diameter.*
							33.3 *the mean diameter.*

	On D.		On C.		On D.		On C.
As	18.95 / 17.15	:	40	::	33.3	:	123.32 *ale gallons.* / 150.55 *wine gallons.*

Required the content in ale gallons, of a vessel in form of the middle frustum of a prolate spheroid; liameter of each end being 20, the middle diameter and the perpendicular depth 25 inches?

Ans. 36.020 *ale gallons.*

Suppose the head diameter of a spheroidal cask be the bung diameter 35, and the length 38 inches; is the content of the cask in wine gallons?

Ans. 144.272 *wine gallons.*

e. This question is also given in Problem XIX., where the con- found to be the same as above.

The diameter of each end of a vessel in the form of ustum of a prolate spheroid, is 40 inches, the mid- liameter 52 inches and the depth 58 inches; what content in malt bushels? *Ans.* 49.484 *malt bushels.*

REMARK.

e content of a vessel in the form of half the middle im of a prolate spheroid, may be found by the fol- g Rule: To twice the square of the greater diame- dd the square of the less diameter; multiply the by the perpendicular depth; divide the product by 15, 882.36, and 8214; and the respective quotients be the content in ale and wine gallons, and malt ls.

may also be observed, that if the content thus found, btracted from the content of the semi-spheroid, the nder will be the content of the segment of the spheroid, cut off by the less diameter of half the mid- rustum. (See the Remark at the end of the last em.)

PROBLEM XV.

nd the content of a vessel in the form of a para- bolic spindle.

RULE.

By the Pen.

ltiply the square of the middle diameter by the

length of the spindle; and that product being divided by 673.22, 551.48, and 5133.75, will give the content in ale and wine gallons, and malt bushels.

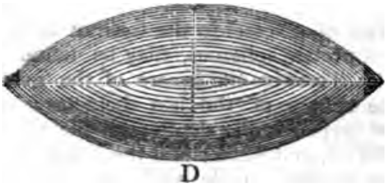
Note 1. A parabolic spindle is equal to $\frac{8}{15}$ of its circumscribing cylinder; hence the above divisors are obtained by multiplying the circular divisors 359.05, 294.12, and 2738, by 15, and dividing the products by 8; or by multiplying the circular divisors by 1.875, which is equal to $\frac{15}{8}$.

2. The gauge-points are the square roots of the divisors; and are found to be 25.9 for ale gallons, 23.5 for wine gallons, and 71.65 for malt bushels.

EXAMPLES.

1. The length A B, of a vessel in the form of a parabolic spindle, is 70, and the middle diameter C D 28 inches; what is the content in ale gallons?

C

A  B

D

Here 28 × 28 = 784, *the square of the middle diameter; and* 784 × 70 = 54880, *the square of the diameter multiplied by the length; then* 54880 ÷ 673.22 = 81.518, *the content in ale gallons.*

By the Sliding Rule.

As the gauge-point on D, is to the length on C; so is the middle diameter on D, to the content on C.

On D.		On C.		On D.		On C.
As 25.9	:	70	::	28	:	81.52 *ale gallons.*

2. The length of a vessel in the form of a parabolic spindle is 65 inches, and its middle diameter 24 inches; what is its content in wine gallons?

Ans. 67.890 *wine gallons.*

3. The length of a vessel in the form of a parabolic spindle is 150 inches, and its middle diameter 56 inches; what is its content in malt bushels?

Ans. 91.628 *malt bushels.*

PROBLEM XVI.

To find the content of a vessel in the form of the middle frustum of a parabolic spindle.

RULE.

By the Pen.

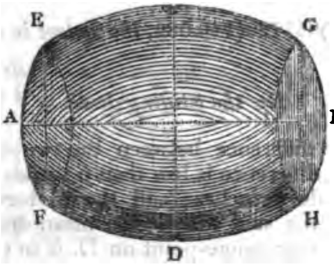
Add 8 times the square of the middle diameter, 3 times the square of the end diameter, and 4 times the product of those diameters into one sum. Multiply this sum by the length; divide the product by 5385.75, 4411.80, and 41070; and the respective quotients will be the content in ale and wine gallons, and malt bushels.

Note 1. A cask in the form of the middle frustum of a parabolic spindle, is called a cask of the *second variety.*

2. The foregoing divisors are found by multiplying the circular divisors by fifteen.

EXAMPLES.

1. The length A B of the vessel E G H F, in the for of the middle frustum of a parabolic spindle, measures 40, the middle diameter C D 36, and the end diameter E F, or G H, 27 inches; what is the content in ale and wine gallons?



Middle Diameter.	*End Diameter.*
Inches.	*Inches.*
36	27
36	27
216	189
108	54
1296 *square.*	729 *square.*
8	3
10368 *eight times the square.*	2187 *three times the square.*

The two Diameters.
Inches.

```
 36
 27
252
72
972 product.
  4
3888 four times the square.
```

```
10368 eight times the square.
 2187 three times the square.
 3888 four times the product.
16443 sum.
   40 the length.
```

Divisor 5385.75)657720(122.122, *the content in ale gallons.*

Divisor 4411.8)657720(149.082, *the content in wine gallons.*

By the Sliding Rule.

Multiply the difference between the two diameters by .62, when that difference is less than 6 inches, but by .64, when it exceeds 6 inches; add the product to the less diameter, and the sum will be the mean diameter; then say, As the circular gauge-point on D, is to the length on C; so is the mean diameter on D, to the content on C.

On A.		On B.		On A.		On B.
As 1	:	9	::	.64	:	5.76 *the product.*
Add						27.0 *the less diameter.*
						32.76 *the mean diameter.*

	On D.		On C.		On D.		On C.
As	18.95	:	40	::	32.76	:	120.0 *ale gallons.*
	17.15						146.5 *wine gallons.*

Note. Here the contents found by the Rule are two ale gallons, and two and a half wine gallons less than those found by the Pen. These differences arise from the mean diameter being rather too small.

2. The head diameter of a cask, in the form of the middle frustum of a parabolic spindle, is 22 inches, the bung diameter 27 inches, and the length 30 inches; what is the content in ale and wine gallons?

Ans. 58.808 *ale gallons, and* 65.688 *wine gallons.*

3. The top diameter, and also the bottom diameter of a vessel in the form of the middle frustum of a parabolic spindle, measures 20 inches, the middle diameter 25 inches, and the depth 27 inches; what is the content in malt bushels? *Ans.* 5.390 *malt bushels.*

PROBLEM XVII.

To find the content of a vessel in the form of a parabolic conoid.

RULE.

By the Pen.

Multiply the square of the diameter of the top of the vessel by half the depth; and divide the product by 359.05 for ale gallons, 294.12 for wine gallons, and 2738 for malt bushels.

Note 1. The above Rule will also give the content of the segment of a paraboloid.

2. A paraboloid is equal to half its circumscribing cylinder.

3. A vessel in the form of a paraboloid, may be placed with its vertex either upwards or downwards. If placed with its vertex upwards, the square of the diameter of the base must be multiplied by

half the perpendicular height; and the product divided by the respectiv divisors, in order to obtain the content. (See Problems V., VII and XI.)

4. Any diameter parallel to the base of a vessel, in the form of paraboloid, may be found by the following Rule: From the perpen dicular altitude of the given vessel, subtract the distance between i base and the required diameter; and reserve the remainder. The say, as the perpendicular altitude of the given vessel, is to the squa of the diameter of its base; so is the the reserved remainder, to th square of the required diameter. Hence the square root of this num ber will be the diameter sought.

EXAMPLES.

1. The diameter A B of the top of a vessel, in the forn of a parabolic conoid, measures 82, and the depth C m 96 inches; what is the content in ale and wine gallons?

A  B

C

Inches.

82 *diameter of the top.*
82 *ditto.*
———
164
656
———
6724 *square of the diameter.*
48 *half the depth.*
———
53792
26896
———
Divisor 259.05)322752(898.905 *ale gallons.*

Divisor 294.12)322752(1097.348 *wine gallons.*

By the Sliding Rule.

As the circular gauge-point on D, is to half the depth on C; so is the diameter of the top of the vessel on D, to the content on C.

$$\begin{array}{lccccc} & \text{On D.} & \text{On C.} & \text{On D.} & \text{On C.} \\ As & \left\{\begin{array}{l} 18.95 \\ 17.15 \end{array}\right\} : & 48 & :: \; 82 \; : & \left\{\begin{array}{l} 898.91 \textit{ ale gallons.} \\ 1097.35 \textit{ wine gallons.} \end{array}\right. \end{array}$$

2. The altitude of a vessel, in the form of a parabolic conoid, is 42 inches, and the diameter of the base 24 inches; what is the content in ale and wine gallons?

Ans. 33.688 *ale gallons, and* 41.126 *wine gallons.*

3. Required the content in malt bushels, of a vessel in the form of the segment of a paraboloid, the depth of which is 18 inches, and the diameter of the top 24 inches? *Ans.* 1.893 *malt bushels.*

REMARK.

Vessels in the form of a hyperbolic conoid are seldom met with in Practice; if, however, they should occur, their contents may be found by the following Rule: To the square of the radius of the base add the square of the diameter taken in the middle between the base and the vertex; then this sum being multiplied by the depth, and the product thus obtained by .5236, will give the content in cubic inches. (See Nesbit's Mensuration, Prob. XIX., Part VII.)

PROBLEM XVIII.

To find the content of a vessel in the form of the frustum of a parabolic conoid.

RULE.

By the Pen.

Multiply the sum of the squares of the top and bottom diameters by the depth of the vessel; divide the product by 718.10, 588.24, and 5476; and the respective quo-

tients will be the content in ale and wine gallons, and malt bushels.

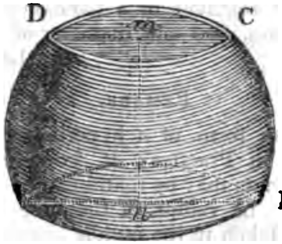
Note 1. The foregoing divisors are obtained by multiplying the circular divisors by two.

2. If the frustums of two equal paraboloids be joined together, at their greater ends, they will form a figure which, by Gaugers, is called a cask of the *third variety*.

☞ *The cut on this page should have appeared on page* 199; *and that on page* 199 *should have been inserted on this page; the mistake, however, was not discovered until page* 199 *was printed.*

EXAMPLES.

1. Required the content of the parabolic frustum A B C D, in ale and wine gallons, and malt bushels; the diameter A B of the bottom being 40 inches, the diameter D C of the top 30 inches, and the perpendicular depth $m\,n$ 18 inches?



The top diameter.	*The bottom diameter.*
Inches.	*Inches.*
30	40
30	40
900 *square.*	1600 *square.*
	900 *square.*
	2500 *sum.*
	18 *depth.*
	20000
	2500

Divisor 718.1)45000(62.665 *ale gallons.*

Also, 45000÷588.24=76.499, *the content in wine gallons;* and 45000÷5476=8.217, *the content in malt bushels.*

By the Sliding Rule.

Multiply the difference of the two diameters by .55,

n that difference is less than 6 inches, but by .57, n it exceeds 6 inches; add the product to the less neter, and the sum will be the mean diameter; then As the circular gauge-point on D, is to the depth on so is the mean diameter on D, to the content on C.

	On A.		On B.		On A.		On B.
As	1	:	10	::	.57	:	5.7 *the product.*
Add	....						30.0 *the less diameter.*
							35.7 *the mean diameter.*

	On D.		On C.		On D.		On C.
As	18.95 17.15 52.32	:	18	::	35.7	:	63.8 *ale gallons.* 78.0 *wine gallons.* 8.4 *malt bushels.*

e. Here the contents found by the Rule exceed those found by en, in consequence of the mean diameter being a little too

What is the content in ale and wine gallons, of a -tun, in the form of the frustum of a paraboloid; the eter of the greater end being 100 inches, that of the nd 50 inches, and the depth 56 inches?

Ans. 974.794 *ale gallons, and* 1189.990 *wine gallons.*

The head diameter of a cask in the form of two frustums of a parabolic conoid is 21 inches, the diameter 28 inches, and the length 36 inches; is the content in ale and wine gallons?

Ans. 61.412 *ale gallons, and* 74.969 *wine gallons.*

Remark.

a vessel in the form of the frustum of a hyperbolic l should be met with, its content may be found by ollowing Rule: To the sum of the squares of the est and least semi-diameters, add the square of the e diameter; then this sum being multiplied by the , and the product thence arising by .5236, will give ntent in cubic inches. (See Nesbit's Mensuration, XX., Part VII.)

PROBLEM XIX.

To find the content of a circular vessel when its sides are a little curved.

RULE.

By the Pen.

To the sum of the squares of the top and bottom diameters, add four times the square of the middle diameter; multiply this sum by the depth; divide the product by 2154.8, 1764.72, and 16428; and the respective quotients will be the content in ale and wine gallons, and malt bushels.

Note 1. The foregoing divisors are found by multiplying the circular divisors by six.

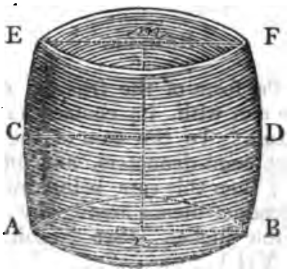
2. As vessels of this kind are seldom perfectly round, it is best to measure two diameters at right angles to each other; and take half their sum for a mean diameter.

3. The Rule given in this Problem is the same, in substance, as that given in Problem IX. See the Scholium in that Problem.

4. This Problem, and some of the following Problems, cannot be easily solved by the Sliding Rule.

EXAMPLES.

1. The diameter A B, of a circular vessel, measures 68, the diameter C D 70, the diameter E F 68, and the depth *m n* 58 inches; what is the content in ale and wine gallons, and malt bushels?



Bottom diameter.	*Top diameter.*	*Middle diameter.*
Inches.	*Inches.*	*Inches.*
63	68	70
63	68	70
189	544	4900 *square.*
378	408	4
3969 *square.*	4624 *square.*	19600 *sq. multiplied by* 4.

4624 *square.*
3969 *square.*
19600 *square multiplied by* 4.

28193 *sum.*
58 *depth.*

225544
140965

Divisor 2154.3)1635194(759.037 *ale gallons.*

Also, 1635194÷1764.72=926.602, *the content in wine gallons; and* 1635194÷16428=99.537, *the content in malt bushels.*

2. The bottom diameter of a vessel is 84, the middle diameter 94, the top diameter 81.5, and the depth 52 inches; what is the content in ale and wine gallons?

Ans. 1183.770 *ale gallons, and* 1445.100 *wine gallons.*

3. The head diameter of a spheroidal cask is 30, the bung diameter 35, and the length 38 inches; what is the content of the cask, in ale and wine gallons?

Ans. 118.181 *ale gallons, and* 144.272 *wine gallons.*

Note. The content of this cask, found by the Rule given in Problem XIV., is the same as above.

PROBLEM XX.

To find the content of a circular vessel, when its sides are much curved, by taking five diameters at equal distances from each other.

RULE.

By the Pen.

Add into one sum, the squares of the top and bottom

diameters, twice the square of the middle diameter, a four times the sum of the squares of the diameters tak at one-fourth and at three-fourths of the depth; multip this sum by the depth; divide the product by 4308. 3529.44, and 32856, and the respective quotients will the content in ale and wine gallons, and malt bushels.

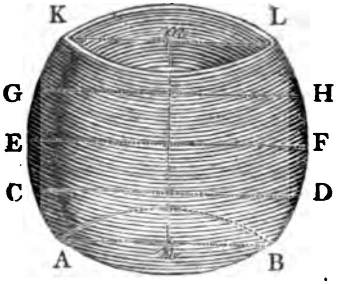
Note 1. The foregoing divisors are found by multiplying the circul divisors by twelve.

2. The preceding Rule is deduced from the method of equi-dista ordinates, described in Problem XX., Part IV.

3. The content of any vessel of an ordinary size may be found b this Problem; but when a vessel is very deep, its sides very muc curved, and great accuracy is required, its content must be found b the next Problem.

EXAMPLES.

1. The diameter A B, of a circular vessel, measures 57 C D 72, E F 80, G H 76, K L 65; and the depth *mn* 4 inches; what is the content in ale and wine gallons, an malt bushels?



Here $57^2+65^2=3249+4225=7474$, *the sum of the squares of the top and bottom diameters;* $80^2\times 2=6400\times 2=12800$, *twice the square of the middle diameter; and* $76^2+72^2\times 4=5776+5184\times 4=10960\times 4=43840$, *four times the sum of the squares of the diameters taken at one-fourth and at three-fourths of the depth; then,* $(7474+12800+43840)\times 48=64114\times 48=3077472$; *and* $3077472\div 4308.6=714.262$, *the content in ale gallons;* 3077472

4=871.948, *the content in wine gallons; and* ÷32856=93.665, *the content in malt bushels.*

: top diameter of a circular guile-tun measures ottom diameter 46, the diameter taken at one- the depth 58, that taken at three-fourths of the , the middle diameter 68, and the depth 60 in- ıat is the content of the vessel in ale gallons?

Ans. 565.572 *ale gallons.*

: top diameter of a circular mash-tun measures bottom diameter 86, the diameter taken at one- ' the depth 95, that at three-fourths of the depth middle diameter 104, and the depth 82 inches; he content of the vessel in malt bushels?

Ans. 276.787 *malt bushels.*

REMARKS.

ontents of Stills, Still-heads, Wash-Backs, Spirit- d every other curved vessel used by Victuallers, ı Brewers, Distillers, &c. &c. may be found either ır the following Problem.

Vash-Back or any other vessel be truly elliptical, s of the horizontal, parallel sections may be ob- y Prob. XVI., Part IV.; but the areas of the al sections of oval vessels which are not truly l, must be found by the method of equi-distant s described in Prob. XX., Part IV. (See Prob- X., Part III., where a method is given to deter- ıether an oval figure be greater or less than a true

PROBLEM XXI.

the content of a circular vessel, when its sides ry much curved, by taking a competent number ı-distant, parallel sections.

RULE I.

By the Pen:

the areas of as many equi-distant, parallel sec-

U

tions as you think necessary, by multiplying the square of the mean diameter of each section by .7854.

Then to the sum of the areas of the two end sections add four times the sum of the areas of all the even sections, and twice the sum of the areas of all the odd sections, not including the sections at the ends; multiply the sum by the common distance of the sections; divide the product by three; and the quotient will be the content in cubic inches.

Divide the content thus found, by 282, 231, and 2150.42; and the respective quotients will be the content in ale and wine gallons, and malt bushels.

Note 1. Always make choice of an odd number of sections, in order that the number of parts into which the vessel is divided, may be equal. Seven or nine will, in general, be sufficient, except when the vessel is very deep, in which case it may be necessary to take eleven, thirteen, &c. as the case may require.

2. The perpendicular depth of the vessel must first be taken, in order to determine the common distance of the ordinates, which may be found by dividing the whole depth by the number of sections minus *one*.

3. Great care must be taken to obtain the diameters of the sections at equal perpendicular distances from each other; for if their distance be measured upon the side of the vessel, it is evident that the process will be incorrect. (See Problem III., Part VI., for the method of quartering circular vessels, taking cross diameters, &c. &c.)

4. The Rule given in this Problem is founded upon the method of equi-distant ordinates, described in Problem XX., Part IV. (See the Scholium to that Problem.)

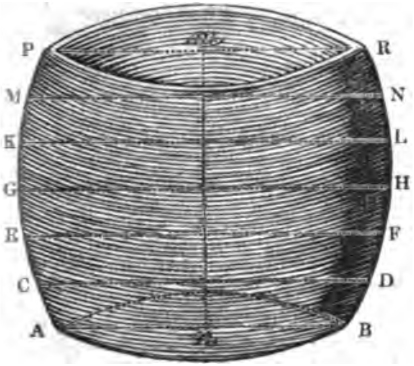
RULE II.

Find the area corresponding to each mean diameter, in the Table of Ale or the Table of Wine Areas, Part VII.; and place those areas in regular succession, opposite to their respective diameters.

Then to the sum of the two extreme areas, add four times the sum of all the even areas, and twice the sum of all the odd areas; multiply this sum by the common distance of the diameters; divide the product by 3; and the quotient will be the content in ale or wine gallons. (See Problem XIII., Part IV.)

EXAMPLES.

hat is the content in ale and wine gallons, of a vessel whose depth *m n*, is 120 inches; and the ameters of seven equi-distant, parallel sections as viz., the diameter of the bottom or first section 4, the second C D 146, the third E F 161, the G H 164, the fifth K L 166, the sixth M N 157, diameter, P R, of the top or seventh section 144



By Rule I.

$24^2 \times .7854 = 15376 \times .7854 = 12076.3104$, *the he first section;* $146^2 \times .7854 = 21316 \times .7854 =$ 4, *the area of the second section;* $161^2 \times .7854$ $\times .7854 = 20358.3534$, *the area of the third sec-* $^2 \times .7854 = 26896 \times .7854 = 21124.1184$, *the area urth section;* $166^2 \times .7854 = 27556 \times .7854 =$ 4, *the area of the fifth section;* $157^2 \times .7854 =$ $854 = 19359.3246$, *the area of the sixth section;* $\times .7854 = 20736 \times .7854 = 16286.0544$, *the area nth or last section.*

$2076.3104 + 16286.0544 = 28362.3648$, *the sum eas of the two end sections;* $(16741.5864 +$ $4 + 19359.3246) \times 4 = 57225.0294 \times 4 =$

228900.1176, *four times the sum of the areas of all the even sections;* and $(20358.3534+21642.4824)\times 2=42000.8358\times 2=84001.6716$, *twice the sum of the areas of all the odd sections;* also, $\frac{120}{6}=20$, *the common distance of the sections;* hence, $(28362.3648+228900.1176+84001.6716)\times\frac{20}{3}=\frac{341264.154\times 20}{3}=\frac{6825283.08}{3}=2275094.36$, *the content in cubic inches;* consequently $\frac{2275094.36}{282}=8067.710$, *the content in ale gallons;* and $\frac{2275094.36}{231}=9848.893$, *the content in wine gallons.*

By Rule II.

	Diameters. Inches.	Areas. Ale Gallons.
First	124	42.8237
Second	146	59.3672
Third	161	72.1925
Fourth	164	74.9080
Fifth	166	76.7462
Sixth	157	68.6499
Seventh	144	57.7518

Then $42.8237+57.7518=100.5755$, *the sum of the two extreme areas;* $(59.3672+74.9080+68.6499)\times 4=202.9251\times 4=811.7004$, *four times the sum of all the even areas;* and $(72.1925+76.7462)\times 2=148.9387\times 2=297.8774$, *twice the sum of all the odd areas;* hence $(100.5754+297.8774+811.7004)\times\frac{20}{3}=\frac{1201.1533\times 20}{3}=\frac{24203.0660}{3}=8067.6886$, *the content in ale gallons, the same as before.*

	Diameters. Inches.	Areas. Wine gallons.
First	124	52.2784
Second	146	72.4744
Third	161	88.1814
Fourth	164	91.4464
Fifth	166	93.6904
Sixth	157	83.8066
Seventh	144	70.5024

52.2784+70.5024=122.7808, *the sum of the two areas;* (72.4744 + 91.4464 + 83.8066) × 4 = 4 × 4 = 990.9096, *four times the sum of all the even* and (88.1314 + 93.6904) × 2 = 181.8218 × 2 = 36, *twice the sum of all the odd areas; hence*

$$08+990.9096+363.6436)\times\frac{20}{3}=\frac{1477.3340\times 20}{3}$$

$$\frac{6.6800}{3}=9848.8933,$$ *the content in wine gallons, the before.*

he bottom or first diameter of a circular vessel is second 109.5, the third 120.8, the fourth 123.2, 124.5, the sixth 117.8, the seventh or top diame- .4, and the depth 114.6 inches; required the con- ale and wine gallons, by the second Rule.
4340.13839 *ale gallons, and* 5298.37756 *wine*

he bottom or first diameter of a circular vessel es 108.4, the second 123.6, the third 130.8, the 136.6, the fifth 136.2, the sixth 131.8, the seventh the eighth 114.8, the ninth or last 96.8, the depth and the common distance of the diameters 18.3 ; required the content in ale and wine gallons, and ishels, by the first Rule.
6476.8996482 *ale gallons,* 7906.8645056 *wine gal- d* 849.36231 *malt bushels.*

REMARK.

etimes the contents of vessels whose sides are , are found by dividing them into a number of

equal or unequal parts, and calculating the content of each part by the Rule for the frustum of a cone. This method, however, is extremely troublesome; and if the vessel be not divided into a great number of parts, so that the altitude of each part may be small, it is also very incorrect. For example, if the vessel belonging to the first Question in the last Problem, be taken as the frustums of six cones, its content will be 8034.0292 ale gallons, and 9807.776 wine gallons, which is too little by 33.6594 gallons of ale, and 41.1173 gallons of wine; hence we perceive the excellency of the Rules given in the last three Problems, for finding the contents of vessels when their sides are curved, and the nature of the curves cannot be determined; that is, when we cannot determine whether a vessel is the frustum of a sphere, the frustum of a spheroid, the frustum of a parabolic spindle, the frustum of a paraboloid, or the frustum of a hyperboloid.

PROBLEM XXII.

To find the content of the ungula or hoof of a cylinder, or the quantity of liquor contained in a cylindrical vessel placed in an inclining position, so that the surface of the liquor intersects the sides of the vessel in an oblique direction.

RULE.

By the Pen.

Multiply the square of the diameter of the base by half the sum of the greatest and least depths of the liquor; divide the product by 359.05 and 294.12, and the respective quotients will be the content in ale and wine gallons.

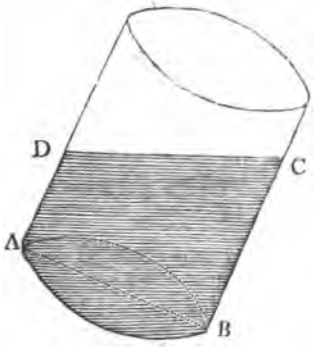
Note 1. The greatest and least depths of the liquor must not be taken perpendicularly to the horizon, but close to the sides of the vessel.

2. If a cylindrical vessel be placed in such an inclining position that the liquor just covers the bottom, it is evident that the hoof will be

cylinder; consequently, the square of the diameter of the base
e multiplied by half the greatest depth of the liquor, and the
t divided by the proper divisors, in order to obtain the content
ınd wine gallons.
'he divisor will be 2738, when the content is required in malt
.

EXAMPLES.

What is the content of the cylindrical hoof A B C D,
and wine gallons; the diameter A B of the base
38.2 inches, the greatest depth B C, of the liquor,
nches, and the least depth A D 18 inches?



re $38.2^2 = 1459.24$, *the square of the diameter of the*
and $\frac{32.6+18}{2} = \frac{50.6}{2} = 25.3$, *half the sum of the*
st and least depths of the liquor; then $1459.24 \times 25.3 =$
$.772$; *and* $36918.772 \div 359.05 = 102.823$, *the content*
e gallons; also, $36918.772 \div 294.12 = 125.522$, *the*
ıt in wine gallons.

The greatest depth of the liquor contained in a
drical vessel, placed in an inclining position, is 58.6
s, and the least depth 18.2 inches; what is the con-
in wine gallons; the diameter of the base being
inches? *Ans.* 722.690 *wine gallons.*

What is the content of a cylindrical ungula, in malt
els; the diameter of its base being 63.8 inches, the

greatest depth 28.8 inches, and the least depth 25.2 inches? *Ans.* 40.138 *malt bushel*

4. The diameter of a cylindrical vessel is 65 inches and it is placed in such a position that the liquor just covers the bottom; how many ale gallons does it contain, the greatest depth of the liquor being 25.8 inches? *Ans.* 151.796 *ale gallon*

PROBLEM XXIII.

To find the content of a cylindrical ungula or hoof, when its base is less than a semi-circle.

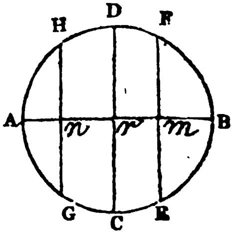
RULE.

By the Pen.

Find the area of the base of the ungula, by Problem XV., Part IV.; and multiply it by the difference between half the diameter of the vessel, and the versed sine of the ungula's base.

Subtract the product thus obtained from $\frac{2}{3}$ of the cube of half the chord; multiply the remainder by the height of the ungula; divide the product by the versed sine of the base; and the quotient will be the content in cubic inches.

Divide this content by 282 and 231, and the respective quotients will be the content in ale and wine gallons.



Note 1. When the base of an ungula is less than a semi-circle, E F will denote the chord, and B m the versed sine; consequently, the difference between half the diameter of the vessel and the versed sine of the ungula's base, will be denoted by B r — B $m = m\,r$.

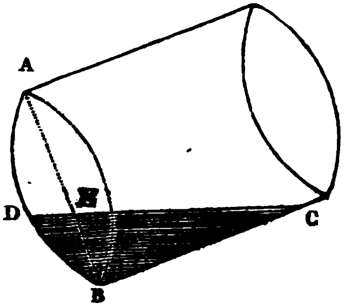
2. When the base of an ungula is a semi-circle, the edge of the liquor will coincide with the diameter C D.

3. When the base of an ungula is greater than a semi-circle, G H will denote the chord, and B n the versed sine; hence the difference between the

ine of the base and half the diameter of the vessel, will be by $Bs - Br = rs$. (See the two following Problems.)

EXAMPLES.

he diameter A B of a cylindrical vessel, is 65 in- ind it is placed in such a position that the chord ingula's base measures 56 inches, and its versed E 16 inches; what is the content of the liquor, in ions, the height B C, of the ungula being 64 in-



ule II., Prob. XV., Part IV., we have $16 \div 65 =$ *tabular height, or quotient of the versed sine divi- the the diameter. The Area Seg. corresponding to tient, is* .150091; *then* $.150091 \times 65^2 = .150091 \times$ 34.134475 *square inches, the area of the base of la.*

$\frac{65}{2} - 16 = 32.5 - 16 = 16.5$, *the difference between diameter of the vessel and the versed sine of the base; and* $634.134475 \times 16.5 = 10463.2188375$, *of the base multiplied by the said difference.*

, $28^3 \times \frac{2}{3} = 21952 \times \frac{2}{3} = \frac{43904}{3} = 14634.666$, *two- the cube of half the chord; and* $14634.666 -$ 188375 = 4171.4478291; *then* $4171.4478291 \times$

$1.4478291 \times 4 = 16685.7913164$, *the content of the*

ungula B C D, in cubic inches; hence 16685.791 ÷ 282 = 59.169, *the content in ale gallons.*

2. What is the content of the hoof of a cylinder, in wine gallons; the diameter of the vessel being 100 inches, the chord of the hoof's base 96 inches, the versed sine 36 inches, and the height of the hoof 108 inches?

Ans. 495.142 *wine gallons.*

Note. The area of the ungula's base was found by the first Rule in Problem XV., Part IV.

PROBLEM XXIV.

To find the content of a cylindrical ungula, when its base is a semi-circle.

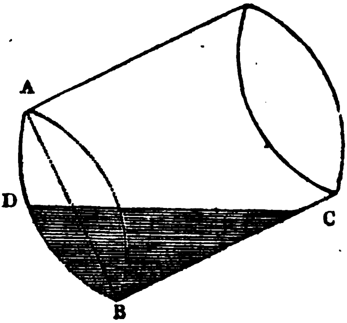
RULE.

By the Pen.

Multiply the square of the diameter of the vessel by the height of the ungula; divide the product by 6, and the quotient will be the content in cubic inches. Divide this content by 282 and 231, and the respective quotients will be the content in ale and wine gallons.

EXAMPLES.

1. The diameter A B of a cylindrical vessel is 65 inches, and it is placed in such a position that the liquor just covers one-half of its bottom; what is the content of the ungula B C D, in ale gallons; its height B C, being 64 inches?



Here $\frac{65\times65\times64}{6}=\frac{4225\times64}{6}=\frac{270400}{6}=45066.666$, *the content in cubic inches; and* $\frac{45066.666}{282}=159.810$, *the content in ale gallons.*

2. A cylindrical vessel is placed in such a position that the liquor just covers one-half of its bottom, and rises up the side 126 inches; what is the content of the ungula, in wine gallons; the diameter of the vessel being 124 inches? *Ans.* 1397.818 *wine gallons.*

PROBLEM XXV.

To find the content of a cylindrical ungula, when its base is greater than a semi-circle.

RULE.

By the Pen.

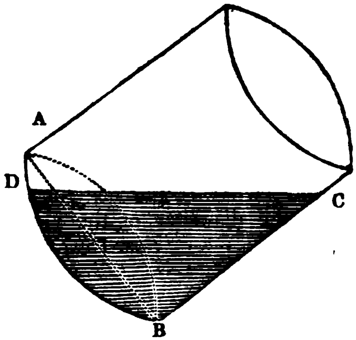
Find the area of the base of the ungula by Problem XV., Part IV.; and multiply it by the difference between the versed sine of the ungula's base, and half the diameter of the vessel.

To the product thus obtained, add ⅔ of the cube of half the chord of the ungula's base; multiply the sum by the height of the ungula; divide the product by the versed sine; and the quotient will be the content in cubic inches.

Divide this content by 282 and 231; and the respective quotients will be the content in ale and wine gallons.

EXAMPLES.

1. The diameter A B of a cylindrical vessel is 65 inches, and it is placed in such a position that the liquor leaves only 20 inches of the diameter of the bottom dry; what is the content of the ungula in ale gallons, the chord of its base being 60 inches, and its height B C, 64 inches?



By Rule I., Prob. XV., Part IV., we have $60 \times 20 \times \frac{2}{3} = 1200$ ×
$\frac{2}{3} = \frac{2400}{3} = 800$, *two-thirds of the product of the chord and ver-sed sine, and* $\frac{20^3}{60 \times 2} = \frac{800}{120} = 66.666$, *the cube of the height divided by twice the chord; then* $800 + 66.666 = 866.66$ *square inches, the area of the remaining segment.*

By Note I., Prob. XIII., Part IV., we have $65 \times 65 \times .7854 = 4225 \times .7854 = 3318.315$ *square inches, the area of the bottom of the vessel; then* $3318.315 - 866.666 = 2451.649$ *square inches, the area of the base of the ungula.*

Now, $65 - 20 = 45$, *the versed sine of the ungula's base and* $65 \div 2 = 32.5$, *half the diameter of the vessel; then* $45 - 32.5 = 12.5$, *the difference between the versed sine and half the diameter; and* $2451.649 \times 12.5 = 30645.6125$ *the area of the ungula's base, multiplied by the said difference.*

Again, $30^3 \times \frac{2}{3} = 27000 \times \frac{2}{3} = \frac{54000}{3} = 18000$, *two-thirds of the cube of half the chord; and* $30645.6125 + 18000 = 48645.6125$; *then* $48645.6125 \times \frac{64}{45} = \frac{3113319.}{45}$ $= 69184.871$, *the content of the ungula, in cubic inches; and* $\frac{69184.871}{282} = 245.336$, *the content in ale gallons.*

The chord of the base of a cylindrical ungula mea-
64 inches, the versed sine 64 inches, and the diame-
the vessel 80 inches; what is the content of the
la in wine gallons; its height being 92 inches?

Ans. 779.775 *wine gallons.*

. The area of the ungula's base was found by the second Rule
lem XV., Part IV.

Remark.

en a vessel in the form of a cylinder, or part of a
er, is laid upon its side, so that the surface of the
is parallel to the axis, and comes in contact with
nds of the vessel, find the area of the circular seg-
of the end, by Problem XV., Part IV.; multiply it
length of the vessel, and the product will be the
t of the ungula in cubic inches.
he vessel be laid in an inclining position, so that
rface of the liquor is oblique to the axis, making
rcular segments of the ends dissimilar figures, the
t of the ungula may be found by Problem IX.,

PROBLEM XXVI.

*d the content of a pyramidical ungula, or the
tity of liquor contained in a vessel in the form of
rustum of a square or rectangular pyramid, when
placed in such a position that the liquor just
rs the bottom, or part of the bottom, and rises up
of the sides in an oblique direction, forming a
e that is called a cuneus or wedge.*

RULE.

By the Pen.

twice the length of the base of the ungula,
e length of the upper edge of the liquor;
y the sum by the breadth of the base, and
oduct by the perpendicular height; and $\frac{1}{6}$ of

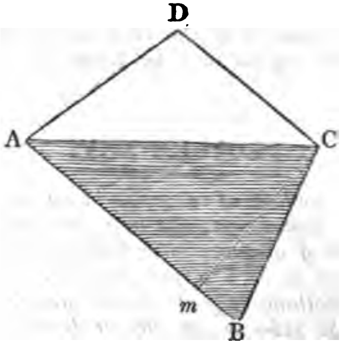
the last product will be the content in cubic inch Divide this content by 282, and 231; and the respecti quotients will be the content in ale and wine gallons.

Note 1. All ungulas made by placing pyramidical or prismoid vessels in a slanting position, so that the liquor covers all, or any p of their bottoms, are in the form of wedges; consequently, the fo going Rule holds good for all ungulas so formed.

2. When the liquor covers only part of the bottom of the vessel, t base of the ungula will be that part of the bottom that is covered the liquor.

EXAMPLES.

1. What is the content, in ale gallons, of the ungu A B C, of the square pyramidical vessel A B C D; th side A B of the greater base being 80 inches, the sid C D of the less base 52 inches, and the perpendicula height C *m* 54 inches?



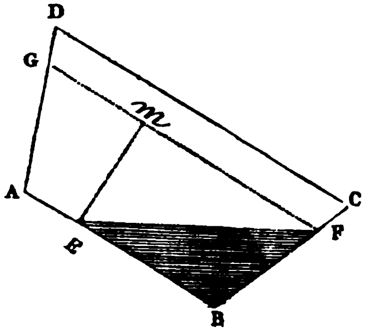
Here the dimensions of the ungula are the same as tho of the pyramidical vessel; hence, by the Rule, we have 80 × 2 + 52 = 160 + 52 = 212, *twice the length of the base adde to the length of the upper edge of the liquor, the*

$$\frac{212 \times 80 \times 54}{6} = 16960 \times \frac{54}{6} = 16960 \times 9 = 152640,$$ *the con-*

tent of the ungula in cubic inches; and $\frac{152640}{282} = 541.27$ *the content in ale gallons.*

ε. It is evident from the foregoing figure, that when the liquor ramidical or prismoidal vessel just covers the whole of the bot- and rises to the top of the vessel, in an oblique direction, it it into two ungulas in the form of wedges, one of which is oc- by the liquor, and the other is empty.

The length of the base A B, of a prismoidal vessel, inches, the breadth of the base 27 inches, the length of the top 62 inches, the breadth of the top 35 in- and the perpendicular height 28 inches; what is ntent of the ungula E B F, in wine gallons; the E B of its base being 30 inches, the breadth of its 27 inches, the length of its edge 33 inches, and pendicular height E *m* 22 inches?



Ans. 39.857 *wine gallons.*

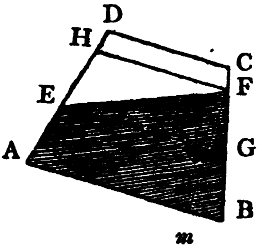
It appears from the foregoing figure, that when the liquor in a ical or prismoidal vessel, covers only part of the bottom, and he top of the vessel in an oblique direction, it divides it into and a prismoid; thus E B F is a wedge, and A E F G a pris- If the liquor does not rise to the top of a pyramidical vessel, empty part of the vessel will be composed of the prismoid , and the frustum G F C D. If the vessel be a prismoid, both ures will be prismoids.

equired the content in ale gallons, of the ungula ectangular, pyramidical vessel; the length of the the ungula being 58 inches, the breadth of the 5 inches, the length of the upper edge of the 38 inches, and the perpendicular height of the un- inches? *Ans.* 172.021 *ale gallons.*

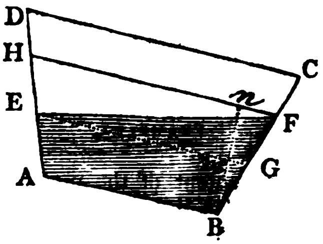
X 2

REMARKS.

1. When liquor intersects obliquely the opposite side of a vessel in the form of the frustum of a square o rectangular pyramid, it disposes itself into a compoun figure consisting of a wedge and the frustum of a pyra mid: Thus in the annexed figure A B C D, if E F denot the surface of the liquor, then the ungula A B F E, i composed of the wedge E G F, and the frustum A B G E F n being the perpendicular height of the wedge; E G on side of its base; and m n the perpendicular height of th frustum; hence the content of the whole ungula may b obtained by finding the contents of the two figures o which it is composed. It may also be observed, that th empty part of the vessel is composed of the wedg E F H, and the frustum H F C D.



2. When liquor intersects obliquely the opposite sides of a vessel in the form of a prismoid, it disposes itself into a compound figure consisting of a wedge and a prismoid: Thus in the annexed figure A B C D, if E F denote the surface of the liquor, then the ungula A B F E, is composed of the wedge E G F, and the prismoid A B G E; B m being the perpendicular height of the prismoid, m n that of the wedge, and E G the length of its base.



3 The content of that part of a compound figure forming the frustum of a pyramid, may be found by Problem VI.; the content of that part forming a prismoid, by Problem IX.; and the content of that part forming a wedge, by the last Problem.

4. The content of the compound figure A B F E, may also be found by subtracting the content of the dry wedge E F H from the content of the frustum or prismoid A B F H. (See Problem XXIX.)

PROBLEM XXVII.

To find the content of a conical ungula, or the quantity of liquor contained in a vessel in the form of the frustum of a cone, when it stands upon its greater base, and in such a position that the liquor just covers the whole of its bottom.

RULE.

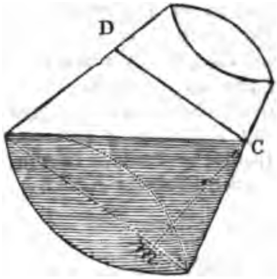
By the Pen.

Multiply the product of the top and bottom diameters by the mean proportional between them; subtract the last product from the cube of the bottom diameter; divide the remainder by the difference of the diameters; multiply the quotient by the perpendicular height of the ungula, and the product thence arising by .0009283, and .001133; and the respective products will be the content in ale and wine gallons.

Note. When the liquor does not rise to the top of the vessel, th diameter at the upper extremity of the liquor, measured parallel t the top of the vessel, must be taken for the top diameter, in order t determine the content of the ungula.

EXAMPLES.

1. What is the content of the ungula A B C, in al gallons; the diameter A B being 60 inches, the diamete C D 40 inches, and the perpendicular height C m 35 inches?



Here $\sqrt{60 \times 40} = \sqrt{2400} = 48.98979$, *the mean proportional between the diameters; and* $48.98979 \times 2400 = 117575.496$, *the product of the two diameters multiplied by the mean proportional between them.*

Also, $60^3 = 216000$, *the cube of the bottom diameter; and* $\frac{216000 - 117575.496}{60-40} = \frac{98424.504}{20} = 4921.2252$; *then* $4921.2252 \times 35 \times .0009283 = 172242.882 \times .0009283 = 159.8930673606$, *ale gallons, the answer required.*

2 Required the content of a conical ungula, in wine gallons; the bottom diameter being 82.6 inches, the top diameter 58.7 inches, and the perpendicular height 47.3 inches. *Ans.* 506.756 *wine gallons.*

PROBLEM XXVIII.

find the content of a conical ungula, or the quantity f liquor contained in a vessel in the form of the rustum of a cone, when it stands upon its less base, nd in such a position that the liquor just covers the hole of its bottom.

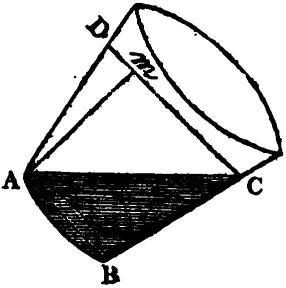
RULE.

By the Pen.

ultiply the product of the two diameters by the ı proportional between them; from the product thus ned subtract the cube of the bottom diameter; di- the remainder by the difference of the diameters; iply the quotient by the perpendicular height of the la, and the product thence arising by .0009283, and 133; and the respective products will be the content e and wine gallons.

EXAMPLES.

What is the content of the ungula A B C, in wine ns; the diameter A B being 30 inches, the diameter 50 inches, and the perpendicular height A m 40 in-



found by subtracting the content of the dry hoof D E F from the con-
tent of the frustum A B E F.

2. What is the content, in wine gallons, of a compound figure consisting of the frustum of a cone and a conical ungula; the bottom diameter being 30 inches, the middle diameter 40 inches, the top diameter 50 inches; and the perpendicular height of the frustum, and also that of the ungula, 20 inches? *Ans.* 141.516 *wine gallons.*

Remark.

If a vessel in the form of the frustum of a cone be laid upon its slant side, so that the surface of the liquor is either parallel or oblique to the axis, and comes in contact with both ends of the vessel, the content of the ungula thus formed, may be found by Problem IX, Part V.

PROBLEM XXX.

To find the quantity of liquor contained in a vessel in the form of the frustum of a cone, when it is placed on its greater end, and in such a position that the liquor, covering only part of its bottom, forms an elliptic ungula.

RULE.

By the Pen.

From the versed sine of the base of the ungula subtract the difference of the diameters of the vessel; divide the remainder by the less diameter; and find the *Area Seg.* answering to the quotient, in the Table of versed sines at the end of Part IV.

Multiply this *Area Seg.* by the cube of the less diameter; also multiply the product thus obtained by the quotient arising from dividing the versed sine of the base of the ungula by the difference between the said versed sine and the difference of the diameters; likewise

the last product by the square root of the said ; and reserve the product.

e the versed sine of the base of the ungula by om diameter of the vessel; find the *Area Seg.* ıg to the quotient;' and multiply it by the cube ›ttom diameter.

the last product subtract the reserved product; the difference by one-third of the perpendicular ' the ungula; divide the product by the differ- :he diameters; and the quotient will be the con- he ungula, in cubic inches.

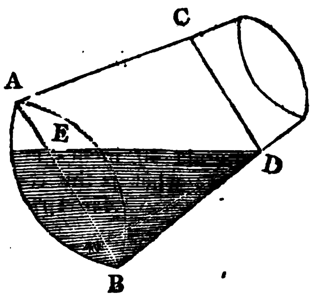
: this content by 282, and 231, and the respective will be the content in ale and wine gallons.

The ungula will always be elliptical when the versed sine ;e is greater than the difference of the diameters of the i this will invariably be the case when the angle B E D is ıe angle B A C.

:ic ungulas are more frequently met with than either para- perbolic ungulas; as a parabolic ungula can be formed only surface of the liquor is parallel to the side A C: and a when the angle B E D is greater than the angle B A C. in Problems XXXII. and XXXIII.)

EXAMPLES.

ıat is the content of the elliptic ungula B D E, allons; the diameter A B being 50 inches, the C D 32 inches, the versed sine B E of the base s, and the perpendicular height D n 48 inches?



Here $50-32=18$, *the difference of the diameters; and* $\frac{34-18}{32}=\frac{16}{32}=.5$, *the quotient obtained by subtracting the difference of the diameters from the versed sine of the base, and dividing the remainder by the less diameter. The Area Seg. answering to this quotient is* .392699.

Now, $32^3=32768$, *the cube of the less diameter; also,* $\frac{34}{34-18}=\frac{34}{16}=2.125$, *the quotient arising from dividing the versed sine of the base by the difference between the said versed sine and the difference of the diameters; and* $\sqrt{2.125}=1.457$, *the square root of the said quotient; then* $.392699\times 32768\times 2.125\times 1.457=12867.960832\times 2.125\times 1.457=27344.416768\times 1.457=39840.815230976$, *the reserved product.*

Again, $50^3=125000$, *the cube of the bottom diameter; and* $\frac{34}{50}=.68$, *the quotient of the versed sine of the base divided by the bottom diameter. The Area Seg. answering to this quotient is* .568732; *then* $.568732\times 125000=71091.5$, *the product arising from multiplying the last Area Seg. by the cube of the bottom diameter.*

Now, $71091.5-39840.815=31250.685$, *the difference between the last product and the reserved product; also* $\frac{48}{3}=16$, *one-third of the perpendicular height of the ungula; then* $31250.685\times\frac{16}{18}=31250.685\times\frac{8}{9}=\frac{250005.48}{9}=27778.386$, *the content of the ungula, in cubic inches; and* $\frac{27778.386}{282}=98.504$, *the content in ale gallons.*

2. The bottom diameter of a conical vessel is 60 inches, the top diameter 50 inches, the versed sine of the base of the elliptic ungula 40 inches, and its perpendicular height 54 inches; what is the content of the ungula in wine gallons? *Ans.* 197.914 *wine gallons.*

PROBLEM XXXI.

l the quantity of liquor contained in a vessel in 'orm of the frustum of a cone, when it is placed s less end, and in such a position that the liquor, ing only part of its bottom, forms an elliptic 'a.

RULE.

By the Pen.

ıe difference of the diameters of the two ends of sel, add the versed sine of the base of the ungula; ide the sum by the greater diameter; then mulhe *Area Seg.* answering to this quotient by the f the greater diameter, and call this product the *nber.*

the versed sine of the base to the difference of meters; divide the versed sine by this sum; and are root of the cube of the quotient call the *second*

de the versed sine by the less diameter; multiply *'a Seg.* answering to the quotient, by the cube of diameter; and call this product the *third num-*

de the perpendicular height of the ungula by three he difference of the diameters; and call the quoe *fourth number.*

ly, from the product of the *first* and *second* numıbtract the *third* number; multiply the difference *fourth* number; and the product will be the conthe ungula, in cubic inches.

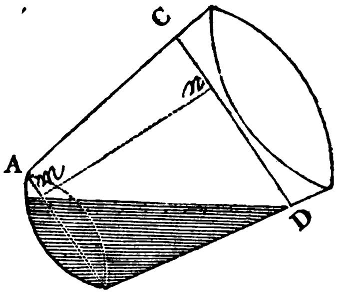
de this content by 282 and 231, and the respective ıts will be the content in ale and wine gallons.

l. When a vessel in the form of the frustum of a cone, is pon its less end, in such a position that the liquor covers t, or the whole of its bottom, the ungula will always be ; that is, the surface of the liquor will form an ellipse when ır covers the whole of the bottom; and an elliptical segment covers only part of the bottom.

2. For the Rule given in this Problem we are indebted to our mu esteemed friend Mr. Joseph Lewthwaite, Teacher of the Mathematic at Halifax.

EXAMPLES.

1. What is the content of the elliptic ungula B D m in wine gallons; the diameter A B being 32 inches, th diameter C D 50 inches, the versed sine B m of the bas 30 inches, and the perpendicular height $m\,n$ 48 inches?



Here $\frac{50-32+30}{50}=\frac{18+30}{50}=\frac{48}{50}=.96$. *The Area Seg. answering to this quotient is* .774861; *and* $.774861\times50^3=.774861\times125000=96857.625$, *the first number.*

Again, $\frac{30}{30+18}=\frac{30}{48}=.625$; *and* $\sqrt{.625^3}=\sqrt{.244140625}=.4941$, *the second number.*

Again, $\frac{30}{32}=.9375$. *The Area Seg. answering to this quotient is* .76496; *and* $.76496\times32^3=.76496\times32768=25066.20928$, *the third number.*

Again, $\frac{48}{18\times3}=\frac{48}{54}=.8888$, *the fourth number.*

Now, $96857.625\times.4941=47857.3525125$, *the product of the first and second numbers; then* $(47857.3525125-25066.20928)\times.8888=22791.1432325\times.8888=$

'68105046, *the content of the ungula, in cubic in-*

and $\frac{20256.768}{231}=87.691$, *the content in wine gallons.*

hat is the content of an elliptic ungula, in ale gallons; diameter of the conical vessel being 110 inches, tom diameter 80 inches, the versed sine of the the ungula 36 inches, and its perpendicular height es? *Ans.* 145.579 *ale gallons.*

PROBLEM XXXII.

l the quantity of liquor contained in a vessel in orm of the frustum of a cone, when it is placed s greater end, and in such a position that the -, covering only part of its bottom, forms a para- ungula.

RULE

By the Pen.

iply the area of the base of the ungula by the diameter of the vessel; divide the product by the ce of the diameters; and reserve the quotient.

iply the difference of the diameters by the less r; extract the square root of the product; and y the said root by $\frac{2}{3}$ of the less diameter.

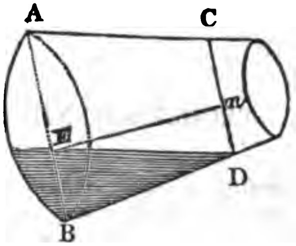
act the last product from the reserved quotient; y the remainder by $\frac{1}{3}$ of the perpendicular height ungula; and the product will be the content in iches.

le this content by 282 and 231, and the respective ts will be the content in ale and wine gallons.

The ungula will always be parabolical when the versed sine se is equal to the difference of the diameters of the vessel; will invariably be the case when the surface of the liquor is the upper side of the vessel, making the angle B E D equal gle B A C.

EXAMPLES.

1. What is the content of the parabolic ungula B D E, in ale gallons; the diameter A B being 50 inches, the diameter C D 32 inches; and the perpendicular height E n 48 inches?



Here $50-32=18=BE$, *the versed sine of the base of the ungula; and* $\frac{18}{50}=.36$. *The Area Seg. answering to this quotient is* $.25455$; *then* $.25455\times 50^2=.25455\times 2500=636.375$, *the area of the base of the ungula; and* $\frac{636.375\times 50}{18}=\frac{31818.75}{18}=1767.708$, *the reserved quotient.*

Again, $\sqrt{18\times 32}\times 32\times\frac{4}{3}=\sqrt{576}\times\frac{128}{3}=24\times\frac{128}{3}=\frac{3072}{3}=1024$.

Then $(1767.708-1024)\times\frac{48}{3}=743.708\times 16=11899.328$, *the content of the ungula in cubic inches; and* $\frac{11899.328}{282}=42.196$, *the content in ale gallons.*

2. What is the content of a parabolic ungula, in wine gallons; the bottom diameter of the conical vessel being 100 inches, the top diameter 74 inches, and the perpendicular height of the ungula 34.5 inches?

Ans. 95.239 *wine gallons.*

PROBLEM XXXIII.

To find the quantity of liquor contained in a vessel in the form of the frustum of a cone, when it is placed upon its greater end, and in such a position that the liquor, covering only part of its bottom, forms a hyperbolic ungula.

RULE.

By the Pen.

Divide one-third of the perpendicular height of the ungula by the difference of the diameters of the vessel; and call the quotient the *first number*.

Multiply the area of the ungula's base by the greater diameter; and call the product the *second number*.

Multiply the less diameter by the versed sine of the ungula's base; divide the product by the axis of the hyberbolic section; multiply the quotient by the area of the hyperbolic section; and call the product the *third number*.

From the second number subtract the third; multiply the remainder by the first number; and the product will be the content of the ungula in cubic inches.

Divide this content by 282, and 231, and the respective quotients will be the content in ale and wine gallons.

Note. The axis of the hyperbola section is the distance between the upper extremity of the surface of the liquor and the middle of the chord of the ungula's base; and the area of the hyperbolic section is the area of the surface of the liquor. (See the definitions of the Conic Sections, Part V.)

To find the area of the hyperbolic section.

RULE.

Multiply the transverse diameter by the absciss; to the product add $\frac{5}{7}$ of the square of the absciss; and multiply the square root of the sum by 21.

Add the product last found to 4 times the square root of the product of the transverse and absciss; and divide the sum by 75.

Divide 4 times the product of the conjugate and absciss by the transverse; then this quotient being multiplied by the former will give the area, *nearly*.

Note 1. In finding the area of the hyperbolic section formed by the surface of the liquor, the absciss mentioned in the last Rule, is the axis of the section; and it may also be observed, that the chord of the ungula's base is the base of the section; and likewise in this case, its double ordinate. (See the definitions of the Conic Sections.)

2. Before we can find the area of the hyperbolic section of the liquor, we must determine the transverse and conjugate diameters.

To find the transverse diameter of the hyperbolic section.

RULE.

Multiply the less diameter of the vessel by the absciss of the hyperbolic section; and reserve the product. From the greater diameter of the vessel subtract the sum of the less diameter and the versed sine of the ungula's base. Divide the reserved product by the remainder; and the quotient will be the transverse diameter required.

To find the conjugate diameter of the hyperbolic section.

RULE.

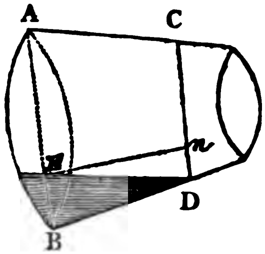
From the greater diameter of the vessel subtract the sum of the less diameter and versed sine of the ungula's base. Divide the said versed sine by the remainder; and extract the square root of the quotient. Multiply this root by the less diameter; and the product will be the conjugate diameter of the section.

Note. The ungula will always be hyperbolical when the versed sine of the base is less than the difference of the diameters of the vessel; and this will invariably be the case when the angle B E D is greater than the angle B A C.

EXAMPLES.

1. What is the content of the hyperbolic ungula

, in wine gallons; the diameter A B being 72 inches, the diameter C D 50 inches, the perpendicular E n 54 inches, the slant height or axis D E of hyperbolic section formed by the surface of the liquor, hes, and the versed sine B E of the ungula's base hes?



CALCULATION.

find the transverse and conjugate diameters of the hyperbolic section.

e $55 \times 50 = 2750$, *the reserved product; and* $72 - (50$ $= 72 - 70 = 2$; *then* $\frac{2750}{2} = 1375$ *inches, the transverse diameter.*

in, $\sqrt{20 \div 2} = \sqrt{10} = 3.1622$; *and* $3.1622 \times 50 =$ *inches, the conjugate diameter.*

To find the area of the hyperbolic section.

e $21\sqrt{1375 \times 55 + 55^2 \times \frac{5}{7}} = 21\sqrt{75625 + 3025 \times \frac{5}{7}}$ $\sqrt{75625 + 15125 \div 7} = 21\sqrt{75625 + 2160.7142} = 21$ $\overline{85.7142} = 278.9 \times 21 = 5856.9.$

in, $(4\sqrt{1375 \times 55} + 5856.9) \div 75 = (4\sqrt{75625} +$ $) \div 75 = (4 \times 275 + 5856.9) \div 75 = (1100 + 5856.9)$ $= 6956.9 \div 75 = 92.758.$

ly, $\frac{158.11 \times 55 \times 4}{1375} \times 92.758 = \frac{34784.2}{1375} \times 92.758 =$

$25.297 \times 92.758 = 2346.499126$ *the area of the section, in square inches.*

To find the area of the circular segment of the base.

Here $\frac{20}{72} = .277\frac{7}{9}$, *the quotient of the versed sine divided by the diameter. The Area Seg. answering to* .277, *is* .177330; *and the Area Seg. answering to* .278, *is* .178225; *their difference is* .000895; *then* $.000895 \times \frac{7}{9} = .000696$, *which being added to* .177330, *gives* .178026, *the Area Seg. corresponding to* $.277\frac{7}{9}$; *hence,* $.178026 \times 72^2 = .178026 \times 5184 = 922.886784$, *the area of the base, in square inches.*

To find the content of the hyperbolic ungula B D E.

Here $\frac{54 \div 3}{72 - 50} = \frac{18}{22} = .8181$, *the first number.*

Again, $922.886784 \times 72 = 66447.848448$, *the second number.*

Also, $\frac{50 \times 20}{55} \times 2346.499126 = \frac{1000}{55} \times 2346.499126 = 18.1818 \times 2346.499126 = 42663.5778091068$, *the third number.*

Lastly, $66447.848448 - 42663.577809 \times .8181 = 23784.270639 \times .8181 = 19457.9118097659$, *the content in cubic inches; and* $\frac{19457.9118}{231} = 84.2333$, *the content in wine gallons.*

2. What is the content of a hyperbolic ungula, in ale gallons; the greater diameter of the conical vessel being 96 inches, the less diameter 60 inches, the versed sine of the ungula's base 30 inches, the absciss 64 inches, and the perpendicular height 63 inches?

Ans. The transverse diameter of the hyperbolic section is 640, *the conjugate* 134.16, *and the area* 3631.120896. *The area of the circular segment of the base is* 1932.50304; *and the content of the ungula* 172.4975 *ale gallons.*

MISCELLANEOUS EXAMPLES,

IN THE

Iensuration of Superficies and Solids,

APPLIED TO

GAUGING.

'he length of a vessel, in the form of a parallelo-n, is 81 inches, and its breadth 64 inches; required rpendicular depth; it being a mean proportional n the other two dimensions. *Ans.* 72 *inches.*

'he area of a trapezium is 59536 square inches; d the side of a square of equal area.

Ans. 244 *inches.*

'he area of the base of a circular vessel is 6432 inches; what is the diameter?

Ans. 90.495 *inches.*

'he diameter of a cylindrical vessel is 78, and pendicular depth 112 inches; required its diago-

Ans. 136.484 *inches.*

'he diagonal of the base of a cubical vessel mea-15 inches; required the side.

Ans. 81.317 *inches.*

'he diagonal of the base of a rectangular cooler es 145 inches, and the length 116 inches; required eadth. *Ans.* 87 *inches.*

'he length of a cask is 45, the bung diameter 36, e head diameter 27 inches; required the diagonal.

Ans. 38.71 *inches.*

The bung diameter is 38.7, the head diameter 29.6, e diagonal 40.3 inches; required the length of the

Ans. 42.78 *inches.*

The content of a cylindrical vessel is 254 ale gal-required the side of a cubical vessel that shall n the same quantity of liquor. *Ans.* 41.52 *inches.*

10. The length of a water cistern in the form of parallelopipedon is 125, its breadth 86, and its depth (inches; required the dimensions of a similar vessel th shall contain 5 times as much.

Ans. The length is 213.74, *the breadth* 147.05, *and t depth* 106.01 *inches, nearly.*

11. The depth of a cylindrical vessel is 254, and it diameter 112 inches; what must be the dimensions of similar vessel that shall contain only ⅓ of the quantity?

Ans. The depth is 176.11, *and the diameter* 77.64 *inches, nearly.*

12. The content of a vessel in the form of a parallelopipedon, is 121500 cubic inches; its length 75, it breadth 45, and its depth 36 inches; required the dimensions of a similar vessel that shall contain 562500 cubic inches.

Ans. The length is 125, *the breadth* 75, *and the depth* 60 *inches.*

13. The side of a square measures 62.8 inches; what is its area in ale gallons? *Ans.* 13.981 *ale gallons.*

14. The length of a rectangle is 115.3 inches, and its breadth 86.4 inches; what is its area in wine gallons?

Ans. 43.125 *wine gallons.*

15. The base of a rhombus is 84.6 inches, and its perpendicular breadth 63.4 inches; required its area in malt bushels? *Ans.* 2.494 *malt bushels.*

16. The base of a triangle measures 123.6 inches, and its perpendicular 85.7 inches; what is its area in ale gallons? *Ans.* 18.781 *ale gallons.*

17. The three sides of a triangle measure 56, 68, and 79 inches respectively; what is the area in wine gallons?

Ans. 8.076 *wine gallons.*

18. The diagonal of a trapezium measures 138.6 inches, one of the perpendiculars 63.8 inches, and the other 56.4 inches; required the area in malt bushels.

Ans. 3.873 *malt bushels.*

19. The two parallel sides of a cooler, in the form of a trapezoid, measure 98 and 124 inches respectively, and the perpendicular distance between them 136 inches; what is the area of the vessel in ale gallons?

Ans. 53.531 *ale gallons.*

'he first side of an irregular pentagon measures
e second 104, the third 87, the fourth 92, and the
2 inches; also, the diagonal from the first to the
igle 168, and that from the third to the fifth 114
now, if the figure be an irregular floor of malt,
any bushels does it contain, if the depth of the
e 6.4 inches? *Ans.* 49.376 *malt bushels.*
The side of the base of an octagonal wine-vat
's 52.6 inches, and the perpendicular 63.7 inches;
.ny gallons of wine does the vessel contain, when
th of the liquor is 84.3 inches?

Ans. 4891.0017 *wine gallons.*

'he side of a regular nonagon measures 87.6 in-
vhat is its area in ale gallons?

Ans. 168.21649 *ale gallons.*

'he diameter of a cylindrical porter-vat is 226.4
and its depth 258.8 inches; how many gallons of
vill it contain? *Ans.* 36945.5116 *gallons.*
The radius of the sector of a circle is 52 inches,
chord of the whole arc 86 inches; what is the
ı in wine gallons? *Ans.* 11.3763 *wine gallons.*
The chord line of a cooler, in the form of the
t of a circle, measures 92 inches, and the versed
inches; how many gallons of ale does the vessel
, when the depth of the liquor is 6.4 inches?

Ans. 59.6608 *ale gallons.*

The transverse diameter of an elliptical vessel,
es 125.4 inches, and the conjugate 82.8 inches;
s the area in malt bushels?

Ans. 3.792 *malt bushels.*

The transverse diameter of an ellipse measures
ches, and the conjugate diameter 52.4 inches; re-
the area, in ale gallons, of a segment cut off by a
rallel to the transverse diameter; the height of
ment being 21.8 inches. *Ans.* 3.76808 *ale gallons.*
The base of a parabola is 98.6 inches, and its
75.4 inches; what is its area in wine gallons?

Ans. 21.455 *wine gallons.*

Required the area and content, in ale gallons, of a
composed of a right-angled triangle, and a circular
ıt described upon the hypothenuse of the triangle;
sed sine of the segment being 25 inches; the base

and perpendicular of the triangle 60 and 80 inches respectively; and the depth of the liquor 9.4 inches.

Ans. The area is 14.13, *and the content* 132.822 *ale gallons.*

30. The base of a curvilineal figure measures 97.6 inches; the first perpendicular ordinate 49.5, the second 51.4, the third 52.2, the fourth 52.6, the fifth 53.2, the sixth 52.7, the seventh 52.3, the eighth 51.5, and the ninth 49.6 inches; required the area of the figure, in wine gallons, by the method of equi-distant ordinates.

Ans. 21.958 *wine gallons*

31. The side of a cubical vessel measures 64.3 inches required its content in ale and wine gallons.

Ans. The content is 942.722 *ale, and* 1150.855 *wine gallons.*

32. The length of a cistern in the form of a parallelopipedon, is 145 inches, its breadth 96 inches, and its depth 84 inches; how many gallons, ale measure, will it contain? *Ans.* 4146.382 *ale gallons.*

33. The side of the base of a heptagonal, prismatic vessel measures 86.4 inches, and its perpendicular depth 73.6 inches; required its content in malt bushels.

Ans. 927.972 *malt bushels.*

34. The perpendicular depth of a cylindrical vessel is 76, and its diagonal 95 inches; required its content in ale gallons. *Ans.* 687.714 *ale gallons.*

35. Each side of the top of a triangular, pyramidical vessel measures 53.8, and its perpendicular depth 62.4 inches; what is its content in wine gallons?

Ans. 112.883 *wine gallons.*

36. What is the content, in malt bushels, of a vessel in the form of the frustum of a pentagonal pyramid; each side of the greater end being 45.7 inches, each side of the less end 34.3 inches, and the perpendicular depth 68.4 inches? *Ans.* 88.1446 *malt bushels.*

37. The slant side of a conical vessel measures 85, and the perpendicular depth 75 inches; what is the content in ale gallons? *Ans.* 445.62 *ale gallons.*

38. The top diameter of a vessel, in the form of the frustum of a cone, measures 126.3, the bottom diameter 158.6, and perpendicular depth 132.7 inches; required the content in wine gallons. *Ans.* 9194.498 *wine gallons.*

What is the content, in malt bushels, of a vessel form of a prismoid; the length of its bottom being and its breadth 42.6 inches; the length of its top and its breadth 54.8 inches; and the perpendicular 112.2 inches? *Ans.* 238.228 *malt bushels.*

The perpendicular depth of a porter-vat, in the f the frustum of an elliptical cone, is 93.6 inches; ansverse and conjugate diameters of the bottom re 99.2 and 75.6 inches; the transverse and condiameters of the top, 134.4 and 100.8 inches; re- the content in ale gallons.

Ans. 2704.794 *ale gallons.*

The inner diameter of a spherical vessel, measures ches; what is the content in wine gallons?

Ans. 326.6026 *wine gallons.*

The top diameter of a vessel in the form of the gment of a sphere is 35.6 inches, and its depth ches; what is the content in ale gallons?

Ans. 35.208 *ale gallons.*

The perpendicular depth of a vessel in the form of eater segment of a sphere, is 32.7 inches; and the er of the base 53.8 inches; what is the content in gallons? *Ans.* 240.155 *wine gallons.*

The top diameter of a vessel, in the form of the m of a sphere, is 72, the bottom diameter 84, and rpendicular depth 60 inches; required the content t bushels. *Ans.* 186.705 *malt bushels.*

The transverse diameter of a vessel, in the form of ate spheroid, is 96, and the conjugate 82 inches; s the content in ale gallons?

Ans. 1198.529 *ale gallons.*

The diameter of the base of a vessel, in the form prolate semi-spheroid, is 40, and its perpendicular 28 inches; what is its content in wine gallons?

Ans. 101.545 *wine gallons.*

The middle diameter of a vessel, in the form of iddle frustum of a prolate spheroid, 48, the end ter 40, and the length 50 inches; what is the content gallons? *Ans.* 288.167 *ale gallons.*

The bottom diameter of a vessel, in the form of he middle frustum of a prolate spheroid, is 72, the

top diameter 54, and the perpendicular depth 80 inches; required the content in wine gallons.

Ans. 1204.406 *wine gallons.*

49. The length of a vessel in the form of a parabolic spindle, is 80 inches, and its middle diameter 50 inches; required its content in ale gallons.

Ans. 297.079 *ale gallons.*

50. The top and bottom diameters of a vessel, in the form of the middle frustum of a parabolic spindle, measure 40 inches, the middle diameter 50 inches, and the perpendicular depth 54 inches; required the content in wine gallons. *Ans.* 401.468 *wine gallons.*

51. The diameter of the top of a vessel, in the form of a paraboloid, is 54 inches, and the perpendicular depth 64 inches; required the content in ale gallons.

Ans. 259.885 *ale gallons.*

52. The diameter of the top of a vessel, in the form of a hyperboloid, is 60 inches, and the middle diameter, and the perpendicular depth, each 42 inches; required the content in wine gallons. *Ans.* 253.6128 *wine gallons.*

53. The bottom diameter of a vessel, in the form of the frustum of a paraboloid, is 72, the top diameter 48, and the perpendicular depth 30 inches; required the content in ale gallons. *Ans.* 312.824 *ale gallons.*

54. The bottom diameter of a vessel, in the form of the frustum of a hyperboloid, is 80, the middle diameter 68, the top diameter 48, and the perpendicular depth 96 inches; required the content in wine gallons.

Ans. 1479.68 *wine gallons.*

55. The bottom diameter of a circular vessel, whose sides are curved, is 42, the middle diameter 47, the top diameter 45, and the perpendicular depth 48 inches; required the content in ale gallons.

Ans. 281.297 *ale gallons.*

56. The bottom diameter of a circular vessel measures 69, the diameter at one-fourth of the depth, from the bottom, 87, the middle diameter 95, the diameter at three-fourths of the depth 90, the top diameter 78, and the depth 80 inches; how many gallons of porter will the vessel contain? *Ans.* 1700.246 *gallons.*

57. What is the content of a circular vessel, in wine

whose perpendicular depth is 90 inches, and the iameters of seven equi-distant, parallel sections as viz. the diameter of the bottom or first section ie second 97.5, the third 107.4, the fourth 109.3, 110.7, the sixth 104.6, and the diameter of the eventh section 96.2 inches?

Ans. 3284.926 *wine gallons.*

Vhat is the content of a cylindrical ungula, in ons; the diameter of its base being 50 inches, the depth 32 inches, and the least depth 23 inches?

Ans. 191.477 *ale gallons.*

Vhat is the content of the hoof of a cylinder, in llons; the diameter of the vessel being 85 inches, sed sine of the hoof's base 36 inches, and the of the hoof 60 inches?

Ans. 248.788 *wine gallons.*

cylindrical vessel is placed in such a position liquor just covers one-half of its bottom, and the side 50 inches; what is the content of the in ale gallons; the diameter of the vessel being es? *Ans.* 106.382 *ale gallons.*

The chord of the base of a cylindrical ungula, s 60 inches, the versed sine 36 inches, and the of the ungula 50 inches; what is its content in llons? *Ans.* 167.580 *wine gallons.*

he length of a cylindrical vessel is 60 inches, its r 50 inches, and the depth of the liquor, when el is laid upon a horizontal plane, 40 inches; how e gallons does it contain?

Ans. 358.284 *ale gallons.*

he length of a cylindrical vessel is 60 inches, neter 50 inches; and the depth of the liquor, at , when the vessel is laid upon an inclined plane, s 40 inches, and at the other 25 inches; how ine gallons does the vessel contain?

Ans. 349.344 *wine gallons.*

Vhat is the content, in ale gallons, of the ungula ctangular, pyramidical vessel; the length of the the ungula being 87 inches, the breadth of the inches, the length of the upper edge of the liquor es, and the perpendicular height 63 inches?

Ans. 584.872 *ale gallons.*

65. The bottom diameter of a conical vessel is 60 inches, the top diameter 50 inches, and the perpendicular depth 40 inches. Now, if this vessel be placed in such a position that the liquor just covers its bottom, and rises up one side to the perpendicular height of 30 inches; required the content of the ungula, in wine gallons.

Ans. 177.7541 *wine gallons.*

66. The top diameter of a conical vessel is 60 inches, the bottom diameter 50 inches, and the perpendicular depth 40 inches. Now, if this vessel be placed in such a position that the liquor just covers the bottom, and rises up one side to the perpendicular height of 20 inches; how many ale gallons must be poured in, so that by changing the position of the vessel, the liquor may still just cover the bottom and rise to the top of the vessel?

Ans. 74.63532 *ale gallons.*

67. The bottom diameter of a conical vessel is 70 inches, the top diameter 60 inches, and the perpendicular depth 50 inches. Now, if this vessel be placed in such a position that the liquor just covers the bottom, and rises to the top of one side; how may ale gallons will fill the vessel, if it be placed with its base upon a horizontal plane? *Ans.* 260.8031 *ale gallons.*

68. The bottom diameter of a conical vessel is 70 inches, the top diameter 75 inches, the versed sine of the base of the elliptic ungula 60 inches, and its perpendicular height 81 inches; what is the content of the ungula in ale gallons? *Ans.* 550.086 *ale gallons.*

69. What is the content of an elliptic ungula, in ale gallons; the top diameter of the conical vessel being 55 inches, the bottom diameter 40 inches, the versed sine of the base of the ungula 18 inches, and its perpendicular height 21 inches? *Ans.* 18.192 *ale gallons.*

70. What is the content of a parabolic ungula, in wine gallons; the bottom diameter of the conical vessel being 50 inches, the top diameter 37 inches, and the perpendicular height of the ungula 17.4 inches?

Ans. 12.0102 *wine gallons.*

71. The head diameter of a cask is 26, the bung diameter 32, and the length 45 inches; required the con-

ale and wine gallons; admitting it to be a cask of ' variety.

The content is 113.800 *ale gallons, and* 138.922 *llons.*

'he head diameter of a cask is 26, the bung dia- 2, and the length 45 inches; required the content nd wine gallons; admitting it to be a cask of the ariety.

The content is 113.198 *ale gallons, and* 138.187 *'lons.*

'he head diameter of a cask is 26, the bung dia- 2, and the length 45 inches; required the content ıd wine gallons; admitting it to be a cask of the riety.

The content is 106.530 *ale gallons, and* 130.048 *'lons.*

'he head diameter of a cask is 26, the bung dia- 2, and the length 45 inches; required the content ıd wine gallons; admitting it to be a cask of the ariety.

The content is 105.779 *ale gallons, and* 129.130 *llons.*

'he area of an equilateral triangle is 45 ale gal- :quired its side. *Ans.* 171.190 *inches.*

'he side of a vessel in the form of a triangular s 50 inches, and its depth 40 inches; what is the 'e, in ale gallons, between its content, and that of lrical vessel that will just contain the prismatic *Ans.* 217.787 *ale gallons.*

'he length of a cooler is 80 inches, and its breadth :s. Now if it be cut by a plane parallel to the side; required the dimensions of the two vessels med, when the area of the greater vessel is 12 ale

The length of each vessel is 60 *inches, the of the greater* 56.4 *inches, and that of the less* 23.6

'he length of a rectangular cistern is 86 inches, lth 64 inches, and its depth 48 inches. Now, if : by a plane parallel to the shorter side; required

the dimensions of the two cisterns thus formed, when the content of the less is 380 ale gallons.

Ans. The length of each cistern is 64 *inches, the breadth of the greater* 51.118 *inches, and that of the less* 34.882 *inches.*

79. The sides of a triangular cooler are 60, 70, and 80 inches, respectively. Now, if it be cut by a plane parallel to the longest side; required the dimensions of the remaining triangle, when the area of the portion parted off is three ale gallons.

Ans. The three sides of the remaining triangle are 45.849, 53.490, *and* 61.132 *inches, respectively.*

80. The sides of a triangular cooler are 60, 70, and 80 inches, respectively. Now, if it be cut by a plane passing from the opposite angle to the middle of the longest side; required the areas, in ale gallons, of the two vessels thus formed.

Ans. The area of each vessel is 3.605 *ale gallons.*

81. If the length of a cistern be 94, and its breadth 72 inches; what must be its depth, in order that it may contain 16 quarters of barley? *Ans.* 40.669 *inches.*

82. The perpendicular depth of a cistern is 42, its breadth 108, and the diagonal of its bottom 180 inches; how many bushels of barley can be steeped in this cistern at one time, allowing $\frac{1}{5}$ of the whole content for the *swell* of the grain? *Ans.* 242.997 *bushels.*

83. The content of a cylindrical vessel is 9743 ale gallons, and the diameter of its base 147 inches; required its perpendicular depth. *Ans.* 161.888 *inches.*

84. The diagonal of the base of a cubical vessel, is 72 inches; required the content in wine gallons.

Ans. 571.268 *wine gallons.*

85. The diameter of the legal Winchester bushel is $18\frac{1}{2}$ inches, and its depth 8 inches; what is the diameter of that bushel whose depth is 9 inches?

Ans. 17.441 *inches.*

86. The bottom diameter of a vessel, in the form of the frustum of a cone, is 150 inches, the top diameter 86 inches, and the slant height 68 inches; what is the content in ale gallons? *Ans.* 2383.846 *ale gallons.*

87. The slant height of a cistern in the form of the frustum of a square pyramid, is 85 inches, the perpendi-

eight 75, and the side of the base 195 inches; the content in malt bushels?

Ans. 513.883 *malt bushels.*

'he content of a vessel in the form of a pentagom, is 8834 wine gallons, and the side of its base ches; required its perpendicular depth.

Ans. 94.005 *inches.*

reservoir measures 182 inches in length, 112 in , and 74 in depth; how long will a person be in with water, by means of a pump; supposing he 6 strokes in a minnte, and lifts 3 pints of water stroke? *Ans.* 9 *hours,* 8.614 *minutes.*

'he bottom diameter of a vessel, in the form of tum of a cone, is 86 inches, the top diameter 62 and the perpendicular depth 58 inches; how allons of ale are contained in the vessel, when the f the liquor is 40 inches?

Ans. 615.401 *ale gallons.*

'he perpendicular depth of a vessel, in the form frustum of a square pyramid, is 65 inches, the the top 68 inches, and the side of the bottom 94 how many wine gallons does the vessel contain, ie depth of the liquor is 45 inches?

Ans. 1412.725 *wine gallons.*

'he perpendicular depth of a vessel, in the form of tum of an elliptical cone, is 65 inches; the transnd conjugate diameters of the bottom measure 86 inches; the transverse and conjugate diameters op, 62 and 42 inches; how many ale gallons does el contain, when the depth of the liquor is 42 in-

Ans. 485.1191 *ale gallons.*

'he perpendicular altitude of a conical vessel, is es, and the diameter of its base 64 inches. Now, essel be cut by a horizontal plane; required the of the two vessels thus formed, in ale gallons, he content of the frustum is to that of the seg- s two to one.

The content of the frustum is 220.552, *and the of the segment* 110.276 *ale gallons.*

'he side of the base of a vessel, in the form of tum of a square pyramid, is 60 inches, the side op 40 inches, and the perpendicular depth 84 in-

ches. Now, if this vessel be cut by a plane parallel to the base; required the contents of the two vessels thus formed, in wine gallons, when the content of the greater is to that of the less, as five to four.

Ans. The content of the greater vessel is 511.784, *and the content of the less* 409.4272 *wine gallons.*

95. The diameter of the base of a conical vessel, is 60 inches, and its perpendicular altitude 72 inches. Now, if this vessel be cut by a plane parallel to the base; required the dimensions of the two vessels thus formed, when their contents are equal.

Ans. The diameter of the section made by the cutting plane is 47.622 *inches, the altitude of the segment* 57.146 *inches, and that of the frustum* 14.854 *inches.*

96. The side of the base of a vessel in the form of the frustum of a square pyramid, is 60 inches, the side of the top 40 inches, and the perpendicular height 50 inches. Now, if this vessel be cut by a plane parallel to the base; required the dimensions of the two vessels thus formed, when their contents are equal.

Ans. The side of the section made by the cutting plane, is 51.924 *inches, the altitude of one of the vessels* 29.812 *inches, and that of the other* 20.188 *inches.*

97. The internal diameter of a spherical vessel is 74 inches; and if it be cut by a horizontal plane, which makes the depth of the less segment 25 inches; required the contents, in ale gallons, of the two vessels thus formed.

Ans. The content of the greater segment is 552.794, *and that of the less* 199.599 *ale gallons.*

98. The diameter of the base of a vessel, in the form of a prolate semi-spheroid, is 50, and its perpendicular altitude 60 inches. Now, if this vessel be cut by a horizontal plane, at the distance of 30 inches from the base; required the contents, in wine gallons, of the two vessels thus formed.

Ans. The content of the frustum is 233.748, *and the content of the segment* 106.249 *wine gallons.*

99. The diameter of the base of a vessel, in the form of a paraboloid, is 50, and its perpendicular altitude 60 inches. Now, if this vessel be cut by a horizontal plane, at the distance of 30 inches from the base; required the

, in wine gallons, of the two vessels thus

The content of the frustum is 191.248, *and the con-*
he segment 63.748 *wine gallons.*

At Konigstein, near Dresden, in Germany, is a
ose head diameter is 25 feet or 300 inches, bung
: 26 feet or 312 inches, and perpendicular altitude
r 336 inches; how many gallons of wine will it
admitting it to be of the third variety?

The content of this enormous cask, is 107010 *wine*
which exceeds the content of the cask at Heidelberg,
$4\frac{1}{2}$ *gallons. (See Example 5, Prob. IV., Part V.)*

The Konigstein cask was begun in the year 1722, and finish-
5, under the direction of General Kyaw; and is considered
largest cask in the world. It consists of 157 staves, each 8
thickness; and one of its heads is composed of 26, and the
8 boards. The top or upper head of this enormous cask is
ıd, and affords sufficient room for twenty persons to regale
s.

Remark.

ıg given copious directions for finding the Areas
tents of all Vessels that can possibly be met with
ice, and illustrated those directions by numerous
es, we come now to describe the methods of *gauging*
ıg the Utensils of Victuallers, Common Brewers,
s, Maltsters, Starch-makers, Soap-makers, Glass-
&c. &c. as practised in the Excise.

PART VI.

The Method of Gauging and Fixing Victuallers' Utensils; of Gauging and Inching Common Brewers' Utensils; and of Gauging and Ullaging Casks. Also, the Method of Gauging and Fixing Maltsters' Utensils; and of Gauging and Inching a Still, and a Distiller's Wash-Back. Likewise, the Method of Gauging and Fixing the Utensils of Starch Makers, Soap Makers, and Glass Makers, as practised in the EXCISE.

SECTION I.

THE METHOD OF GAUGING AND FIXING VICTUALLERS' UTENSILS, AS PRACTISED IN THE EXCISE.

PROBLEM I.

To gauge and fix a mash-tun, in the form of the frustum of a cone.

To take the dimensions.

With the Dimension Cane, or any other convenient instrument, take the perpendicular depth of the vessel.

Also, with the Diameter Rule, take two cross diameters of the top, at right-angles to each other; and likewise two cross diameters of the bottom. Add these diameters together; divide the sum by 4, and take the quotient for a mean diameter.

Note 1. A mean diameter may also be found by taking two cross

it one-half of the perpendicular depth, and dividing their sum

ls in the form of the frustum of a cone, are seldom per-lar; hence the propriety of taking cross diameters, in order ne the mean diameter.

Diameter Rule will take any dimension from 24 to 47 in-either of the diameters exceed 47 inches, the Dimension be used, which will take any diameter from 36 to 120

To find the area and content.

RULE.

the square of the mean diameter by 289, and ent will be the area in mash-tun gallons.

ly this area by the depth of the vessel, or by the ' the grains, generally called *goods;* and the ict will be the content in mash-tun gallons.

If the area be multiplied by the depth of the vessel, the ill be the whole content of the vessel; but if multiplied th of the *goods*, it is evident the product will be the content s, in mash-tun gallons.

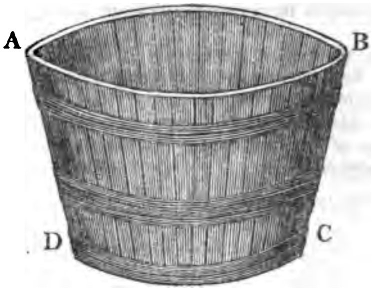
-tuns are generally in the form of the frustum of a cone; netimes placed on their less, and sometimes on their greater whatever position, however, a victualler's round mash-tun t is always gauged according to the foregoing directions. gh this method is not mathematically correct, yet it is con-irate enough for victuallers' mash-tuns; because they are nall vessels, and no duty arises from mash-tun gauges.

h-tuns of Common Brewers are gauged and fixed in a very anner. (See the next Section.)

ne divisors and gauge-points used in this and the follow-ns, may be found in the Table of Factors, Part IV.

EXAMPLES.

the following figure A B C D, represent a mash-the form of the frustum of a cone, whose per-ir depth is 40.5 inches, the cross diameters of the and 64.4 inches, and the cross diameters of the 5.2 and 45.6 inches; required its area and con-ash-tun gallons.



By the Pen.

To find the area.

Inches.
65.2 } *top cross diameters.*
64.4 }
45.2 } *bottom ditto.*
45.6 }
4)220.4 *sum.*
55.1 *mean diameter.*
55.1 *ditto.*
551
2755
2755
Divisor 289)3036.01(10.505 *mean area in gallons.*

To find the content.

Mash-tun gallons.
10.51 *area.*
40.5 *depth.*
5255
4204
425.655 *content in gallons.*

BY THE SLIDING RULE.

To find the area.

ne circular gauge-point on D, is to unity on C; so nean diameter on D, to the area on C. Or, as the cular divisor on A, is to the mean diameter on B; e mean diameter on A, to the area on B.

On D.		On C.		On D.		On C.
As 17.07	:	1	::	55.1	:	10.51 *area.*

Or,

On A.		On B.		On A.		On B.
s 289	:	55.1	::	55.1	:	10.51 *area.*

To find the content.

he circular gauge-point on D, is to the depth on is the mean diameter on D, to the content on C. unity on A, is to the area on B; so is the depth on he content on B.

n D.		On C.		On D.		On C.
7.07	:	40.5	::	55.1	:	425.66 *content.*

Or,

n A.		On B.		On A.		On B.
ls 1	:	10.51	::	40.5	:	425.66 *content.*

1. The dimensions and area thus found are transferred to the n of the dimension book, at the end of this section; and the rocess is called *gauging* and *fixing* the mash-tun.

Practice, the area must always be found to three places of s; and in transferring to the dimension book, if the third place above, the second is called one more; but if the third place be , it is rejected.

is customary in the practice of gauging to take the depth of ls to the nearest whole inch, and to enter the content in gal- ence, if the first decimal, in the content, be .5 or above, it is gallon, but if it be under .5, it is rejected; consequently the ontent would be called 426 gallons.

The perpendicular depth of a circular mash-tun, in rm of the frustum of a cone, is 50.5 inches, the diameters taken at one-half of the perpendicular , are 73.4 and 72.6 inches; required the area and it of the vessel, in mash-tun gallons.

. *The area is* 18.439, *and the content* 931.22 *mash- llons.*

A a

Note. In the real practice of gauging, the content is always found from the depth of the goods; but in the foregoing and following examples, we have found the whole content of the vessel. The process, however, in both cases is the same.

REMARKS.

1. By the Rule for the frustum of a cone, Prob. VIII., Part V., we find the content of the mash-tun in the first example, to be 429.82 gallons; hence the content found by this Problem is too little by 4 gallons.

2. It frequently happens that the surface of the grains is several inches below the top of the tun, when it is gauged by the Officer. Now, when this is the case, the mean area multiplied by the depth of the grains will evidently give the content too much when the vessel stands upon its less end, and too little when it stands upon its greater end: for example, suppose the depth of the grains in the foregoing mash-tun, question the first, to be only 32 inches; then we have $10.51 \times 32 = 336$ gallons, the content found by using the mean area obtained when the vessel is supposed to be full. But this mean area is evidently too great; for the diameter at the top of the grains, found by Prob. XIV. is only 60.8 inches; hence we have $(60.8+45.4) \div 2 = 106.2 \div 2 = 53.1$, the mean diameter; and $(53.1 \times 53.1) \div 289 = 2819.61 \div 289 = 9.76$ gallons, the mean area; then $9.76 \times 32 = 312$, the content in gallons; consequently, the former content is too much by 24 gallons.

3. Again, let the same vessel be inverted, so as to stand upon its greater base, and let the depth of the grains be 32 inches, as before; then it is evident that the content will still be 336 gallons, by using the mean area obtained when the vessel is supposed to be full. But this mean area is evidently too little; for the diameter at the top of the grains, found by Prob. XIV., is 49.4 inches; hence we have $(64.8+49.4) \div 2 = 114.2 \div 2 = 57.1$, the mean diameter; and $(57.1 \times 57.1) \div 289 = 3260.41 \div 289 = 11.28$ gallons, the mean area; then $11.28 \times 32 = 361$, the content in gallons; consequently, the former content is too little by 25 gallons.

4. Notwithstanding the method of gauging and fixing victuallers' mash-tuns, as described in the foregoing Problem, has been generally adopted in practice, yet it is necessary to observe that by a late Act of the 1st and 2nd Geo. IV.

2, the greatest accuracy is required in gauging nputing the contents of mash-tuns.

, as it frequently happens that victuallers use more : one brewing than at another, it is evident, as beserved, that a mean fixed area multiplied by the of the goods cannot always give the true content h-tun gallons; these irregularities, however, may y obviated, by taking diameters in the middle of en inches, and fixing the mash-tun in the same as a guile-tun. This method becomes absolutely y, when the mash-tun has curved sides.

PROBLEM II.

ge and fix a mash-tun in the form of the frustum of a square pyramid.

To take the dimensions.

any convenient instrument, take the perpendicuh of the tun.

measure each side of the top; divide the sum of mensions by 4; and the quotient will be a mean on. Find a mean dimension of the bottom in the anner; then half the sum of these two mean dis will be the mean dimension of the middle.

The mean dimension of the middle may also be found by ; the sides of the vessel at half of the perpendicular depth. els that assume the form of the frustum of a square pyraeldom perfectly square; hence the necessity of measuring all n order to determine the mean dimensions.

To find the area and content.

RULE.

ply the mean dimension of the middle of the itself; divide the product by 227, and the quol be the area in mash-tun gallons.

ly this area by the perpendicular depth; and uct will be the content in mash-tun gallons.

EXAMPLES.

the following figure A B C D E F G, represent tun, in the form of the frustum of a square

and bottom for a mean length, and half the sum of the breadths for a mean breadth; then multiply the mean length by the mean breadth, divide the product by 227, and the quotient will be the mean area, in mash-tun gallons.

2. If a mash-tun be a parallelopipedon, the area of its base may be found by Prob. II.; if it be a prism, by Prob. IX.; and if it be a cylinder, by Prob. XIII., Part IV.

PROBLEM III.

To gauge and fix a copper with a falling crown.

To take the dimensions.

The most eligible method of taking the dimensions of circular vessels, is to quarter them, in order to obtain cross diameters as correctly as possible.

First, take the diameter of the top of the vessel, which multiply by .707, or .7; and the product will be the side of the inscribed square. (See Rule III., Prob. XIII., Part IV.)

Apply the side of the square thus found, four times to the top of the copper, by your dimension cane, or tape; and mark, with chalk, each angle of the inscribed square; and thus will the top of the vessel be quartered.

With a steady hand, draw a chalk line, from each quartering point, down the inside of the vessel, to the bottom; taking care that these lines be equally distant from each other, not only at the bottom of the vessel, but also at every horizontal section; otherwise the vessel will not be truly quartered in every part.

Place your dimension cane, or other instrument, in a perpendicular direction, with its end resting upon the bottom of the copper, half-way between the centre of the crown and the side; at the same time laying a rod or holding a cord diametrically across the top, so as to come in contact with the dimension cane; and thus you will have the perpendicular depth of the copper, upon the dimension cane, where it intersects the rod or cord at the top.

Now, in order to take cross diameters in the middle of

en inches, mark the dimension cane with chalk, 5, 25, 35 inches, &c.; and place that instrument in ne manner in which it stood in taking the depth; ring a rod horizontally in contact with the cane, at es from the bottom, so that the rod and cane may ight angles; and let the end of the rod, at the ime, touch one of the quartering lines; and there mark with chalk, on the inside of the vessel; and ill have one point at which a diameter must be

he same at 15, at 25 inches, &c. from the bottom; u will obtain all the points in one of the quarter-es, at which diameters must be taken.

se points may then be transferred to the other three ing lines, by a pair of *compasses* or a *dipping-piece;* ence you may proceed to take cross diameters as directed; always beginning at the bottom.

1. The sum of each pair of cross diameters must be divided by ler to obtain mean diameters.

hen the sides of a copper or other vessel, are much curved, at accuracy is required, cross diameters must be taken in the f every 6 or 8 inches; but in *practice*, 10 inches is the most nt number, for in multiplying the area, corresponding to each ameter, by 10, it is only necessary to remove the decimal point e towards the right-hand.

To find the area and content.

RULE I.

l the area corresponding to each mean diameter, in able of *Ale Areas*, Part VII.; then multiply each y its respective depth; and the sum of the products e the whole content required. (See Prob. XIII., V.)

RULE II.

ide the square of the first mean diameter by ; and the quotient will be the area of the first ntal section, in ale gallons.

d the area of each horizontal section in the same r; then multiply each area by its respective per-

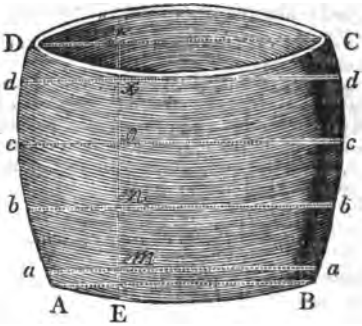
pendicular depth; and the sum of the products will be the whole content of the copper.

Note 1. If it be required to find the quantity of liquor contained in a copper, at any assigned depth, suppose 24.8 inches, when the mean diameters are taken in the middle of every 10 inches, remove the the decimal points, in the first and second areas, one place towards the right; and multiply the third area by 4.8, and the sum of the three products will be the content of the copper, at 24.8 inches of the perpendicular depth.

2. It is scarcely necessary to observe that the depth of the liquor must always be taken half-way between the side of the vessel and the centre of the crown, as before directed.

EXAMPLES.

1. Let the following figure A B C D represent a copper with a falling crown; it is required to gauge and fix it, according to the method practised in the Excise.



To take the dimensions.

First, take the diameter C D of the top of the copper, which you will find to be 49 inches; then, $49 \times .7 = 34.3$ inches, the side of the inscribed square.

With this number quarter the top of the copper, as before directed; and draw four chalk lines from the quartering points to the bottom.

Measure the diameter A B, which you will find to be 40 inches; and $40 \div 4 = 10$. Set off 10 inches from A to E; and E will be the point that is half-way between the centre of the crown and the side of the vessel; hence we

nd the perpendicular depth E r, to be 40 in-

; made marks, with chalk, upon the dimension
, 15, 25, and 35 inches from the bottom, place
ument perpendicularly at E; then bring a rod
with it at m, 5 inches from the bottom, forming
ontal line $m\,a$, and mark the side of the copper

same at n, o, and x; namely, at 15, 25, and 35
om the bottom; and you will obtain the points
d.

transfer these points to the other three quarter-
; take the necessary cross diameters; and enter
mensions in your Note Book.

; the depth of the copper is exactly 40 inches, the upper
l, falls at 5 inches from the top; but if the depth had been
hen the upper diameter would have fallen at 4 inches from
amely, in the middle of the last 8 inches of the depth. If
ad been 42 inches, the upper diameter might have been
nches from the top; but if 44 inches, then it would have
ent to take five diameters, in which case the upper diame-
ll at 2 inches from the top; namely, in the middle of the
hes of the depth; and in like manner for any other depths.

BY THE TABLE OF ALE AREAS.

To find the area.

; found the area corresponding to each mean
as directed in Rule I.; those areas may be en-
the Note Book, opposite to their respective dia-
s below.

NOTE BOOK.

B.'s Copper, No. 1, gauged Jan. 23, 1821.

ns es.	Depths from the bottom.	Cross Diameters.		Sum.	Mean Diameters.		Areas.
	35	50.7	50.5	101.2	50.6	*dd*	7.13
	25	51.6	51.4	103.0	51.5	*cc*	7.39
	15	48.4	48.6	97.0	48.5	*bb*	6.55
	5	44.5	44.7	89.2	44.6	*aa*	5.54

BY THE SLIDING RULE.

In order to *prove* or *check* the results obtained by the Table of Ale Areas, it will be necessary to find the area of each section by the Sliding Rule, which may be done by either of the following proportions: As the circular divisor, 359.05, on A, is to the mean diameter on B; so is the mean diameter on A, to the area on B. Or, As the circular gauge-point, 18.95, on D, is to unity on C; so is the mean diameter on D, to the area on C. (See Prob. XIII., Part IV.)

$$\begin{array}{cccc} \text{On A.} & \text{On B.} & \text{On A.} & \text{On B.} \\ \textit{As}\ 359.05 : & \left\{\begin{array}{c} 50.6 \\ 51.5 \\ 48.5 \\ 44.6 \end{array}\right\} :: & \left\{\begin{array}{c} 50.6 \\ 51.5 \\ 48.5 \\ 44.6 \end{array}\right\} : & \left\{\begin{array}{l} 7.13\ \textit{section dd.} \\ 7.39\ \textit{section cc.} \\ 5.55\ \textit{section bb.} \\ 5.54\ \textit{section aa.} \end{array}\right. \end{array}$$

Or,

$$\begin{array}{cccc} \text{On D.} & \text{On C.} & \text{On D.} & \text{On C.} \\ \textit{As}\ 18.95 : & 1 : & \left\{\begin{array}{c} 50.6 \\ 51.5 \\ 48.5 \\ 44.6 \end{array}\right\} : & \left\{\begin{array}{l} 7.13\ \textit{section dd.} \\ 7.39\ \textit{section cc.} \\ 6.55\ \textit{section bb.} \\ 5.54\ \textit{section aa.} \end{array}\right. \end{array}$$

Note. The depth, mean diameters, and areas, are transferred to the Specimen of the Dimension Book, at the end of this section.

REMARK.

It is immaterial whether we begin at the top or the bottom section to find the areas. In the last example, we have begun at the uppermost section, and found the areas downwards, in succession; but in most of the following Problems, the areas of the sections are found in the same order that the dimensions are taken. It may, however, be observed, that it is found most convenient in the practice of *gauging* and *fixing* Victuallers' Utensils, to enter the dimensions and areas, in the Officer's Dimension Book, in such a manner that the dimensions of the uppermost section and its area may stand at the top of the

book; and the other dimensions and areas follow in succession. (See the Note Book; and also the Specimen of the Dimension Book at the end of this Section.)

To find the content of the foregoing copper.

As the diameters are taken in the middle of every 10 *inches, we have (by removing each decimal point one place towards the right)* 55.4 *ale gallons, the content of the first* 10 *inches from the bottom;* 65.5, *the content of the second* 10 *inches;* 73.9, *the content of the third* 10 *inches; and* 71.3, *the content of the fourth* 10 *inches: Then,*

Ale Gallons.

55.4
65.5
73.9
71.3
266.1 *whole content.*

2. How many ale gallons are contained in the foregoing copper, when the mean depth of the liquor is 24.8 inches?

CONTENTS.

Ale Gallons.

	55.4	*first* 10 *inches*
	65.5	*second* 10 *inches*
$7.39 \times 4.8 =$	35.472	*upper* 4.8 *inches.*
	156.372	*Ans.*

3. The first mean diameter of a copper, taken at the distance of 3 inches from the bottom, measures 48.4 inches; the second at 9 inches from the bottom, 54.6 inches; the third at 15 inches from the bottom, 57.6 inches; the fourth at 21 inches from the bottom, 59.2 inches; the fifth at 27 inches from the bottom, 56.8 inches; and the sixth at 34.2 inches from the bottom, namely, in the middle of the last 8.4 inches of the depth, measures 54.4 inches; required the area of each section, and the

whole content in ale gallons; the mean depth being 38.4 inches.

Ans. By Rule II., the area of the first section is found to be 6.52, *the area of the second section* 8.30, *the area of the third section* 9.24, *the area of the fourth section* 9.76, *the area of the fifth section* 8.98, *and the area of the sixth section* 8.24 *ale gallons.*

Also, the content of the first 6 *inches from the bottom, is* 39.12; *the content of the second* 6 *inches,* 49.8; *the content of the third* 6 *inches,* 55.44; *the content of the fourth* 6 *inches,* 58.56; *the content of the fifth* 6 *inches,* 53.88; *and the content of the last* 8.4 *inches,* 69.216 *ale gallons; hence the whole content of the copper is* 326.016 *ale gallons.*

Remark.

The coppers of Victuallers have, in general, *falling* crowns; but should you meet with a Victualler's copper that has a *rising* crown, you must proceed, in every respect, as directed in the last Problem; namely, by quartering the copper; taking cross diameters; measuring the mean depth half-way between the side of the copper and the centre of the crown, &c. &c.

PROBLEM IV.

To gauge and fix an under back.

Under backs have, in general, either rectangular, circular, or elliptical bases; and perpendicular sides.

To take the dimensions.

If the back be rectangular, with some convenient instrument take the mean length and breadth; if it be circular, take cross diameters, from which find the mean diameter; and if it be elliptical, measure the transverse and conjugate diameters. Also, take the perpendicular depth, and enter all the dimensions in your Note Book.

To find the area and content.

RULE.

If the back be rectangular, divide the product of the length and breadth by 282; if it be circular, divide the square of the mean diameter by 359.05; and if it be elliptical, divide the product of the two diameters by 359.05; and the respective quotients will be the area of the base in ale gallons. Multiply this area by the perpendicular depth, and the product will be the content in ale gallons.

Note. When the sides of an under-back are not perpendicular to the base, it must be gauged in the same manner as a guile-tun. (See Problems VI., VII., VIII., IX., and X., of this Section.)

EXAMPLES.

1. The mean length of a rectangular under-back, is 58.6 inches, the mean breadth 40.4 inches, and the perpendicular depth 25.8 inches; required the area and content, in ale gallons.

BY THE PEN.

To find the area.

```
             Inches.
              58.6 length.
              40.4 breadth.
             ----
             2344
           2344
           -------
Divisor 282)2367.44(8.395 ale gallons.
```

To find the content.

```
Ale gallons.
     8.4 area.
    25.8 depth.
   -----
     672
    420
   168
   ------
   216.72 content.
```

BY THE SLIDING RULE.

The proportions for rectangular, circular, and elliptical vessels, may be seen in Problems II., XIII., and XVI., Part IV.; and Problems II., and IV., Part V.

To find the area.

	On A.		On B.		On A.		On B.
As	282	:	58.6	::	40.4	:	8.40 *area.*

To find the content.

	On A.		On B.		On A.		On B.
As	1	:	8.4	::	25.8	:	216.72 *content.*

Note. The dimensions and area are transferred to the Specimen of the Dimension Book, at the end of this Section.

2. Required the area and content of a cylindrical under-back, whose mean diameter is 38.6, and depth 32.4 inches.

Ans. The area is 4.15, *and the content* 134.46 *ale gallons.*

3. Required the area and content of an elliptical under-back, whose transverse diameter is 58.6, conjugate 40.4, and depth 28.6 inches.

Ans. The area is 6.59, *and the content* 188.474 *ale gallons.*

REMARK.

Circular and elliptical backs or coolers are not unfrequently met with in Victuallery; and are gauged and fixed precisely in the same manner as under-backs. A constant dipping place is found as directed in the next Problem.

Note. Elliptical coolers are generally wider at the top than at the bottom; hence it is necessary to take the transverse and conjugate diameters at such a distance from the base, as will give the mean area of the liquor.

PROBLEM V.

To gauge and fix a rectangular back or cooler.

To take the dimensions.

(See the following figure.)

With your dimension-tape, take the breadth of the end A D. Double the tape, and extend it from A to E, at which place make a mark, on the cooler, with chalk; and you will have divided the end A D into two equal parts. Find the middle of the end B C, and also the middle of the sides A B and C D, in the same manner; then with your dimension-cane or other proper instrument, measure the mean length E F, and the mean breadth G H; and enter the dimensions in your Note Book.

Now, as coolers are always fixed in an inclining position, in order that the wort may drain off, it is necessary to take the depth of the liquor in several places, and divide the sum of these depths by their number for a mean depth. If ten dips be taken, the division may be performed by removing the decimal point one place towards the left.

Note. When a cooler is placed in such a situation that you cannot get round it, a plank may be thrown across it, to enable you to take dips in various parts of the cooler, in order to obtain the mean depth with as much accuracy as possible.

To find a constant dipping-place.

As it would be very inconvenient for an Officer to determine the mean depth of the wort, as directed in the last article, every time he has to gauge a cooler, it is necessary to find a constant dipping-place, which may be done in the following manner: Having obtained the mean depth, as already directed, try in different parts of the cooler, until you find a place where the depth of the liquor exactly coincides with the mean depth; then make a mark upon the side of the cooler, with white paint, composed of white lead and oil, as at x; and you will

have a constant dipping-place, which will always give you the mean depth, whatever liquor may be in the cooler.

For the sake of convenience the constant dipping-place should not be far from the side of the cooler; but should you not be able to find a suitable place near the side, fix upon the most convenient part of the cooler, and observe whether the depth taken there be more or less than the mean depth. If it be less, the deficiency must be added; but if it be more, the excess must be subtracted. For example, suppose the mean depth to be 4.5, and the depth at the constant dipping-place 4.2; then .3 must be added; but if the depth at x be 4.7, then .2 must be subtracted.

The number to be added must be marked upon the side of the cooler, with paint, thus, +.3; and the number to be subtracted thus, −.2. The first of these numbers is read, *plus three-tenths;* and the second, *minus two-tenths*.

Note. If the cooler be empty, it will be necessary to cover its bottom with water, in order to find a *mean* dipping-place.

To find the area and content.

RULE.

Multiply the mean length by the mean breadth, divide the product by 282; and the quotient will be the area in ale gallons. Multiply this area by the mean depth, and the product will be the content in ale gallons.

EXAMPLES.

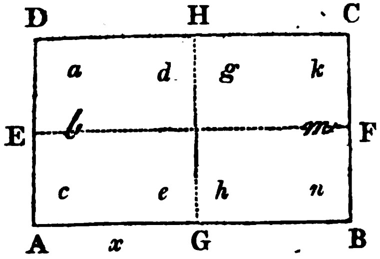
1. The mean length E F of a cooler is 116.8 inches, the mean breadth G H 68.2 inches, and the depths of the wort, taken in ten different places, as below; required the area and content in ale gallons.

Depth in Inches.

At a	=	4.3
b	=	4.2
c	=	4.2
d	=	4.5
e	=	4.4
g	=	4.6
h	=	4.5
k	=	4.9
m	=	4.8
n	=	4.6

10)45.0 *sum.*

4.5 *mean depth.*



BY THE PEN.

To find the area.

Inches.

116.8 *length.*
68.2 *breadth.*

2336
9344
7008

Divisor 282)7965.76(28.247 *area in ale gallons.*

To find the content.

Ale Gallons.
28.25 *area.*
4.5 *depth.*
14125
11300
127.125 *content.*

BY THE SLIDING RULE.

To find the area.

As the square divisor on A, is to the length on B; *so* is the breadth on A, to the area on B.

On A.		On B.		On A.		On B.
As 282	:	116.8	::	68.2	:	28.25 *ale gallons.*

To find the content.

As unity on A, is to the area on B; so is the depth on A, to the content on B.

On A.		On B.		On A.		On B.
As 1	:	28.25	::	4.5	:	127.13 *ale gallons.*

Note 1. The length, breadth, and area are transferred to the specimen of the Dimension Book, at the end of this Section.

2. It appears from the foregoing depths that the constant dipping-place may be either at *d* or *h*; but as both these places would be inconvenient for that purpose, it will be necessary to fix upon one nearer to the side of the cooler, as at *x*.

3. The method of deducting the *heat* from *warm* wort is described in Problem XIII.

2. If the mean depth of the wort, in the foregoing cooler, be 6.4 inches; how many ale gallons does it contain? *Ans.* 180.8 *ale gallons.*

3. The mean length of a victualler's cooler, is 98.3 inches, the mean breadth 74.8 inches, and the mean depth

of the wort 7.2 inches; required the area and content in ale gallons.

Ans. The area is 26.07, *and the content* 187.704 *ale gallons.*

REMARK.

The bases of coolers are more commonly rectangles than any other figures; if, however, the base of a cooler be a rhombus or rhomboides, its area may be found by Problem III.; if a triangle, by Problem IV. or V.; if a trapezium, by Problem VI.; if a trapezoid, by Problem VII.; if an irregular polygon, by Problem VIII.; if a regular polygon, by Problem IX. or X.; if a circle, by Problem XIII.; if the sector of a circle, by Problem XIV.; if the segment of a circle, by Problem XV.; if an ellipse, by Problem XVI.; if the segment of an ellipse, by Problem XVII.; if a parabola, by Problem XVIII.; if a compound figure, by Problem XIX.; and if an oval greater or less than a true ellipse, or any other indetermined, curvilineal figure, its area may be found by the method of equi-distant ordinates, as described in Problem XX., Part IV.

Note. When victuallers' coolers are in the form of ovals greater or less than true ellipses, it becomes absolutely necessary to *gauge* and *fix* them by the method of equi-distant ordinates. But after the numerous examples that have been given in Problem XX., Part IV., it would be quite superfluous again to introduce this subject. It is only necessary to observe, that any figure in that Problem may be considered as the base of a cooler; and that the method of taking the dimensions and of finding the area, in the *real* Practice of Gauging, is precisely the same as there described. The manner of entering the dimensions and area, in the Officer's Dimension Book, is shown at the end of this Section.

PROBLEM VI.

To gauge and fix a guile-tun in the form of the frustum of a cone.

To take the dimensions.

First quarter the tun, as directed in Problem III.;

then take cross diameters in the middle of every 6, 8, or 10 inches; also, measure the perpendicular depth, and enter all the dimensions in your Note Book.

Note 1. If it be thought more convenient, both the top and bottom of the tun may be quartered, as directed in Problem III., Part VI.; care, however, must be taken to make the quartering points at the top and bottom, correspond with each other, in position, which may be done by setting off the two first points upon the joint made by any two adjoining staves. The quartering lines may then easily be drawn upon the sides of the tun, or struck by a chalk-line.

It may also be observed, that the perpendicular distances between the horizontal sections, must be *set off* upon the quartering lines in the same manner as directed in that Problem.

2. When the sides of the tun are much inclined, it will be necessary to take cross diameters in the middle of every 6 or 8 inches, in order to obtain the content with more accuracy.

To find the area and content.

RULE.

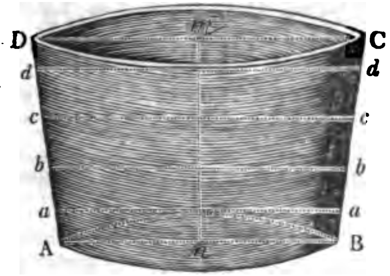
Find the area corresponding to each mean diameter, in the Table of *Ale Areas*, Part VII.; then multiply each area by its respective depth; and the sum of the products will be the whole content required. (See Prob. XIII., Part IV.)

Note. The area and content may also be found by Rule II., Prob. III.

EXAMPLES.

1. Let the following figure A B C D represent a guile-tun in the form of the frustum of a cone, whose perpendicular depth *m n* is 42 inches; the mean diameter *a a*, taken at 5 inches from the bottom, 63.7 inches; the diameter *b b*, taken at 15 inches from the bottom, 66.2 inches; the diameter *c c*, taken at 25 inches from the bottom, 68.7 inches; and the diameter *d d*, taken at 36 inches from the bottom, 71.4 inches; required the area of each section, and the whole content in ale gallons.

Note. The first three diameters are taken in the middle of every 10 inches; but the fourth diameter is taken in the middle of the last 12 inches of the depth.



BY THE TABLE OF ALE AREAS.

To find the area.

Having found the area of each section, as directed in the Rule, the dimensions and areas will appear in the Note Book, as exhibited below.

NOTE BOOK.

A. B.'s Guile Tun, No. 1, gauged Jan. 28, 1821.				
Divisions in Inches.	Depths from the Bottom.	Mean Diameters.		Areas.
12	36	71.4	*dd*	14.20
10	25	68.7	*cc*	13.14
10	15	66.2	*bb*	12.21
10	5	63.7	*aa*	11.30

BY THE SLIDING RULE.

As the circular divisor, 359.05, on A, is to the mean diameter on B; so is the mean diameter on A, to the area on B. Or, As the circular gauge-point, 18.95, on D, is to unity on C; so is the mean diameter on D, to the area on C. (See Prob. XIII., Part IV.; and also Prob. III., Part VI.)

$$\begin{array}{lcccc} & \text{On A.} & \text{On B.} & \text{On A.} & \text{On B.} \\ \textit{As} & 359.05 : & \begin{Bmatrix}71.4\\68.7\\66.2\\63.7\end{Bmatrix} :: & \begin{Bmatrix}71.4\\68.7\\66.2\\63.7\end{Bmatrix} : & \left\{\begin{array}{l}14.20 \textit{ section dd.}\\13.14 \textit{ section cc.}\\12.21 \textit{ section bb.}\\11.30 \textit{ section aa.}\end{array}\right. \end{array}$$

Note. The depth, mean diameters, and areas are transferred to the specimen of the Dimension-Book, at the end of this Section.

To find the content.

Contents.

	Ale gal.	
	113.0	*first* 10 *inches.*
	122.1	*second* 10 *inches.*
	131.4	*third* 10 *inches.*
$14.2 \times 12 =$	170.4	*upper* 12 *inches.*
	536.9	*whole content.*

Note. The bottom diameter of the foregoing guile-tun, is 62.4 inches, the top diameter 72.9 inches, and the perpendicular depth 42 inches; from which dimensions we find the *true* content, by Rule I., Prob. VIII., Part V., to be 536.4 ale gallons. Hence it appears that the Rule in this Problem gives the content only half a gallon too much, in 536 gallons.

2. How many ale gallons are contained in the foregoing guile-tun, when the depth of the liquor is 32.4 inches? *Ans.* 398.036 *ale gallons.*

3. The perpendicular depth of a guile-tun is 38 inches; the mean diameter taken at 5 inches from the bottom, 51.1 inches; the diameter taken at 15 inches from the bottom, 53.2 inches; the diameter taken at 25 inches from the bottom, 55.3 inches; and that taken at 34 inches from the bottom, 57.2 inches; required the area of each section, and the whole content of the guile-tun, in ale gallons.

Note. The first three diameters are taken in the middle of every 10 inches; but the fourth diameter is taken in the middle of the last 8 inches of the depth.

Ans. The area of the first section is 7.27, *the second*

section 7.88, *the third section* 8.52, *and the area of the fourth section* 9.11 *ale gallons; hence we find the whole content of the tun to be* 309.58 *ale gallons.*

PROBLEM VII.

To gauge and fix a circular guile-tun, with curved sides.

To take the dimensions.

Quarter the tun, as directed in Problem III.; then take cross diameters in the middle of every 6, 8, or 10 inches; also, measure the perpendicular depth, and enter all the dimensions in your Note Book.

Note. When the sides are much curved, cross diameters must be taken in the middle of every 6 or 8 inches.

To find the area and content.

RULE.

Find the area corresponding to each mean diameter, in the Table of *Ale Areas*, Part VII.; then multiply each area by its respective depth; and the sum of the products will be the whole content of the tun. (See Prob. XIII., Part IV.)

Note 1. The area and content may also be found by Rule II., Prob. III.

2. A circular guile-tun is sometimes made of a hogshead, puncheon, or pipe, standing upon one end; the other end being sawn off, or taken out.

EXAMPLES.

1. Let the following figure A B C D, represent a circular guile-tun, whose perpendicular depth is 40 inches; the mean diameter, taken at 4 inches from the bottom, 46.3 inches; the diameter taken at 12 inches from the bottom, 49.6 inches; the diameter taken at 20 inches from the bottom, 53.2 inches; the diameter taken at 28 inches from the bottom, 54.4 inches; and that taken at 36 inches from the bottom, 50.8 inches; required the

area of each section, and the whole content, in ale gallons.

BY THE TABLE OF ALE AREAS.

To find the area.

Having found the area of each section, as directed in the Rule, the dimensions and areas will appear in the Note Book, as below.

NOTE BOOK.

A. B.'s Guile Tun, No. 2, gauged Jan. 23, 1821.

Divisions in Inches.	Depths from the Bottom.	Mean Diameters.	Areas.
8	36	50.8	7.19
8	28	54.4	8.24
8	20	53.2	7.88
8	12	49.6	6.85
8	4	46.3	5.97

BY THE SLIDING RULE.

As the circular divisor, 359.05, on A, is to the mean diameter on B; so is the mean diameter on A, to the area on B. Or, As the circular gauge-point, 18.95, on D, is to

unity on C; so is the mean diameter on D, to the area on C. (See Prob. XIII., Part IV.; and also Probs. III. and VI., Part VI.)

$$\begin{array}{ccccccc} & \text{On A.} & \text{On B.} & & \text{On A.} & & \text{On B.} \\ \text{As } 359.05 : & \left\{\begin{array}{c}50.8\\54.4\\53.2\\49.6\\46.3\end{array}\right\} & :: & \left\{\begin{array}{c}50.8\\54.4\\53.2\\49.6\\46.3\end{array}\right\} & : & \left\{\begin{array}{l}7.19 \textit{ fifth section.}\\8.24 \textit{ fourth section.}\\7.88 \textit{ third section.}\\6.85 \textit{ second section.}\\5.97 \textit{ first section.}\end{array}\right. \end{array}$$

Note. The dimensions and areas are transferred to the specimen of the Dimension Book, at the end of this Section.

BY THE PEN.

To find the content.

Areas. *Ale gallons.*		*Depths.* *Inches.*		*Contents.* *Ale gallons.*
5.97	×	8	=	47.76 *first* 8 *inches.*
6.85	×	8	=	54.80 *second* 8 *inches.*
7.88	×	8	=	63.04 *third* 8 *inches.*
8.24	×	8	=	65.92 *fourth* 8 *inches.*
7.19	×	8	=	57.52 *fifth* 8 *inches.*
				289.04 *whole contents.*

BY THE SLIDING RULE.

As one on A, is to the area on B; so is the depth on A, to the content on B.

$$\begin{array}{ccccccc} & \text{On A.} & \text{On B.} & & \text{On A.} & & \text{On B.} \\ \text{As } 1 : & \left\{\begin{array}{c}5.97\\6.85\\7.88\\8.24\\7.19\end{array}\right\} & :: & \left\{\begin{array}{c}8\\8\\8\\8\\8\end{array}\right\} & : & \left\{\begin{array}{l}47.76 \textit{ first 8 inches.}\\54.80 \textit{ second 8 inches.}\\63.04 \textit{ third 8 inches.}\\65.92 \textit{ fourth 8 inches.}\\57.52 \textit{ fifth 8 inches.}\end{array}\right. \end{array}$$

289.04 *whole content.*

2. Required the number of ale gallons contained in the foregoing guile-tun, when the depth of the liquor is 38.4 inches. *Ans.* 218.336 *ale gallons.*

3. The perpendicular depth of a circular guile-tun, made of a wine-pipe, is 42 inches; the mean diameter, taken at 4 inches from the bottom, 28.4 inches; the diameter taken at 12 inches from the bottom, 31.2 inches; the diameter taken at 20 inches from the bottom, 32.6 inches; the diameter taken at 28 inches from the bottom, 31.5 inches; and that taken at 37 inches from the bottom, 29.8 inches; required the area of each section, and the whole content in ale gallons.

Ans. The area of the first section is 2.25, *the second* 2.71, *the third* 2.96, *the fourth* 2.76, *and the fifth* 2.47 *ale gallons; hence we find the whole content to be* 110.14 *ale gallons.*

PROBLEM VIII.

To gauge and fix a guile-tun in the form of the frustum of an elliptical cone.

To take the dimensions.

By Problem XXIX., Part III., or by repeated trials, as directed in Note 1, Problem XX., Part IV., find the transverse and conjugate diameters of the bottom of the tun. Also, find the extremities of the transverse and conjugate diameters of the top of the tun; then, draw lines, with chalk, up the inside of the vessel, from the extremities of the bottom diameters to the extremities of the top diameters; and the tun will be *truly* quartered.

Next, make marks upon the quartering lines, in the middle of every 6, 8, or 10 inches, as directed in Problem III., Part VI.; measure the transverse and conjugate diameters of each section; also take the perpendicular depth of the vessel, and enter all the dimensions in your Note Book.

To find the area and content.

RULE.

Multiply the transverse diameter of the first section, by the conjugate; divide the product by 359.05; and the quotient will be the area in ale gallons. Find the area of each section in the same manner; then, multiply each area by its respective depth; and the sum of the products will be the whole content of the tun.

EXAMPLES.

1. The perpendicular depth of a guile-tun in the form of the frustum of an elliptical cone, is 30 inches; the transverse and conjugate diameters, taken at 5 inches from the bottom, measure 34.2 and 25.3 inches; at 15 inches from the bottom, 40.5 and 29.6 inches; and at 25 inches from the bottom, 46.4 and 33.8 inches; required the area of each section, and the whole content of the tun, in ale gallons.

BY THE PEN.

To find the area.

Here $34.2 \times 25.3 \div 359.05 = 865.26 \div 359.05 = 2.409$ *ale gallons, the area of the first section;* $40.5 \times 29.6 \div 359.05 = 1198.80 \div 359.05 = 3.338$ *ale gallons, the area of the second section; and* $46.4 \times 33.8 \div 359.05 = 1568.32 \div 359.05 = 4.367$ *ale gallons, the area of the third section.*

BY THE SLIDING RULE.

As the circular divisor on A, is to the transverse diameter on B; so is the conjugate diameter on A, to the area on B.

	On A.	On B.		On A.		On B.	
As	359.05 :	34.2 40.5 46.4	::	25.3 29.6 33.8	:	2.41 3.34 4.37	*first section.* *second section.* *third section.*

The dimensions and areas will appear in the Note Book, as below.

NOTE BOOK.

A. B.'s Guile Tun, No. 3, gauged April 12, 1821.

Divisions in Inches.	Depths from the Bottom.	Transverse and Conjugate Diameters.	Areas.
10	25	T. 46.4 C. 33.8	4.37
10	15	T. 40.5 C. 29.6	3.84
10	5	T. 34.2 C. 25.3	2.41

Note. The dimensions and areas are transferred to the specimen of the Dimension Book, at the end of this Section.

To find the content.

Contents.
Ale gallons.
24.1 *first* 10 *inches.*
33.4 *second* 10 *inches.*
43.7 *third* 10 *inches.*
101.2 *whole content.*

2. Required the number of ale gallons contained in the foregoing guile-tun, when the depth of the liquor is 23.4 inches. *Ans.* 72.358 *ale gallons.*

3. The perpendicular depth of an elliptical guile-tun is 40 inches; the transverse and conjugate diameters, taken at 5 inches from the bottom, measure 49.6 and 37.8 inches; at 15 inches from the bottom, 55.4 and 41.7 inches; at 25 inches from the bottom, 61.6 and 46.2 inches; and at 35 inches from the bottom, 67.3 and 50.5

inches; required the area of each section, and the whole content of the tun, in ale gallons.

Ans. The area of the first section is 5.22, the second 6.48, the third 7.93, and the fourth 9.47 ale gallons; hence we find the whole content to be 290.5 ale gallons.

PROBLEM IX.

To gauge and fix a guile-tun with an elliptical base and a circular top, generally called a cylindroid.

To take the dimensions.

Find the transverse and conjugate diameters of the bottom of the tun, as in the last Problem. Also, quarter the top, as directed in Problem III.; taking care to make the quartering points at the top and bottom, correspond with each other, in position.

Having thus quartered the tun, measure transverse and conjugate diameters in the middle of every 6, 8, or 10 inches; also take the perpendicular depth of the vessel, and enter all the dimensions in your Note Book.

To find the area and content.

Divide the product of the two diameters of the first section, by 359.05; and the quotient will be the area in ale gallons. Find the area of each section by a similar process; then multiply each area by its corresponding depth, and the sum of the products will be the content required.

Note. As the base of a cylindroid is an ellipse and the top a circle, it is evident that the horizontal sections taken in the middle of every 6, 8, or 10 inches, cannot be *true* ellipses; consequently the areas found by the above Rule, will not be *mathematically* correct; although near enough the truth for all practical purposes. (See Remarks at the end of Problem IX., Part V.)

EXAMPLES.

1. The perpendicular depth of a guile-tun in the form

of a cylindroid, is 30 inches; the transverse and conjugate diameters, taken at 5 inches from the bottom, measure 44.3 and 37.6 inches; at 15 inches from the bottom, 39.8 and 35.9 inches; and at 25 inches from the bottom, 35.6 and 33.8 inches; required the area of each section, and the whole content of the tun, in ale gallons.

BY THE PEN.

To find the area.

Here $44.3 \times 37.6 \div 359.05 = 1665.68 \div 359.05 = 4.639$ *ale gallons, the area of the first section;* $39.8 \times 35.9 \div 359.05 = 1428.82 \div 359.05 = 3.979$ *ale gallons, the area of the second section; and* $35.6 \times 33.8 \div 359.05 = 1203.28 \div 359.05 = 3.351$ *ale gallons, the area of the third section.*

BY THE SLIDING RULE.

As the circular divisors on A, is to the transverse diameter on B; so is the conjugate diameter on A, to the area on B.

$$\begin{array}{lcccccc} & \text{On A.} & \text{On B.} & & \text{On A.} & & \text{On B.} \\ \textit{As} & 359.05 : & \left\{\begin{array}{c}44.3\\39.8\\35.6\end{array}\right\} & :: & \left\{\begin{array}{c}37.6\\35.9\\33.8\end{array}\right\} & : & \left\{\begin{array}{l}4.64\ \textit{first section.}\\3.98\ \textit{second section.}\\3.35\ \textit{third section.}\end{array}\right. \end{array}$$

The dimensions and areas will appear in the Note Book, as below.

NOTE BOOK.

A. B.'s Guile Tun, No. 4, gauged April 18, 1821.

Divisions in Inches.	Depths from the Bottom.	Transverse and Conjugate Diameters.	Areas.
10	25	T. 35.6 C. 33.8	3.35
10	15	T. 39.8 C. 35.9	3.98
10	5	T. 44.3 C. 37.6	4.64

To find the content.

Contents.
Ale Gallons.
46.4 *first* 10 *inches.*
39.8 *second* 10 *inches.*
33.5 *third* 10 *inches.*
119.7 *whole content.*

2. How many gallons are contained in the foregoing guile-tun, when the depth of the liquor is 26.7 inches?

Ans. 108.645 *ale gallons.*

3. The perpendicular depth of a guile-tun in the form of a cylindroid, is 40 inches; the transverse and conjugate diameters, taken at 5 inches from the bottom, measure 62.7 and 55.4 inches; at 15 inches from the bottom, 58.8 and 53.6 inches; at 25 inches from the bottom, 54.9 and 51.8; and at 35 inches from the bottom 50.8 and 49.7 inches; required the area of each section, and the whole content of the tun, in ale gallons.

Ans. The area of the first section is 9.67, *the second* 8.78, *the third* 7.92, *and the fourth* 7.08 *ale gallons; hence we find the whole content to be* 334 *ale gallons.*

PROBLEM X.

To gauge and fix a guile-tun in the form of the frustum of a square pyramid.

To take the dimensions.

With some convenient instrument take the perpendicular depth of the tun; also, measure the side of the tun in the middle of every 6, 8, or 10 inches; and enter all the dimensions in your Note Book.

Note 1. The perpendicular distances between the horizontal sections, must be *set off* upon the inside of the vessel, as directed in Problem III., Part VI.

2. As vessels that assume the form of the frustum of a square pyramid, are seldom perfectly square, it is necessary to measure each side, and divide the sum of the sides by 4, to find the mean side.

3. Guile Tuns with circular or elliptical bases are generally termed *Rounds*; and those with square or rectangular bases, are denominated *Squares*. (See Remarks at the end of Prob. VI., Sect. II., Part VI.)

To find the area and content.

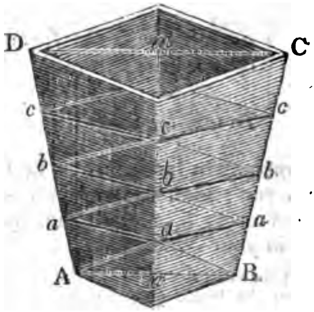
RULE.

Multiply the side of the first horizontal section by itself; divide the product by 282, and the quotient will be the area in ale gallons.

Find the area of each section in the same manner; then multiply each area by its respective perpendicular depth; and the sum of the products will be the whole content, in ale gallons.

EXAMPLES.

1. The perpendicular depth *m n*, of the guile-tun A B C D, in the form of a frustum of a square pyramid, is 32 inches; the mean side *a a*, 5 inches from the bottom, 33.2; the side *b b*, 15 inches from the bottom, 40.4; and the side *c c*, 26 inches from the bottom, 48.6 inches; required the area of each section, and the whole content, in ale gallons.



BY THE PEN.

To find the area.

Inches.
33.2 mean side a a.
33.2 ditto.

```
   664
  996
 996
```

Divisor 282)1102.24(3.908 *area in ale gallons.*

Also, 40.4×40.4÷282=1632.16÷282=5.787 *ale gallons, the area of the second section; and* 48.6×48.6÷282 =2361.96÷282=8.375 *ale gallons, the area of the third section.*

BY THE SLIDING RULE.

As the square divisor on A, is to the mean side on B; so is the mean side on A, to the area on B. Or, as the square gauge-point on D, is to unity on C; so is the mean side on D, to the area on C.

$$\begin{array}{lllll} & \text{On A.} & \text{On B.} & \text{On A.} & \text{On B.} \\ \textit{As } 282 & : & \left\{\begin{array}{l}33.2\\40.4\\48.6\end{array}\right\} :: \left\{\begin{array}{l}33.2\\40.4\\48.6\end{array}\right\} : & \left\{\begin{array}{l}3.91 \textit{ first section.}\\5.79 \textit{ second section.}\\8.38 \textit{ third section.}\end{array}\right. & \end{array}$$

The dimensions and areas will appear in the Note Book, as below.

NOTE BOOK.

A. B.'s Sq. Guile Tun, No. 1, gauged March 6, 1821.

Divisions in Inches.	Depths from the Bottom.	Mean Sides.	Areas.
12	26	48.6	8.38
10	15	40.4	5.79
10	5	33.2	3.91

Note. The dimensions and areas are transferred to the specimen of the Dimension Book, at the end of this Section.

To find the content.

	Contents.	
	Ale gallons.	
	39.1	*first* 10 *inches.*
	57.9	*second* 10 *inches.*
$8.38 \times 12 =$	100.56	*upper* 12 *inches.*
	197.56	*whole content.*

2. How many ale gallons are contained in the foregoing guile-tun, when the depth of the liquor is 27.8 inches? *Ans.* 162.364 *ale gallons.*

3. The side of a square, pyramidical guile-tun, at 5 inches from the bottom, measures 49.2; at 15 inches from the bottom, 52.3; at 25 inches from the bottom, 55.2; at 35 inches from the bottom, 58.4; at 46 inches from the bottom, 61.2 inches; required the area of each section, and the whole content, in ale gallons; its perpendicular depth being 52 inches.

Note. The first four dimensions are taken in the middle of every 10 inches; but the fifth dimension is taken in the middle of the last 12 inches of the depth.

Ans. The area of the first section is 8.58, *the second section* 9.70, *the third section* 10.81, *the fourth section* 12.09, *and the fifth section* 13.28 *ale gallons; hence we find the whole content of the tun to be* 571.16 *ale gallons.*

PROBLEM XI.

To gauge and fix a guile-tun in the form of the frustum of a rectangular pyramid, or a prismoid.

To take the dimensions.

With some convenient instrument take the perpendicular depth of the vessel; also, measure the mean length

and breadth in the middle of every 6, 8, or 10 inches; and enter all the dimensions in your Note Book.

Note. Measure both sides of the vessel, divide the sum by 2, and take the quotient for the mean length. The mean breadth must be found in the same manner.

To find the area and content.

RULE.

Multiply the mean length of each section by the mean breadth; divide each product by 282; and the quotients will be the areas of the respective sections, in ale gallons. Multiply the area of each section by its respective perpendicular depth; and the sum of the products will be the whole content of the tun, in ale gallons.

EXAMPLES.

1. The perpendicular depth of a guile-tun, in the form of the frustum of a rectangular pyramid, is 30 inches; the mean length and breadth, taken at 5 inches from the bottom, measure 36 and 26 inches; at 15 inches from the bottom, 42 and 32 inches; and at 25 inches from the bottom, 48 and 38 inches; required the area of each section, and the whole content, in ale gallons.

BY THE PEN.

To find the area.

Inches.
36 *length.*
26 *breadth.*
216
72
Divisor 282)936(3.319 *area in ale gallons.*

Also, $42 \times 32 \div 282 = 1344 \div 282 = 4.765$ *ale gallons, the area of the second section; and* $48 \times 38 \div 282 = 1824 \div 282 = 6.468$ *ale gallons, the area of the third section.*

BY THE SLIDING RULE.

As the square divisor on A, is to the length on B; so is the breadth on A, to the area on B.

	On A.		On B.		On A.		On B.	
As	282	:	36	::	26	:	3.32	*first section.*
			42		32		4.77	*second section.*
			48		38		6.47	*third section.*

The dimensions and areas will appear in the Note Book, as below.

NOTE BOOK.

A. B.'s Sq. Guile Tun, No. 2, gauged March 12, 1821.

Divisions in Inches.	Depths from the Bottom.	Mean Lengths and Breadths.	Areas.
10	25	L. 48 B. 38	6.47
10	15	L. 42 B. 32	4.77
10	5	L. 36 B. 26	3.32

Note. The dimensions and areas are transferred to the specimen of the Dimension Book, at the end of this Section.

To find the content.

Contents. Ale gallons.	
33.2	*first* 10 *inches.*
47.7	*second* 10 *inches.*
64.7	*third* 10 *inches.*
145.6	*whole content.*

2. How many ale gallons are contained in the foregoing guile-tun, when the depth of the liquor is 26.3 inches? *Ans.* 121.661 *ale gallons.*

3. The perpendicular depth of a guile-tun, in the form of a prismoid, is 40 inches; the mean length and breadth, taken at 5 inches from the bottom, measure 56.4 and 41.2 inches; at 15 inches from the bottom, 60.8 and 44.8 inches; at 25 inches from the bottom, 65.7 and 48.6; and at 35 inches from the bottom, 71.2 and 51.9; required the area of each section, and the whole content in ale gallons.

Ans. The area of the first section is 8.24, *the second* 9.66, *the third* 11.32, *and the fourth* 13.10 *ale gallons: hence we find the whole content of the tun to be* 423.2 *ale gallons.*

REMARK.

Having given the methods of gauging and fixing all kinds of guile-tuns that generally occur in victuallery, it only remains to observe, that if a guile-tun should be met with whose base is a triangle, a trapezium, a trapezoid, &c. &c. such dimensions must be taken as will give the areas of horizontal sections in the middle of every 6, 8, or 10 inches of the perpendicular depth. Then multiply each area by its corresponding depth; and the sum of the products will be the whole content of the tun. (See the Remark at the end of Prob. V.)

PROBLEM XII.

The method of gauging by-tubs.

By-tubs are such vessels as are not fixed in the Officer's Dimension Book. They are generally small, of various and irregular forms; and used by victuallers for cooling their wort, or for working small quantities of beer, when the fixed vessels are too full.

To take the dimensions.

Whatever be the form of the vessel, such mean dimen-

sions must be taken as are most likely to give the true content of the liquor. Thus, if the vessel be a parallelopipedon, measure the mean length and breadth; if it be a cylinder, measure the mean diameter; if it be the frustum of a right cone, measure the mean diameter, at half the perpendicular depth of the liquor, in order to reduce the vessel to a cylinder; and if it be the frustum of an elliptical cone, measure the mean transverse and conjugate diameters, at half the perpendicular depth of the liquor; and when their difference is not great, take half their sum for a mean circular diameter, thus reducing the vessel to a cylinder. Also, in every case, take the mean perpendicular depth of the liquor; and enter all the dimensions in a vacant column of your Survey Book.

Note 1. In order to find a mean diameter at half the perpendicular depth of the liquor contained in a vessel in the form of the frustum of a right cone, standing upon its less base, place a rod perpendicularly to the bottom of the vessel, and touching the side; then the distance between the other side of the vessel and the rod, measured at the surface of the liquor, will be the mean diameter sought.—By a similar process you may obtain the mean transverse and conjugate diameters, at half the perpendicular depth of the liquor contained in a vessel in the form of the frustum of an elliptical cone.

2. The foregoing method of reducing a vessel in the form of the frustum of a right cone, or an elliptical cone to a cylinder, is not mathematically correct; but it is generally adopted in practice, because the content of the liquor can be found by the Sliding Rule, without finding the area of the vessel.

To find the content.

RULES.

BY THE PEN.

1. When the vessel is a parallelopipedon, multiply the mean length, breadth, and depth continually together; divide the last product by 282; and the quotient will be the content in ale gallons.

2. When the vessel is a cylinder, or the frustum of a cone reduced to a cylinder, multiply the square of the mean diameter by the depth of the liquor; divide the

product by 359; and the quotient will be the content, in in ale gallons.

3. When the vessel is the frustum of an elliptical cone, and not reduced to a cylinder, multiply the product of the mean diameters by the depth of the liquor; divide this product by 359; and the quotient will be the content in ale gallons.

BY THE SLIDING RULE.

1. *When the vessel is a parallelopipedon.*

As the square divisor 282, on A, is to the length on B; so is the breadth on A, to the area on B. Then, as one on A, is to the area on B; so is the depth on A, to the content on B.

2. *When the vessel is a cylinder.*

As the circular gauge-point 18.95, on D, is to the depth on C; so is the mean diameter on D, to the content on C.

3. *When the vessel is the frustum of an elliptical cone.*

As the circular divisor 359, on A, is to the transverse diameter on B; so is the conjugate diameter on A, to the area on B. Then, as one on A, is to the area on B; so is the depth on A, to the content on B.

EXAMPLES.

1. The mean length of a by-tub is 32.5, the mean breadth 28.3, and the mean depth of the liquor 14.6 inches; how many gallons of ale does the vessel contain?

BY THE PEN.

Rule I.

```
                    Inches.
                    32.5 length.
                    28.3 breadth.
                    ----
                    975
                  2600
                  650
                  ---------
                  919.75 product.
                     14.6 depth.
                 --------
                 551850
               367900
               91975
               ---------
Divisor  282)13428.350(47.618 content in ale gallons.
             1128
             ----
              2148
              1974
              ----
               1743
               1692
               ----
                 515
                 282
                 ----
                 2330
                 2256
                 ----
                   74
```

BY THE SLIDING RULE.

Rule 1.

On A.		On B.		On A.		On B.
As 282	:	32.5	::	28.3	:	3.26 *area*.

And,

On A.		On B.		On A.		On B.
As 1	:	3.26	::	14.6	:	47.62 *content*.

2. The mean diameter of a cylindrical by-tub is 28.8; and the mean depth of the liquor 18.2 inches; what is the content in ale gallons?

BY THE PEN.

Rule II.

Here $28.8 \times 28.8 \times 18.2 = 829.44 \times 18.2 = 15095.808$, the square of the mean diameter multiplied by the depth of the liquor; and $15095.808 \div 359 = 42.049$, the content in ale gallons.

BY THE SLIDING RULE.

Rule II.

	On D.		On C.		On D.		On C.
As	18.95	:	18.2	::	28.8	:	42.05 *content.*

3. The mean diameter of a by-tub in the form of the frustum of a right cone, is 25.6, and the depth of the liquor 22.7 inches; what is the content in ale gallons?

Ans. 41.439 *ale gallons.*

4. The mean transverse diameter of a by-tub, in the form of the frustum of an elliptical cone, is 45.3, and the mean conjugate diameter 36.8 inches; what is the content in ale gallons, when the depth of the liquor is 12.7 inches? *Ans.* 58.973 *ale gallons.*

REMARKS.

1. In some places it is customary to cool wort in brass pans or kettles, with falling crowns. In such cases, find the mean diameter as before directed; and take the mean depth of the liquor half way between the side of the vessel and the centre of the crown.

2. Sometimes wort is cooled in *wooden bowls*, somewhat resembling the segment of a sphere. The practical method of gauging such vessels, as by-tubs, is to take the diameter at the surface of the liquor, and half the greatest depth; and then compute the content, from these dimensions, as directed for a cylinder, in the second Rule. No general Rules can however be laid down that will give the contents of irregular vessels with accuracy; and as by-tubs assume such a variety of forms, nothing but experience will enable the Officer to adopt such methods of gauging them as the nature of the vessels may seem to require.

Note. By experiments made by filling several *wooden bowls* with water, from a known measure, we have found that the foregoing Rule gives the content too little, and the Rule for the segment of a sphere gives sometimes too much, and sometimes too little; the following method, however, gives *very nearly* the true content: Take the diameter of the *bowl*, at the surface of the liquor, and also its greatest depth: To this depth, add two-tenths, for every inch; and take half the sum for the mean depth; and the vessel will be reduced to a cylinder, *nearly*.

EXAMPLE. The diameter of the top of a *wooden bowl*, is 18.2 inches, and its greatest depth 5.9 inches; what is its content in ale gallons?

SOLUTION. Here $\frac{5.9 + 1.18}{2} = \frac{7.08}{2} = 3.54$, the mean depth; then, $(18.2 \times 18.2 \times 3.54) \div 359 = (331.24 \times 3.54) \div 359 = 1172.5896 \div 359 = 3.2662$, the content in ale gallons.

Note. The number to be added to the depth, may always be obtained by multiplying the depth by two-tenths; thus we have $5.9 \times .2 = 1.18$, the number added to the depth, in the above solution.

PROBLEM XIII.

To deduct the heat out of victuallers' warm worts.

It has been found by experience that 10 gallons of hot wort will only measure 9 gallons when cold; therefore, by act of parliament, an allowance of one gallon in every ten must be made, when the wort is gauged hot.

CASE I.

When the number of warm gallons are known.

By Subtraction.

RULE.

By the Pen.

Write down the number of warm gallons, found by gauge; from this subtract one-tenth of itself; and the remainder will be the number of gallons when cold.

By Multiplication.

RULE.

Multiply the number of warm gallons by .9, and the product will be the number of gallons when cold.

EXAMPLES.

1. If a gauge of warm wort be 184.8 gallons; how many gallons must be charged with the duty?

By Subtraction.	*By Multiplication.*
Gallons.	*Gallons.*
184.8	184.8
18.48	.9
166.32 *Ans.*	166.32 *Ans.*

Note. The trader must be charged with 166.5 gallons, as .32 is nearer .5 than 0. See the Remark at the end of this Problem.

By the Sliding Rule.

RULE.

As the gauge-point 1.11 on A, is to unity on B; so is the warm gallons on A, to the cold gallons on B.

Note. The gauge-point 1.11, is found by dividing unity by nine-tenths.

	On A.		On B.		On A.		On B.
As	1.11	:	1	::	184.8	:	166.5 *ale gallons.*

2. Suppose a gauge of warm wort, in a cooler, to be 225.6 gallons; upon how many gallons must the duty be charged? *Ans.* 203 *gallons.*

CASE II.

When the area of a fixed utensil, and the depth of the liquor are known, but not the number of warm gallons.

RULE.

By the Pen.

Multiply the area by the depth; divide the product by 1.11; and the quotient will be the number of cold gallons.

Note. Here it may be observed that the area multiplied by the depth evidently gives the number of warm gallons; hence the number of cold gallons may be obtained by case the first; the last Rule, however, is well adapted to the Sliding Rule.

EXAMPLES.

1. If the area of a cooler be 23.25 gallons, and the depth of the warm wort 5.6 inches; for how many gallons must the victualler be charged?

Ale Gallons.

```
                  23.25 area.
                    5.6 depth.
                 -------
                 13950
                11625
                -------
Divisor 1.11)130.200(117.29 Ans.
             111
             ---
              192
              111
              ---
               810
               777
               ---
                330
                222
                ----
                1080
                 999
                ----
                  81
```

By the Sliding Rule.

As the divisor 1.11 on A, is to the area on B; so is the depth on A, to the answer on B.

	On A.		On B.		On A.		On B.
As	1.11	:	23.25	::	5.6	:	117.5 *Ans.*

Note. A brass pin should be put upon the line A, at the gauge-point 1.11, for the convenience of casting warm gauges.

2. The area of a cooler is 26.45 gallons, and the depth of the warm wort 6.8 inches; required the number of gallons, when the wort is cold. *Ans.* 162.036 *ale gallons.*

3. The area of the first horizontal section of a guile-tun in the form of the frustum of a cone, is 4.9 ale gallons, and the corresponding depth of warm wort 8 inches; the area of the second section, is 5.25 ale gallons, and its corresponding depth 7.4 inches; how many gal-

lons will the vessel contain, when the heat has evaporated?
Ans. 70.315 *ale gallons.*

CASE III.

When the warm wort is in a circular by-tub.

By the Sliding Rule.

RULE.

As the warm gauge-point 20 on D, is to the depth of the liquor on C; so is the mean diameter on D, to the net gallons on C.

Note 1. If the square of the diameter of a cylindrical vessel, be multiplied by the depth of the warm wort, and the product divided by 400; the quotient will be the number of gallons, when the wort is cold; hence the warm circular gauge-point, given in the last Rule, is found by extracting the square root of 400, the warm circular divisor.

2. If the length of a rectangular cooler be multiplied by its breadth, and this product by the depth of the warm wort; the quotient arising from dividing the last product by 313.33, will be the number of gallons, when the wort is cold; hence, if we extract the square root of this warm square divisor, we obtain 17.7, the warm square gauge-point.

3. The warm square divisor 313.33, is found by dividing 282 by .9; and the warm circular divisor 400, is obtained by dividing 359.05 by .9; the last quotient, however, is only 398.944; but for expedition, in practice, it may be called 400.

EXAMPLES.

1. If the mean diameter of a circular by-tub be 36.4 inches, and the depth of the warm wort 14.8 inches; to what number of gallons will the wort gauge, when it is cold?

By the Sliding Rule.

	On D.		On C.		On D.		On C.
As	20	:	14.8	::	36.4	:	49 *ale gallons.*

2. The mean diameter of a circular by-tub is 27.8 inches, and the depth of the warm wort 23.6 inches; required the number of gallons, when the wort is cold?
Ans. 45.6 *ale gallons.*

Remark.

In charging the duty upon ale and beer, when the decimal parts of a gallon are less than .25, they are rejected; but if .25 and under .75, they are called half a gallon; and when .75 or above, they are called a whole gallon.

PROBLEM XIV.

Given the top and bottom diameters, and the perpendicular depth of a vessel in the form of the frustum of a cone, to determine the intermediate diameters, at any assigned depth.

RULE.

Divide the difference between the top and bottom diameters by the perpendicular depth of the vessel; multiply the assigned depth by the quotient; add the product to the less diameter; and the sum will be the diameter at the assigned depth, if that depth be measured from the less base. But if the depth be taken from the greater base, then subtract the said product from the greater diameter; and the remainder will be the diameter required.

EXAMPLES.

1. If the top diameter of a vessel in the form of the frustum of a cone be 40 inches, the bottom diameter 30 inches, and the perpendicular depth 28 inches; required the intermediate diameters, at every 4 inches of its perpendicular depth, measured from the less base?

Here $\frac{40-30}{28}=\frac{10}{28}=.357$, *the common factor or multiplier; then* $4\times.357+30=1.428+30=31.428$ *the diameter at 4 inches from the bottom;* $8\times.357+30=2.856+30=32.856$, *the diameter at* 8 *inches from the bottom;* $12\times.357+30=4.284+30=34.284$, *the diameter at* 12 *inches from the bottom;* $16\times.357+30=5.712+30=35.712$, *the diameter at* 16 *inches from the bottom,* $20\times.357+30=7.140+30=37.140$, *the diameter at* 20 *inches from the bot-*

m; and $24 \times .357 + 30 = 8.568 + 30 = 38.568$ *the diame-r at 24 inches from the bottom.*

2. The top diameter of a vessel in the form of the rustum of a cone, measures 42 inches, the bottom diameter 58 inches, and the perpendicular depth 35 inches; is required to determine the intermediate diameters at very 5 inches of its depth, measured from the greater ase.

Ans. The diameter at 5 inches from the bottom, is 55.715; *t* 10 *inches,* 53.430; *at* 15 *inches,* 51.145; *at* 20 *inches,* 8.860; *at* 25 *inches,* 46.570; *and at* 30 *inches,* 44.290 *ches.*

REMARK.

The last Problem may be well applied in *gauging* and *ching* a Common Brewer's large guile-tun, in the form f the frustum of a cone; for if we take cross diameters t the top of the vessel, and at every ten inches of the erpendicular depth; and from these cross diameters nd mean diameters; we may then determine the intermediate diameters, between every 10 inches, with the reatest accuracy; and consequently, *inch* the guile-tun ith the utmost correctness, by finding the area corresponding to each diameter, either by the Pen, the Sliing Rule, or the Table of Ale Areas.

Or, when diameters are taken in the middle of every) inches, the Rule may be well applied as a check, in e following manner: Measure the top and bottom diaeters, and also the perpendicular depth; and then dermine the intermediate diameters in the middle of every) inches; and if they correspond with those found by measurement, it is evident that the vessel is the true istum of a cone, and that all the operations have been rformed with accuracy.

PROBLEM XV.

ven the top and bottom diameters, and the perpendicular depth of a vessel in the form of the frustum of a semi-sphere, to determine the intermediate diameter, at any assigned depth from the top of the vessel.

To find the diameter of the sphere of which the frustum is a part.

RULE I.

From the square of half the greater diameter of the frustum, subtract the sum of the squares of half the less diameter and the perpendicular depth; divide the remainder by twice the said depth; and the quotient will be the distance of the greater diameter from the centre of the sphere. To the square of this distance add the square of half the greater diameter; multiply the square root of the sum by two, and the product will be the diameter of the sphere.

Note. When the greater diameter of the frustum is equal to the diameter of the sphere, the square of half the greater diameter of the frustum will be equal to the sum of the squares of half the less diameter and the perpendicular depth; hence we may always know when the greater diameter of the frustum passes through the centre of the sphere. *(See the following Figure.)*

To find the height of the spherical segment cut off by the less diameter of the frustum.

RULE II.

From half the diameter of the sphere subtract the distance between its centre and the less diameter of the frustum; and the remainder will be the height of the segment.

Note. When the greater diameter of the frustum is equal to the diameter of the sphere; the height of the segment must be found by subtracting the height of the frustum from half its greater diameter.

To find the diameter at any assigned depth from the top of the frustum.

RULE III.

To the altitude of the segment cut off by the top diameter, add the assigned depth of the frustum; subtract the sum from the diameter of the sphere; multiply the remainder by the said sum; and twice the square root of the product will be the diameter required.

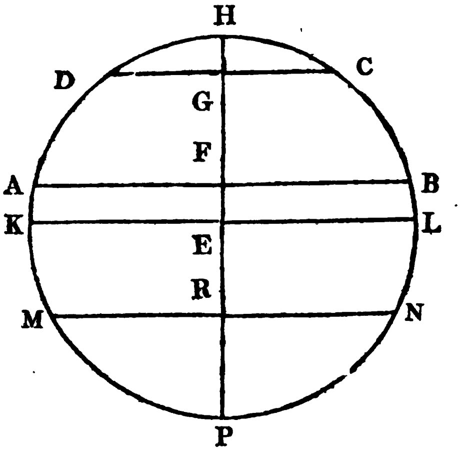
In this manner proceed until you have found the diameters at every inch of the depth, if necessary.

Note. When it is required to find the intermediate diameters at every inch of the perpendicular depth of the frustum, 1 inch will be the first assigned depth, 2 inches the second, 3 inches the third, &c. &c.; which numbers must be added successively to the height of the segment.

EXAMPLES.

1. The greater diameter of a vessel, in the form of the frustum of a semi-sphere, is 98.4, the less diameter 60.6, and the perpendicular depth 30.2 inches; required the intermediate diameters at every inch of the first 5 inches of the perpendicular depth from the top of the vessel.

TO FIND THE DIAMETER OF THE SPHERE.



Let KHL denote the middle section of the semi-sphere, and ABCD that of the given vessel; then by the question, we have the greater diameter AB $= 98.4$, *the less diameter DC* $= 60.6$, *and the perpendicular depth FG* $= 30.2$.

By Rule I., we have, $49.2^2 = 2420.64$, *the square of half the greater diameter; and* $30.3^2 + 30.2^2 = 918.09 + 912.04 = 1830.13$, *the sum of the squares of half the less diameter*

and the perpendicular depth; then $\frac{2420.64-1830.13}{30.2\times 2}=\frac{590.51}{60.4}=9.77$ *inches* $=EF$, *the distance of the greater diameter of the frustum from the centre of the sphere.*

Again, $\overline{9.77^2+49.2^2}=95.4529+2420.64=2516.0929$; *and* $2\sqrt{2516.0929}=50.16\times 2=100.32$ *inches,* $=KL$ *or* HP, *the diameter of the sphere.*

TO FIND THE HEIGHT OF THE SPHERICAL SEGMENT.

Here $30.2+9.77=39.97$ *inches* $=EG$, *the distance between the centre of the sphere and the less diameter of the frustum; then, by Rule II., we have* $50.16-39.97=10.19$ *inches* $=GH$, *the height of the segment* DHC.

TO FIND THE INTERMEDIATE DIAMETERS.

By Rule III., we have $10.19+1=11.19$, *the altitude of the segment added to the assigned depth; then,* $\overline{100.32-11.19}\times 11.19=89.13\times 11.19=997.3647$; *and* $2\sqrt{997.3647}=31.58\times 2=63.16$ *inches, the diameter at* 1 *inch from the top of the given vessel.*

Again, $10.19+2=12.19$, *the altitude of the segment added to the assigned depth; then* $\overline{100.32-12.19}\times 12.19=88.13\times 12.19=1074.3047$; *and* $2\sqrt{1074.3047}=32.77\times 2=65.54$ *inches, the diameter at* 2 *inches from the top of the vessel.*

By a similar process, we find 67.88 *inches to be the diameter at* 3 *inches;* 69.90 *inches, at* 4 *inches; and* 71.92 *inches, at* 5 *inches from the top of the given vessel.*

Note. By proceeding in the same manner, any diameter of the frustum M N C D, or of the segment D P C, may be easily obtained.

2. The top diameter of a vessel, in the form of the frustum of a semi-sphere, is 24 inches, the bottom diameter 36 inches, and the perpendicular depth 8 inches; required the intermediate diameters, at every inch of the perpendicular depth, from the top of the vessel.

Ans. The first intermediate diameter is 26.32, *the second* 28.34, *the third* 30.08, *the fourth* 31.60, *the fifth* 32.94, *the sixth* 34.101, *and the seventh* 35.10 *inches.*

3. The top diameter of the upper part of a still, in the form of the frustum of a semi-sphere, is 28 inches, the bottom diameter 55.6 inches; and the perpendicular depth 12 inches; required the intermediate diameters, at every inch of the perpendicular depth, from the top of the frustum.

Ans. The first intermediate diameter is 31.92, *the second* 35.32, *the third* 38.30, *the fourth* 40.98, *the fifth* 43.40, *the sixth* 45.60, *the seventh* 47.62, *the eighth* 49.48, *the ninth* 51.18, *the tenth* 52.76, *and the eleventh* 54.22 *inches.*

GENERAL RULES.

1. *Given the top and bottom diameters, and the perpendicular depth of a vessel in the form of the frustum of a semi-sphere, to determine the intermediate diameter, at any assigned depth from the bottom of the vessel.*

RULE.

(See the last figure.)

Find the diameter H P of the sphere, and the versed sine F H of the less segment A H B, as directed in the last Problem; then from the diameter of the sphere subtract the versed sine of the less segment; and the difference will be the versed sine F P of the greater segment A P B.

To the versed sine of the greater segment add the assigned depth, and from the versed sine of the less segment subtract the said depth; multiply the sum by the remainder; and twice the square root of the product will be the diameter of the vessel, at the assigned depth from the bottom diameter A B.

2. *Given the top diameter and perpendicular depth of a vessel in the form of the less segment of a sphere, to determine the intermediate diameter at any assigned depth from the top of the vessel.*

RULE.

(See the last figure.)

Divide the square of half the top diameter M N, by the versed sine or perpendicular depth R P, of the less segment M P N; and the quotient will be the versed sine R H, of the greater segment M H N.

To the versed sine of the greater segment add the assigned depth, and from the versed sine of the less segment subtract the said depth; multiply the sum by the remainder; and twice the square root of the product will be the diameter of the vessel, at the assigned depth from the top diameter M N.

3. *Given the top diameter and perpendicular depth of a vessel in the form of the less segment of a sphere, to determine the intermediate diameter, at any assigned depth from the bottom of the vessel.*

RULE.

(See the last figure.)

Divide the square of half the top diameter by the perpendicular depth; to the quotient add the said depth; and the sum will be the diameter of the sphere.

From the diameter of the sphere subtract the assigned depth from the bottom of the segment; multiply the remainder by the said depth; and twice the square root of the product will be the diameter required.

Note. The diameter at any part of a vessel in the form of a sphere or globe, may be found by the last Rule.

REMARK.

The last Problem may be well applied in *gauging* and *inching* any vessel either in the form of a sphere, or of the frustum or segment of a sphere; for having determined the diameters at every inch of the perpendicular depth, we may hence find the area corresponding to each diameter, either by the Pen, the Sliding Rule, or the Tables of Ale and Wine Areas, in Part VII.

A SPECIMEN OF A DIMENSION BOOK,

Containing the Dimensions and Areas of A. B.'s Mash-Tuns, Copper, Under-Back, Coolers, and Guile Tuns, as determined in the foregoing Problems.

40.5 1 M. T.	55.6 2 M.T.	40.0 Copper	Areas.	25.8 ⊙ U. B.	1 Back.	42.0 1 Rd.	Areas.
55.1	Sq. 38.8	50.6	7.13	L. 58.6	L. 116.8	71.4	14.20
		51.5	7.39	B. 40.4	B. 68.2	68.7	13.14
		48.5	6.55			66.2	12.21
10.51	6.63	44.6	5.54	8.40	28.25	63.7	11.30

DIMENSION BOOK CONTINUED.

40.0 2 Rd.	Area.	30.0 3 Rd.	Areas.	32.0 1 Sq.	Areas.	30.0 2 Sq.	Areas.
50.8	7.19	T. 46.4 C. 33.8	4.37	48.6 48.6	8.38	48.0 38.0	6.47
54.4	8.24						
53.2	7.88	T. 40.5 C. 29.6	3.34	40.4 40.4	5.79	42.0 32.0	4.77
49.6	6.85						
46.3	5.97	T. 34.2 C. 25.3	2.41	33.2 33.2	3.91	36.0 26.0	3.32

DIMENSION BOOK CONTINUED.

2 Back; 9 Ordinates, 14 inches equidistant = 112 *inches.*

Transverse.	1	2	3	4	Conj. 5	6	7	8	9	Sum of Seg.	Total Area.
122.0	31.2	50.8	62.0	68.2	71.0	68.8	63.0	52.0	32.0	10.0	24.153

REMARKS.

The foregoing method of entering the dimensions and reas of Victualler's Utensils, in the Officer's Dimension ook, is the same that is practised in the Excise. The rst column contains the dimensions and area of the Round [ash Tun, given in the first Problem of this Section. .t the top is placed the depth; under it the diameter; nd at the bottom the area. The second column contains

the dimensions and area of the Square Mash Tun, given in the second Problem. In the third column we have the depth and diameters of the Copper; and in the fourth column, the areas correponding to those diameters, as found in the third Problem.

The fifth column contains the dimensions and area of the Underback; the sixth contains the length, breadth, and area of the first Back or Cooler; and in the seventh and eighth columns are placed the dimensions and areas of the first Round Guile-Tun.

The second part of the Table contains the dimensions and areas of the second and third Rounds; and of the first and second Squares. The dimensions and areas of the fourth Round, are not inserted; because the method of entering them is precisely the same as that of the third Round.

The third part of the Table, contains the dimensions and area of the second Back or Cooler, gauged and fixed by *equi-distant ordinates.* This is the seventh Example in the twentieth Problem of Part IV.; and agreeably to an observation made in a Note, at the end of the fifth Problem of this Section, it is considered as the base of a Victualler's Cooler.

Note. When the bottom of a Cooler is a triangle, the base and half the perpendicular may be entered in the Officer's Dimension Book; when it is a trapezium, the diagonal and half the sum of the two perpendiculars may be entered; and when an irregular polygon of several sides, many experienced Officers make a small sketch of the figure, at the top of the Dimension Book; and enter the dimensions upon the respective parts of the figure, and the area below it. (See Examples and Notes in Problems IV., V., VI., VII., and VIII., Part IV.; and also Problem VII., Section II., Part VI.)

SECTION II.

THE METHOD OF GAUGING AND INCHING COMMON BREWERS' UTENSILS, AS PRACTISED IN THE EXCISE.

PROBLEM I.

To gauge and inch a mash-tun in the form of the frustum of a cone, standing upon its greater base.

To take the dimensions.

With any convenient instrument, measure the perpendicular depth of the tun. Then quarter it as directed in Problem III., Sect. I.; and take cross diameters in the middle of every ten inches, from the bottom upwards, if you intend to *table* the tun for *wet* inches; but from the top downwards, when the tun is to be *tabled* for *dry* inches; so will the odd inches fall at the top of the vessel in the first case, and at the bottom in the second. From the cross diameters find mean diameters; and enter all the dimensions in your Note Book.

To find the area and content.

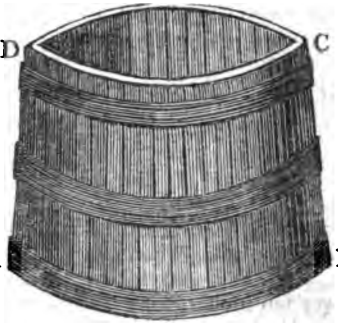
RULE.

Divide the square of each mean diameter by 289; and the quotients will be the areas of the respective sections, in mash-tun gallons. Multiply these areas by their corresponding depths; and the sum of the products will be the whole content of the tun.

Note. As 8 gallons make one bushel, and 8 bushels one quarter, any area or content in gallons, may be reduced to bushels by dividing it by 8; and if the quotient thus found, be divided by 8 also, we shall obtain the area or content in quarters. The same numbers must also be used in Addition and Subtraction of quarters, bushels, and gallons.

EXAMPLES.

1. Let the following figure A B C D represent a common brewer's mash-tun in the form of the frustum of a cone standing upon its greater base; it is required to *gauge* and *inch* it, as practised in the Excise.



Measure the perpendicular depth of the tun; then quarter it, and take cross diameters, from which find mean diameters, as before directed; and enter all the dimensions in your Note Book, as below.

NOTE BOOK.

A.B.'s Round Mash Tun, No.1, gauged July 20, 1821.

Divisions in Inches.	Depths from the Bottom.	Cross Diameters.		Sum of Ditto.	Mean Diameters.
10	45	61.9	61.7	123.6	61.8
10	35	63.8	63.6	127.4	63.7
10	25	66.4	65.6	132.0	66.0
10	15	68.4	68.8	137.2	68.6
10	5	71.3	71.1	142.4	71.2

BY THE PEN.

To find the area and content.

Inches.		*Inches.*		*Divisor.*		*M. T. Gallons.*
71.2	×	71.2	÷	289	=	17.541 *area of the 1st section.*
68.6	×	68.6	÷	289	=	16.283 *area of the 2nd section.*
66.0	×	66.0	÷	289	=	15.072 *area of the 3rd section.*
63.7	×	63.7	÷	289	=	14.040 *area of the 4th section.*
61.8	×	61.8	÷	289	=	13.215 *area of the 5th section.*

8)761.51 *whole content.*

8)95 *bu.* 1.51 *gallons.*

11 *qr.* 7 *bu.* 1.51 *gallons.*

Note 1. In adding the areas together, the decimal point is removed one place towards the right; hence we obtain the whole content of the tun, in gallons. We then divide twice by 8, in order to reduce the content into quarters, bushels, and gallons. We also obtain the content of every 10 inches of the perpendicular depth, by removing the decimal point one place towards the right, in each area.

2. The areas of the horizontal sections may be found by the Sliding Rule, as directed in the last Section.

DIMENSION BOOK.

From the Note Book, and from the foregoing areas and contents, we form the Dimension Book, as below.

A. B.'s Round Mash Tun, No. 1, gauged July 20, 1821.

Depths in Inches.	Mean Diameters.	Areas in Gallons.	Contents in Gallons.	Areas in Q.	B.	G.	Contents in Q.	B.	G.
10	61.8	13.215	132.15	0	1	5.215	2	0	4.15
10	63.7	14.040	140.40	0	1	6.040	2	1	4.40
10	66.0	15.072	150.72	0	1	7.072	2	2	6.72
10	68.6	16.283	162.83	0	2	0.283	2	4	2.83
10	71.2	17.541	175.41	0	2	1.541	2	5	7.41
50	Whole Depth.		761.51	Whole content.			11	7	1.51

To prove the foregoing Table.

RULE.

In order to prevent mistakes, it is necessary to prove the Tabling at every 10 inches of the depth, in the following manner: Compare the content of the first 10 inches, with the corresponding content in the Dimension Book; and if they agree, the work is right. Again, to the content of the first 10 inches, add the content of the second 10 inches; and the sum will be the content of the first 20 inches. To this content add the content of the third 10 inches; and you will obtain the content at 30 inches; &c. &c. as in the following work.

Proof.

Q.	B.	G.	
2	5	7.41	*content of the first* 10 *inches.*
2	4	2.83	*content of the second* 10 *inches.*
5	2	2.24	*content of the first* 20 *inches.*
2	2	6.72	*content of the third* 10 *inches.*
7	5	0.96	*content at* 30 *inches*
2	1	4.40	*content of the fourth* 10 *inches.*
9	6	5.36	*content at* 40 *inches.*
2	0	4.15	*content of the last* 10 *inches.*
11	7	1.51	*content at* 50 *inches.*

Note. When cross diameters are taken in the middle of every 6, 8, 12, &c. inches; the Tabling must of course be proved at those depths.

To form the following Table Book.

RULE.

From the foregoing Table, take the numbers corresponding to each inch, and place them successively one after another, in proper columns; and when the first place of decimals is five-tenths or upwards, call it *one* gallon; but if it be under five-tenths, it must be rejected.

COMMON BREWER'S TABLE BOOK.

A. B.'s Round Mash Tun, No. 1.															
et In- es.	Contents in			Wet In-ches.	Contents in			Wet In-ches.	Contents in			Wet In-ches.	Contents in		
	Q.	B.	G.		Q.	B.	G.		Q.	B.	G.		Q.	B.	G.
[illegible]	0	2	2	14	3	6	1	27	6	7	4	40	9	6	5
[illegible]	0	4	3	15	4	0	1	28	7	1	3	41	10	0	3
[illegible]	0	6	5	16	4	2	1	29	7	3	2	42	10	2	0
[illegible]	1	0	6	17	4	4	1	30	7	5	1	43	10	3	5
[illegible]	1	3	0	18	4	6	2	31	7	6	7	44	10	5	2
[illegible]	1	5	1	19	5	0	2	32	8	0	5	45	10	6	7
[illegible]	1	7	3	20	5	2	2	33	8	2	3	46	11	0	5
[illegible]	2	1	4	21	5	4	1	34	8	4	1	47	11	2	2
[illegible]	2	3	6	22	5	6	0	35	8	5	7	48	11	3	7
[illegible]	2	5	7	23	5	7	7	36	8	7	5	49	11	5	4
[illegible]	3	0	0	24	6	1	7	37	9	1	3	50	11	7	2
[illegible]	3	2	0	25	6	3	6	38	9	3	1				
[illegible]	3	4	0	26	6	5	5	39	9	4	7				

. Suppose the Excise Officer found the depth of the *goods*, in
going mash-tun, to be 32 inches; then, by the above Table
he content of this gauge is 8 quarters, 0 bushels, and 4 gallons;
this part of gauging is intended for a check upon the Trader,
icer may expect to find about 24 barrels of liquor, when he
the wort; because, on an average, one quarter of malt is con-
to produce about 3 barrels of ale. However, as some Brewers
their ale stronger than others; and as bad malt generally
s more grains and less wort than good; much dependence
be placed on mash-tun gauges.

EXAM. 2.

perpendicular depth of a mash-tun in the form of
ustum of a cone, standing upon its greater base, is
hes; the mean diameter taken at 5 inches from the
ı, is 90.4; at 15 inches, 88.2; at 25 inches, 86.1;
inches, 83.8: and 46 inches from the bottom, the
diameter is 81.6 inches; it is required to find the
f each section, the content of each division, and to
e tun as practised in the Excise.

ANSWER.

Areas.		Contents.	
Mash-tun gallons.		Mash-tun gallons.	
28.277	*first section*........	282.77	*first division.*
26.917	*second section*......	269.17	*second division.*
25.651	*third section*.........	256.51	*third division.*
24.299	*fourth section*.......	242.99	*fourth division.*
23.140	*fifth section*.........	276.48	*fifth division.*
	Whole content......	1327.92	*sum.*

Note. In the above answer, only the areas of the sections, and the contents of the divisions are inserted; but in the Key to this Work, a Dimension Book is formed, and the content of the vessel is given at every inch of its perpendicular depth, as required in the Question.

REMARK.

A mash-tun in the form of a parallelopipedon, or a cylinder, may be *inched* by finding the area of its base, and proceeding with this area according to the foregoing directions; and if a mash-tun be the frustum of a pyramid, or a prismoid, take mean dimensions in the middle of every ten inches, find the areas of the different sections, with which proceed as directed in the last Problem.

PROBLEM II.

To gauge and inch a copper with a rising crown.

PRELIMINARY OBSERVATIONS.

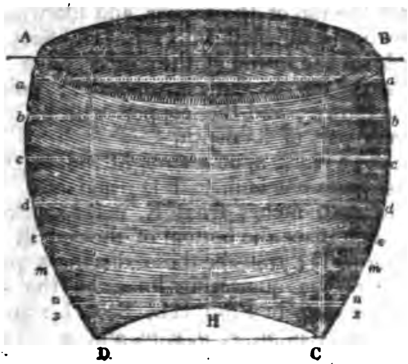
Coppers used by Common Brewers, have generally bottoms somewhat resembling the segment of a sphere. When the convex part projects upwards, the copper is said to have a *rising crown;* but when it projects downwards, leaving a concave space within, it is denominated a *falling crown.* Coppers with horizontal bottoms are seldom met with; and it may also be remarked that small coppers have generally falling crowns; while those of pretty large dimensions, are commonly made with rising crowns. To the mouth of some coppers is attached a copper-hoop, called a *curb.* The curb rises several inches

above the top of the copper; and is generally of a considerably greater diameter; consequently, it is necessary to take its dimensions as a distinct part of the copper.

EXAMPLES.

EXAM. 1.

Let the following figure A B C D represent a common brewer's copper with a *rising* crown; it is required to *gauge* and *inch* it, as practised in the Excise.



To take the dimensions.

Lay a straight pole or rod diametrically across the top the copper; and mark it with chalk, where it is intersected by the sides, as at A and B. Also, mark the middle of the line A B, as at F; measure the distance from F to the centre of the crown at H; and you will have the perpendicular depth of the copper = 57.8 inches. Let fall a plumb-line from the diameter A B, to the angle of the bottom at D; and mark A B, where the plumb-line and diameter intersect each other, as at E. Measure the depth E D = 65 inches; then E D − F H = − 57.8 = 7.2 inches, the height of the crown or spherical segment of the bottom.

Again, let fall the plumb-line to C; mark A B at G;

and E G will be = C D = 54 inches, the diameter of the spherical segment of the bottom.

Set off 7.2 inches, the height of the crown, perpendicularly from D to x; and by repeating the process, make a sufficient number of marks upon the sides of the copper, so that through these marks a circle may be drawn by the hand, round the inside of the copper, on a level with the top of its crown; that is, at the perpendicular distance of 57.8 inches from the diameter A B, at the copper's mouth.

Measure the diameter A B = 80 inches, and also the the diameter $x\,x$, at the top of the crown = 63 inches; then quarter the top of the copper, and likewise the circle at the crown, as directed in Prob. III., Sect. I. Draw four lines down the inside of the copper, from the angular points at the top to the corresponding points in the circle at the crown; and the copper will be truly quartered in every part of its depth.

Next, make a mark upon a straight rod, at the distance of 65 inches from one end, = E D; and at 4, 13, 23, 33, 42, 49, and 54.9 inches from E, make marks upon the same rod; then place the bottom of the rod in the angle of the copper at D; transfer those distances perpendicularly to one of the quartering lines, on the side of the copper, as directed in Prob. III., Sect. I.; and you will obtain the points a, b, c, d, e, m, and n. Transfer the same distances to the other three quartering lines; and you will have all the points, on the quartering lines, at which cross diameters must be taken.

Lastly, measure the cross diameters, from which find mean diameters; and enter all the dimensions in your Note Book, as follows.

NOTE BOOK.

A. B.'s Copper No. 1, gauged July 20, 1821.

Divisions in Inches.	Depths from the Top.	Cross Diameters.		Sum of Ditto.	Mean Diameters.	
8	4	81.4	81.2	162.6	81.3	*aa*
10	12	82.2	82.6	164.8	82.4	*bb*
10	22	82.6	83.0	165.6	82.8	*cc*
10	32	82.1	82.3	164.4	82.2	*dd*
8	42	76.5	76.3	152.8	76.4	*ee*
6	49	72.6	72.8	145.4	72.7	[illegible]
5.8	54.9	68.2	68.4	136.6	68.3	[illegible]
	57.8	63.3	63.1	126.4	63.2	*xx*
Height of the crown.				7.2	54.0	CD

ote I. A common Brewer's copper is generally *tabled* for dry es; consequently, the cross diameters must be taken from the top nwards, in order that the odd inches and tenths may fall at the om of the copper. When it is intended to table a copper for wet es, then the diameters must be taken from the bottom towards the of the vessel. (See Prob. III.)

When the copper is large, a lighted candle may be placed upon the re of its crown, and the day-light excluded from above, by ring the top of the copper; then, if a plumb-line be held suc-ively between the candle and each of the quartering points at the the quartering lines may easily be traced down the sides of the er by means of the shadow produced by the candle and the

The quartering points at the bottom may also be found without help of a candle; for if the rod A B, be laid upon two of the site quartering points at the top, and a plummet suspended from as to come in contact with the side of the copper, on a level with op of the crown, we shall thus obtain the quartering point at x, h corresponds with that at A. In like manner the other three tering points at the bottom may be obtained. Hence the copper be quartered without drawing a circle round the inside, upon a with the top of the crown.

To find the area and content.

RULE.

ind the area corresponding to each mean diameter, in Table of *Ale Areas*, Part VII., then multiply each area

by its respective depth; and the sum of the products will be the whole content of the copper.

To find the quantity of liquor that will cover the crown

RULE I.

By Prob VIII., Part V., find the content of the frustum C D $x\,x$; also, by Prob. XI., of the same Part, find the content of the segment C H D; and the difference of these contents will be the quantity of liquor required.

Note. As the altitude of the frustum C D $x\,x$, is small, it may be reduced to a cylinder by finding a mean diameter; and then its content may easily be obtained by Prob. IV., Part V.

RULE II.

From the Table of Ale Areas, take the area corresponding to the diameter $x\,x$, at the top of the crown; and also that corresponding to the number which expresses the height of the crown; then from the first area subtract $1\frac{1}{3}$ of the latter area; multiply the remainder by half the height of the crown; and the product will be the number of gallons required.

RULE III.

Pour into the copper as much water as will just cover the crown; then draw *off* the water into a vessel of a known measure, or into one that may be easily gauged; and you will obtain the *exact* number of gallons required.

Note 1. A circle may be easily traced, at the surface of the water, with chalk, before the water is drawn off, if the copper be not very large; and this circle may then be quartered as before directed. (See the method of taking the dimensions.)

2. It is unnecessary, to measure the diameters C D and $x\,x$, or the height of the crown; when the quantity of liquor that will cover it, is ascertained by the last Rule.

3. The two first Rules will always give nearly the true quantity of liquor that will cover the crown, when it is the segment of a sphere; but as it cannot be easily determined what figure the crown of a copper assumes, the third Rule is always preferred in Practice.

Operation of finding the quantity of liquor to cover the crown.

By Rule II.

		Inches.	*Ale gallons.*
Diameter $\times$ $\times$	=	63.2	11.1596 *area.*
Height of the crown	=	7.2	.1444 } *area.*
			.0481 } $\frac{1}{3}$ *ditto.*
			.1925 *sum.*
			10.9671 *difference.*
Half the height of the crown			3.6 *inches.*
			658026
			329013
Liquor to cover the crown.................			40.48156 *product.*

Note 1. By the first Rule, the quantity of liquor necessary to cover the crown, is found to be 39.1 gallons; but by covering the crown with water, as directed in the third Rule, the quantity is found to be gallons.

2. As 9 gallons make one firkin, and 4 firkins one barrel, any area or content in gallons may be reduced to firkins by dividing it by 9; and the quotient thus found be divided by 4, we shall obtain the area or content in barrels. The same numbers must also be used in Addition and Subtraction of barrels, firkins, and gallons. (See Table II. Part IV.)

DIMENSION BOOK.

Having found the areas of the several sections, and the contents of the different divisions, as before directed; we hence form the Dimension Book, as follows.

A. B.'s Copper, No. 1, gauged July 20, 1821.

Depths in Inches.	Mean Diameters.	Areas in Gallons.	Contents in Gallons.	Areas in B.	F.	G.	Contents in B.	F.	G.
8	81.3	18.409	147.272	0	2	0.409	4	0	3.272
10	82.4	18.910	189.100	0	2	0.910	5	1	0.100
10	82.8	19.094	190.940	0	2	1.094	5	1	1.940
10	82.2	18.818	188.180	0	2	0.818	5	0	8.180
8	76.4	16.256	130.048	0	1	7.256	3	2	4.048
6	72.7	14.720	88.320	0	1	5.720	2	1	7.320
5.8	68.3	12.992	75.354	0	1	3.992	2	0	3.354
57.8	Depth	Crown	38.000	Per Measure			1	0	2.000
Whole Content			1047.214	Whole Content			29	0	3.214

Note. The contents in the fourth column, were found by multiplying the areas in the third column, by their respective depths in the first column.

To find the content of the foregoing copper, at every dry inch of its perpendicular depth.

RULE.

From the whole content, reduced to barrels, firkins, and gallons, subtract the area of the first section; and the remainder will be the content, when one inch is dry. From this remainder subtract the same area, and you will obtain the content when two inches are dry. Proceed in this manner until you have *inched* the whole tun; taking care to change the area when you arrive at 8 inches, at 18 inches, &c. of the perpendicular depth from the top of the copper, as in the following Table.

A TABLE SHOWING THE METHOD OF INCHING THE FOREGOING COPPER.

Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
Full	29	0	3.214	15	21	1	2.572	30	13	1	5.266
	0	2	0.409		0	2	0.910		0	2	0.818
1	28	2	2.805	16	20	3	1.662	31	12	3	4.448
	0	2	0.409		0	2	0.910		0	2	0.818
2	28	0	2.396	17	20	1	0.752	32	12	1	3.630
	0	2	0.409		0	2	0.910		0	2	0.818
3	27	2	1.987	18	19	2	8.842	33	11	3	2.812
	0	2	0.409		0	2	1.094		0	2	0.818
4	27	0	1.578	19	19	0	7.748	34	11	1	1.994
	0	2	0.409		0	2	1.094		0	2	0.818
5	26	2	1.169	20	18	2	6.654	35	10	3	1.176
	0	2	0.409		0	2	1.094		0	2	0.818
6	26	0	0.760	21	18	0	5.560	36	10	1	0.358
	0	2	0.409		0	2	1.094		0	2	0.818
7	25	2	0.351	22	17	2	4.466	37	9	2	8.540
	0	2	0.409		0	2	1.094		0	2	0.818
8	24	3	8.942	23	17	0	3.372	38	9	0	7.722
	0	2	0.910		0	2	1.094		0	1	7.256
9	24	1	8.032	24	16	2	2.278	39	8	3	0.466
	0	2	0.910		0	2	1.094		0	1	7.256
10	23	3	7.122	25	16	0	1.184	40	8	1	2.210
	0	2	0.910		0	2	1.094		0	1	7.256
11	23	1	6.212	26	15	2	0.090	41	7	3	3.954
	0	2	0.910		0	2	1.094		0	1	7.256
12	22	3	5.303	27	14	3	7.996	42	7	1	5.698
	0	2	0.910		0	2	1.094		0	1	7.256
13	22	1	4.392	28	14	1	6.902	43	6	3	7.442
	0	2	0.910		0	2	0.818		0	1	7.256
14	21	3	3.482	29	13	3	6.084	44	6	2	0.186
	0	2	0.910		0	2	0.818		0	1	7.256

TABLE CONTINUED.

Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.
45	6	0	1.930	50	3	3	7.794	55	2	0	2.378
	0	1	7.256		0	1	5.720		0	1	3.992
46	5	2	3.674	51	3	2	2.074	56	1	2	7.386
	0	1	5.720		0	1	5.720		0	1	3.992
47	5	0	6.954	52	3	0	5.354	57	1	1	3.394
	0	1	5.720		0	1	3.992		0	1	1.394
48	4	3	1.234	53	2	3	1.362	57.8	1	0	2.000
	0	1	5.720		0	1	3.992	Crown	1	0	2.000
49	4	1	4.514	54	2	1	6.370				
	0	1	5.720		0	1	3.992				

Note. The number 1 F. 1.394 G. in the third line from the end of the above Table, is obtained by taking $\frac{8}{10}$ of 1 F. 3.992 G., the area of the seventh section.

To prove the foregoing Table.

RULE.

From the whole content of the copper, subtract the content of the first division, taken from the Dimension Book; and the remainder will be the content at eight inches from the top. From this content take the content of the second division, and the remainder will be the content at eighteen inches from the top, &c. &c. as in the following work.

PROOF.

B.	F.	G.	
29	0	3.214	*whole content of the copper.*
4	0	3.272	*content of the first division.*
24	3	8.942	*content at eight inches from the top.*
5	1	0.100	*content of the second division.*
19	2	8.842	*content at eighteen inches from the top.*
5	1	1.940	*content of the third division.*
14	1	6.902	*content at twenty-eight inches from the top.*
5	0	8.180	*content of the fourth division.*
9	0	7.722	*content at thirty-eight inches from the top.*
3	2	4.048	*content of the fifth division.*

B.	F.	G.	
5	2	3.674	*content at forty-six inches from the top.*
2	1	7.320	*content of the sixth division.*
3	0	5.354	*content at fifty-two inches from the top.*
2	0	3.354	*content of the seventh division.*
1	0	2.000	*content at the top of the crown.*
1	0	2.000	*content to cover the crown, per measure.*

Note. The following Table Book is formed in the same manner as lirected in the last Problem.

COMMON BREWER'S TABLE BOOK.

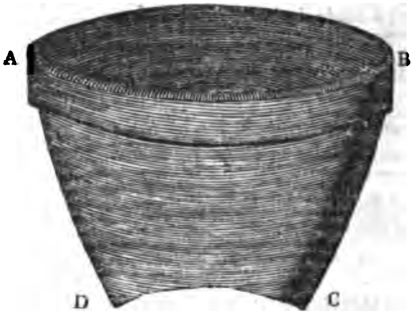
A. B.'s Copper, No. 1.

Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.
Full	29	0	3	15	21	1	3	30	13	1	5	45	6	0	2
1	28	2	3	16	20	3	2	31	12	3	4	46	5	2	4
2	28	0	2	17	20	1	1	32	12	1	4	47	5	0	7
3	27	2	2	18	19	3	0	33	11	3	3	48	4	3	1
4	27	0	2	19	19	0	8	34	11	1	2	49	4	1	5
5	26	2	1	20	18	2	7	35	10	3	1	50	3	3	8
6	26	0	1	21	18	0	6	36	10	1	0	51	3	2	2
7	25	2	0	22	17	2	4	37	9	3	0	52	3	0	5
8	25	0	0	23	17	0	3	38	9	0	8	53	2	3	1
9	24	1	8	24	16	2	2	39	8	3	0	54	2	1	6
10	23	3	7	25	16	0	1	40	8	1	2	55	2	0	2
11	23	1	6	26	15	2	0	41	7	3	4	56	1	2	7
12	22	3	5	27	14	3	8	42	7	1	6	57	1	1	3
3	22	1	4	28	14	1	7	43	6	3	7	57.8	1	0	2
4	21	3	3	29	13	3	6	44	6	2	0	crown	1	0	2

Note. The copper may be tabulated for *wet* inches, by Prob. III., of s Section.

EXAM. 2.

Let the following figure A B C D represent a copper :h a *rising* crown, and a *curb*; it is required to *gauge* l *inch* it, as practised in the Excise.



Directions, for taking the dimensions, &c.

Take the dimensions precisely as directed in the last Example; only observing to measure the first pair of cross diameters exactly in the middle of the curb, if its depth be an integral number of inches. If, however, the depth of the curb consists of inches and tenths, the tenths must be considered as belonging to the second division, and must be measured as part of its perpendicular depth; consequently, the odd tenths that may be in the whole depth of the copper, will fall at the bottom, as they always should, when we intend to *table* for dry inches.

Having taken the diameters; found the areas of the several sections; the contents of the different divisions; and the quantity of liquor to cover the crown; we hence form the Dimension Book, as below.

DIMENSION BOOK.

A. B.'s Copper, No. 2. gauged July 20, 1821.

Divisions in Inches.	Depths from the top.	MeanDiameters.	Areas in Gallons.	Depths of the Divisions.	Contents in Gallons.	Areas in B.	F.	G.	Contents in B.	F.	G.
11	5.5	87.6	21.3722	×11 =	235.0942	0	2	3.3722	6	2	1.0942
10	16	79.4	17.5583	×10 =	175.5830	0	1	8.5583	4	3	4.5830
10	26	73.5	15.0458	×10 =	150.4580	0	1	6.0458	4	0	6.4580
8	35	65.2	11.8396	× 8 =	94.7168	0	1	2.8396	2	2	4.7168
8	43	57.8	9.3046	× 8 =	74.4368	0	1	0.3046	2	0	2.4368
6.6	50.3	52.7	7.7350	× 6.6=	51.0510	0	0	7.7350	1	1	6.0510
53.6	Depth.	Crown per measure.			18.7500	. . . . .			0	2	0.7500
Whole content of the copper.					800.0898	. . . . .			22	0	8.0898

By way of practice, the Learner is required to *tabulate* the foregoing copper, for *dry* inches. (See the Key to this Work, where the content is given at every inch from the top to the bottom of the copper.)

Note 1. The depth of the curb is 11.6 inches; consequently, the first mean diameter is taken at 5.5 from the top; namely, in the middle of the eleven inches; and the six-tenths are considered as belonging to the second division of the copper.

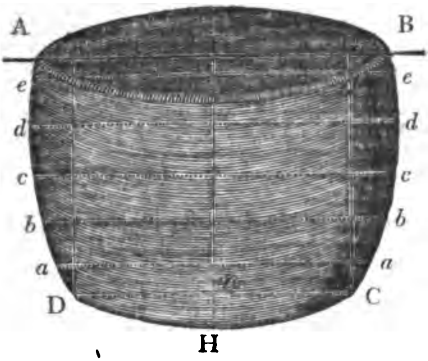
2. When the depth of a curb exceeds 10 or 12 integral inches, it will sometimes be necessary, for the sake of accuracy, to divide it into two or more divisions, by measuring mean diameters in the middle of every 6, 7, 8, or 9 inches, as the case may require.

PROBLEM III.

To gauge and inch a copper with a falling crown.

EXAMPLE.

Let the following figure A B C D, represent a copper with a falling crown; it is required to *gauge* and *inch* it, as practised in the Excise.



To take the dimensions, &c.

By the help of a plumb-line find the depth E D or G C, from the diameter A B, to the angle where the body of

the copper and its bottom are united; and you will have the perpendicular depth of that part of the copper which is to be *tabulated*.

By the same method determine the depth F H, from the middle of the diameter A B, to the centre of the crown; then F H minus E D = H *n*, the depth of the crown C H D.

Next, measure the diameters A B and C D, by the help of which quarter the copper. Also take cross diameters, from which find mean diameters; and enter the dimensions, areas, &c. in your Dimension Book, as below.

DIMENSION BOOK.

A. B.'s Copper, No. 3, gauged Aug. 16, 1821.

Divisions in Inches.	Depths from the Bottom.	Mean Diameters.	Areas in Gallons.	Contents in Gallons.	Areas in B.	F.	G.	Contents in B.	F.	G.
10	45	*ee* =78.2	17.032	170.32	0	1	8.032	4	2	8.32
10	35	*dd* =75.8	16.002	160.02	0	1	7.002	4	1	7.02
10	25	*cc* =73.4	15.005	150.05	0	1	6.005	4	0	6.05
10	15	*bb* =70.6	13.882	138.82	0	1	4.882	3	3	3.82
10	5	*aa* =64.3	11.515	115.15	0	1	2.515	3	0	7.15
50	Diam. Crown.	CD=58.4 H*n*= 8.2	Crown	40.11	. . .			1	0	4.11
Whole Depth.	Whole content of the Copper.			774.47	. . .			21	2	0.47

To find the content of the foregoing copper, at every wet inch of its perpendicular depth.

RULE.

To the content of the crown, reduced to barrels, firkins, and gallons, add the area of the first section; and the sum will be the content at one inch deep. To this content add the same area, and you will obtain the content at 2 inches deep, &c. &c. as in the following Table.

A TABLE

SHOWING THE METHOD OF INCHING THE FOREGOING COPPER.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
Crown	1	0	4.110	4	2	1	5.170	8	3	2	6.230
	0	1	2.515		0	1	2.515		0	1	2.515
1	1	1	6.625	5	2	2	7.685	9	3	3	8.745
	0	1	2.515		0	1	2.515		0	1	2.515
2	1	3	0.140	6	3	0	1.200	10	4	1	2.260
	0	1	2.515		0	1	2.515	*	0	1	4.882
3	2	0	2.655	7	3	1	3.715	11	4	2	7.142
	0	1	2.515		0	1	2.515		0	1	4.882

In this manner the content may be obtained at every inch of the depth; and the Learner is required to continue the above process, and form a Table Book.

Note 1. The crown of the foregoing copper is considered as the segment of a sphere. The content by Rule I., Prob. XI., Part V., is found in the Key; in which Work the copper is tabulated for every inch of its depth.

2. In order to prove the process of tabling, add the content of the first division to the content of the crown; and you will have the content at 10 inches. To this content add the content of the second division; and the sum will be the content at 20 inches, &c. &c.

PROBLEM IV.

To gauge and inch a copper-back.

PRELIMINARY OBSERVATIONS.

A copper-back is a vessel, with a flat bottom, fixed horizontally over the top of a copper; and when several kinds of wort are made at the same brewing, one kind is conveyed into the copper back, while another is boiling in the copper.

By this means the steam which rises from the copper,

A TABLE SHEWING THE METHOD OF INCHING THE FOREGOING COPPER-BACK.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
1	1	3	8.878	5	9	3	8.390	9	17	3	7.902
	1	3	8.878		1	3	8.878		1	3	8.878
2	3	3	8.756	6	11	3	8.268	10	19	3	7.780
	1	3	8.878		1	3	8.878		1	3	8.878
3	5	3	8.634	7	13	3	8.146	11	21	3	7.658
	1	3	8.878		1	3	8.878		1	3	8.878
4	7	3	8.512	8	15	3	8.024	12	23	3	7.536
	1	3	8.878		1	3	8.878		1	3	8.878

In this manner the content may be obtained at every *wet* inch; and the Learner is required to continue the above process, and form a Table Book. (See the Key to this Work.)

REMARKS.

1. Whatever form a copper-back may assume, its base must be divided into such geometrical figures as may be the most easily measured by the Rules given in the respective Problems of Part IV.

2. If a copper-back be the frustum of a pyramid or cone, dimensions must be taken in the middle of every 10 inches of its perpendicular depth; and it must then be tabulated in the same manner as a guile-tun. (See Problems IX., X., and XI. of this Section.)

PROBLEM V.

To gauge and inch an underback in the form of a cylinder.

Directions for taking the dimensions, &c.

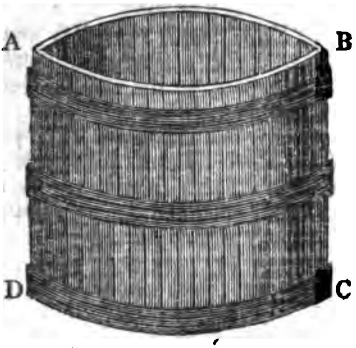
As cylindrical underbacks are seldom perfectly round, cross diameters must be measured in several places; and their sum divided by their number may be taken as a

mean diameter. The square of this mean diameter divided by 359.05, will give the area, (or rather the content) in ale gallons, at one inch deep; and if this content be reduced to barrels, firkins, and gallons, and added to itself, the sum will be the content at two inches deep; and thus, by continual addition, the content of the vessel may be obtained at every wet inch of its depth.

Note. The area answering to the mean diameter may also be found in the Table of Ale Areas, in Part VII.; and if it be multiplied by the mean depth of the vessel, the product will be the content in ale gallons.

EXAMPLE.

Let the following figure A B C D, represent a common brewer's cylindrical underback, whose depth is 50 inches, and the cross diameters taken in several places, as below; it is required to tabulate the vessel, for wet inches, as practised in the Excise.



Diameter.		Inches.
First	....	62.6
Second	....	62.5
Third	....	62.4
Fourth	....	63.8
Fifth	....	62.3
Sixth	....	62.6
Divide by	....	6)376.2 *sum.*
Mean diameter	....	62.7 *quotient.*

To find the area and content.

Here $62.7 \times 62.7 \div 359.05 = 3931.29 \div 359.05 = 10.949$ *ale gallons,* $= 1$ *firkin,* 1.95 *gallons, the mean area; and* $10.95 \times 50 = 547.5$, *the content in ale gallons.*

A TABLE

SHEWING THE METHOD OF INCHING THE FOREGOING UNDERBACK.

A. B.'s Underback, gauged Aug. 16, 1821.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
1	0	1	1.95	5	1	2	0.75	9	2	2	8.55
	0	1	1.95		0	1	1.95		0	1	1.95
2	0	2	3.90	6	1	3	2.70	10	3	0	1.50
	0	1	1.95		0	1	1.95		0	1	1.95
3	0	3	5.85	7	2	0	4.65	11	3	1	3.45
	0	1	1.95		0	1	1.95		0	1	1.95
4	1	0	7.80	8	2	1	6.60	12	3	2	5.40
	0	1	1.95		0	1	1.95				

In this manner the content may be obtained at every inch of the depth; and the Learner is required to continue the above process, and form a Table Book. (See the Key to this Work.)

e. The process of *tabling* may be proved at every 10 inches, by plying the area in gallons, by 10; and reducing the product to s, firkins, and gallons; then if the content thus found, at 10 , be added to itself; the sum will be the content at 20 inches, tc.

Remark.

1 underback in the form of a parallelopipedon may *ched* by finding the area of its base, and proceeding this area as directed in the last Problem; and if an rback be the frustum of a pyramid, or a prismoid, it be *inched* by finding the areas of a competent num- of horizontal sections, and proceeding with these in the same manner as directed for the frustum of a in the first Problem of this Section. (See Problems ., IX., X., and XI.)

PROBLEM VI.

To gauge and inch a restangular hop-back.

hop-back is a vessel with a false bottom, in which great number of small holes. Into this vessel the r is received from the copper; and passing through oles in the false bottom, it is conveyed to the cooler, ıg the hops behind. Sometimes the liquor is re- l in the hop-back a considerable time, in order to he convenience of the Brewer. In such a case it be necessary to take a gauge of the liquor, while in op-back; this method, however, is not practised in ;xcise, when it can be avoided, without any danger e Revenue.

Directions for taking the dimensions, &c.

easure several lengths, and divide their sum by their er for a mean length. Find a mean breadth in the manner; then multiply the mean length by the breadth; divide the product by 282; and the quo- will be the area in ale gallons. Reduce this area barrels, firkins, and gallons; and then tabulate the l, for *wet* inches, as directed in Problems IV. and V.

e. If the area be multiplied by the depth, the product will be ntent in ale gallons. (See Remarks at the end of this Problem.)

EXAMPLE.

The mean length of a rectangular hop-back, is 125.6 inches, the mean breadth 84.7 inches, and the depth 26 inches; it is required to tabulate the vessel for *wet* inches, as practised in the Excise.

To find the area and content.

Here $125.6 \times 84.7 \div 282 = 10638.32 \div 282 = 37.72$ *gallons* $= 1$ *barrel,* 0 *firkins, and* 1.72 *gallons, the mean area of the vessel; and* $37.72 \times 26 = 980.72$ *gallons* $= 27$ *barrels,* 0 *firkins, and* 8.72 *gallons, the content.*

DIMENSION BOOK.

A. B.'s Hop-Back, guaged Aug. 16, 1821.

Depth.	Length.	Breadth.	Area in Galls.	Content in Gallons.	Area in B.	F.	G.	Contents in B.	F.	G.
26	125.6	84.7	37.72	980.72	1	0	1.72	27	0	8.72

THE METHOD OF TABULATING THE FOREGOING HOP-BACK, FOR WET INCHES.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
1	1	0	1.72	5	5	0	8.60	9	9	1	6.48
	1	0	1.72		1	0	1.72		1	0	1.72
2	2	0	3.44	6	6	1	1.32	10	10	1	8.20
	1	0	1.72		1	0	1.72		1	0	1.72
3	3	0	5.16	7	7	1	3.04	11	11	2	0.92
	1	0	1.72		1	0	1.72		1	0	1.72
4	4	0	6.88	8	8	1	4.76	12	12	2	2.64
	1	0	1.72		1	0	1.72		1	0	1.72

By proceeding in this manner, the content at every wet inch may be obtained; and the Learner is required to continue the process, and form a Table Book. (See the Key to this Work.)

find the depth to be allowed for the hops, false bottom, &c.

RULE.

ind the content of the liquor, hops, and false bottom, e hop-back; then draw off the liquor into a cooler, her vessel; and find the content of the liquor only. n the content of the liquor, hops, and false bottom, act the content of the liquor; and the remainder be the content of the hops and false bottom. Divide content, in inches, by the area of the hop-back, in s; and the quotient will be the deduction that must ade in the depth, when a gauge of wort is taken in op-back.

EXAMPLE.

e length of a hop-back is 114 inches, its breadth ches, and the depth of the liquor 15.5 inches. Now, wort be drawn off into a cooler whose length mea- 152 inches, the breadth 126 inches, and the depth e liquor 7 inches; what deduction must be made depth, when a gauge is taken in the hop-back, in to make a proper allowance for the hops and false n? *Ans.* 1.5 *inches.*

REMARKS.

The mean depth of the liquor in the hop-back, must und in the same manner as directed for a cooler; the different *dips* cannot be taken with a *dipping-* a small *wire* must be used, that will easily pass gh the holes in the false bottom.

The constant *dipping-place* can be only at that end ner of the hop-back, where the liquor is drawn off he cooler; and as the back is deeper there than in ther place, in consequence of the inclination of the , it is evident that a deduction must always be made depth, whenever a gauge of wort is taken in the ack. Now, if the deduction for *hops* and *false bot-* added to the deduction for *drip;* the sum will be hole deduction required, which must be marked the side of the back, as directed in Problem V., of st Section.

1. Here it may be necessary to observe, that when a hop-back d and fixed as directed in the last Problem, the deduction for d false bottom will only be correct when the same quantity of used as was contained in the back when it was fixed. Conse-, when more hops are used, the deduction should be increased;

but when a smaller quantity is employed, the deduction should b diminished in taking the gauge. (For the method of deducting th heat from warm wort, see Problem XII.)

2. When a hop-back is in the form of a cylinder, or the frustum of a pyramid or cone, it must be gauged and inched, in the same manne as an under-back, or a guile-tun. (See Problems V., IX., X., and XI.

PROBLEM VII.

To gauge and tenth a back or cooler.

Directions for taking the dimensions, &c.

Whatever be the form of the cooler, such dimensions must be taken as will give the area of its base, in ale gallons, as directed in Part IV.; and as this area or content is always considered to be one inch deep, it is evident that if we divide it by 10, the quotient will be the area or content at one-tenth of an inch. Reduce this content to barrels, firkins, and gallons, and add it to itself, and the sum will be the content at two-tenths of an inch; and thus by continual addition we may obtain the content of the cooler at every tenth of an inch of its depth.

Note 1. When brackets are placed at the corners of the cooler, to strengthen the sides, the sum of their areas must be deducted from the area of the cooler.

2. A constant *dipping-place* may be found as directed in Problem V. of the last Section. (See also the Remark at the end of that Problem.)

EXAMPLES.

EXAM. 1.

The dimensions of a common brewer's rectangular cooler are given below; it is required to *tenth* it to the depth of three inches, as practised in the Excise.

DIMENSION BOOK.

A. B.'s Cooler, No. 1, guaged Aug. 16, 1821.

Depth.	Length.	Breadth.	Area in Gallons.	Content in Gallons.	Area in B.	F.	G.	Content in B.	F.	G.
6	220.8	174.3	136.473	818.838	3	3	1.473	22	2	8.838

BY THE PEN.

To find the area and content.

By Prob. 5, *Sect. I., Part VI., we have* 220.8×174.3÷282=38485.44÷282=136.473 *gallons*=3 *barrels,* 3 *fir-*

s, and 1.473 *gallons, the area of the cooler; and* 136.473
)=818.838 *gallons*=22 *barrels,* 2 *firkins,* 8.38 *gallons,*
content of the cooler; which areas and contents are
ısferred to the Dimension Book, as above.

Note. The area of the cooler at one inch deep, is 136.473 ale gal-
. This being divided by 10, gives 13.6473 ale gallons, the area of
cooler, at one-tenth of an inch; and if this be reduced, we obtain
kin and 4.6473 gallons, the numbers to be used in *tenthing* the
er, as in the following Table.

TABLE SHEWING THE METHOD OF TENTHING THE FOREGOING COOLER.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Content in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
0.1	0	1	4.6473	1.2	4	2	1.7676	2.3	8	2	7.8879
	0	1	4.6473		0	1	4.6473		0	1	4.6473
0.2	0	3	0.2946	1.3	4	3	6.4149	2.4	9	0	3.5352
	0	1	4.6473		0	1	4.6473		0	1	4.6473
0.3	1	0	4.9419	1.4	5	1	2.0622	2.5	9	1	8.1825
	0	1	4.6473		0	1	4.6473		0	1	4.6473
0.4	1	2	0.5892	1.5	5	2	6.7095	2.6	9	3	3.8298
	0	1	4.6473		0	1	4.6473		0	1	4.6473
0.5	1	3	5.2365	1.6	6	0	2.3568	2.7	10	0	8.4771
	0	1	4.6473		0	1	4.6473		0	1	4.6473
0.6	2	1	0.8838	1.7	6	1	7.0041	2.8	10	2	4.1244
	0	1	4.6473		0	1	4.6473		0	1	4.6473
0.7	2	2	5.5311	1.8	6	3	2.6514	2.9	10	3	8.7717
	0	1	4.6473		0	1	4.6473		0	1	4.6473
0.8	3	0	1.1784	1.9	7	0	7.2987	3.0	11	1	4.4190
	0	1	4.6473		0	1	4.6473	3.0	11	1	4.4190
0.9	3	1	5.8257	2.0	7	2	2.9460	6.0	22	2	8.8380
	0	1	4.6473		0	1	4.6473				
1.0	3	3	1.4730	2.1	7	3	7.5933				
	0	1	4.6473		0	1	4.6473				
1.1	4	0	6.1203	2.2	8	1	3.2406				
	0	1	4.6473		0	1	4.6473				

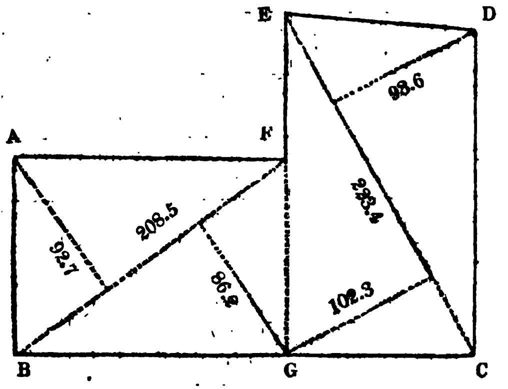
COMMON BREWER'S TABLE BOOK.

A. B.'s Cooler, No. 1.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
0.1	0	1	5	1.1	4	0	6	2.1	7	3	8
0.2	0	3	0	1.2	4	2	2	2.2	8	1	3
0.3	1	0	5	1.3	4	3	6	2.3	8	2	8
0.4	1	2	1	1.4	5	1	2	2.4	9	0	4
0.5	1	3	5	1.5	5	2	7	2.5	9	1	8
0.6	2	1	1	1.6	6	0	2	2.6	9	3	4
0.7	2	2	6	1.7	6	1	7	2.7	10	0	8
0.8	3	0	1	1.8	6	3	3	2.8	10	2	4
0.9	3	1	6	1.9	7	0	7	2.9	11	0	0
1.0	3	3	1	2.0	7	2	3	3.0	11	1	4

EXAM. 2.

Let the following figure A B C D E F, represent an irregular cooler, whose depth is 5 inches; it is required to *gauge* and *tenth* it, as practised in the Excise.



Directions for taking the dimensions, &c.

Produce the line E F to G; and the cooler will be divided into the two trapeziums A B G F, and E G C D. Measure the diagonals and perpendiculars; find the double area of each trapezium; and half the sum of these double areas will be the whole area of the cooler, at one inch deep. Divide this area by 10, and you will have the area at one-tenth of an inch; then reduce this area to barrels, firkins, and gallons; and proceed to *tenth* the cooler as shown in the last example. (See Prob. VIII., Part IV.)

Note. In the practice of gauging, the dimensions of an irregular figure are most easily entered upon a sketch, as in the last Example. (See Note 2, Exam. 4, of the Problem to which we last referred.)

BY THE PEN.

To find the area.

By Prob. VIII., Part IV., we have $(92.7+86.2)\times 208.5 = 178.9\times 208.5 = 37300.65$ *double the area of the trapezium A B G F. Again,* $(102.3+93.6)\times 223.4 = 195.9\times 223.4 = 43764.06$ *double the area of the trapezium E G C D.*

Then, $\dfrac{37300.65+43764.06}{2} = \dfrac{81064.71}{2} = 40532.355$ *square inches, the area of the whole cooler; and* $40532.355 \div 282 = 143.731$, *the area in ale gallons, at one inch deep; consequently,* 14.3731 *gallons* = 1 *firkin, and* 5.3731 *gallons, the area at one-tenth of an inch; hence, we may proceed to tenth the cooler.*

To find the content.

Here $143.731\times 5 = 718.655$ *ale gallons* = 19 *barrels,* 3 *firkins, and* 7.655 *gallons, the whole content of the cooler.*

COMMON BI[LL]

A. B.				
Wet Inches.	Contents in B.	F.	G.	Wet Inches
0.1	0	1	5	1.1
0.2	0	3	0	1.2
0.3	1	0	5	1.3
0.4	1	2	1	1.4
0.5	1	3	5	1.5
0.6	2	1	1	1.6
0.7	2	2	6	1.7
0.8	3	0	1	1.8
0.9	3	1	6	1.9
1.0	3	3	1	2.0

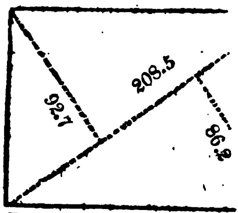
... may be entered

Aug. 16, 1821.

... cooler given in the last Exam... way of practice, the Learner ... Table-Book, as shown in

Let the following fig[ure] ... irregular cooler, whose d[imensions] ... to *gauge* and *tenth* it, as p[...]

A



B

yet given for finding a *true* dipping-place, rrect; as it is founded on Mathematical

)BLEM VIII.

ile-tun in the form of a parallelo-pipedon.

taking the dimensions, &c.

hs, in different parts of the vessel; y their number for a mean length. . the same manner; then multiply mean breadth; divide the pro- uotient will be the area, or rather is at one inch deep.

y the depth, and the product will f the tun. From this content, re- s, and gallons, subtract the *mean* r will be the content at one dry is content subtract the same area, e content at 2 dry inches, &c. &c. bottom of the vessel. (See the copper in Problem II.)

he area by 10, it is evident that the pro- inches of the tun; and if this content be ntent, the remainder will be the content, ence, the *tabling* may be proved at every

ted for wet inches, by Prob. IV., of this

EXAMPLE.

guile-tun in the form of a paral- 4.6 inches, its breadth A E 83.4 3 C 72 inches; it is required to nches, as practised in the Excise.

DIMENSION BOOK.

The dimensions, area, and content may be entered as below.

A. B.'s Cooler, No. 2, gauged Aug. 16, 1821.										
Depth.	Diagonals.	Sum of Perps.	Area in Gallons.	Content in Gallons.	Area in B.	F.	G.	Content in B.	F.	G.
5	208.5 223.4	178.9 195.9	143.731	718.655	3	3	8.731	19	3	7.655

Note. In the Key to this Work, the Cooler given in the last Example, is *tenthed* to its whole depth; and, by way of practice, the Learner is required to repeat the process, and form a Table-Book, as shown in the first Example.

REMARK.

Sometimes it is necessary to find a *new* dipping-place, in consequence of the bottom of a cooler becoming very much *warped* with the heat of the wort. In this case, the best method is to pour into the cooler a certain number of gallons of water, so as completely to cover every part of its bottom; then if these gallons be reduced to inches, and divided by the area of the cooler, in inches, the quotient will be the *true mean* depth of the water that is in the cooler; hence, by repeated trials you may find a place where the depth of the water exactly corresponds with the *mean* depth found as above directed. Or, you may add or subtract a few tenths, as directed in Prob. V., of the first Section.

EXAMPLE.

Suppose the area of a Cooler to be 2 barrels, 1 firkin, and 4 gallons; and admit that 6 barrels of water be poured into the cooler, in order to find a *new* dipping-place; required the *true* mean depth of the liquor?

SOLUTION. Here $6\times4\times9\times282=24\times9\times282=216\times282=60912$, the cubic inches in 6 barrels; also, 2 barrels, 1 firkin, and 4 gallons$=23970$ cubic inches; then $60912\div23970=2.54$ inches, the *true* mean depth required.

Note. Of all the methods yet given for finding a *true* dipping-place, s appears to be the most correct; as it is founded on Mathematical nciples.

PROBLEM VIII.

gauge and inch a guile-tun in the form of a parallelopipedon.

Directions for taking the dimensions, &c.

Measure several lengths, in different parts of the vessel; divide their sum by their number for a mean length. d a mean breadth in the same manner; then multiply mean length by the mean breadth; divide the pro- t by 282, and the quotient will be the area, or rather content in ale gallons at one inch deep.

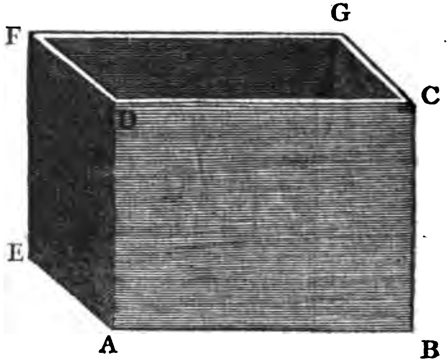
Multiply this area by the depth, and the product will he whole content of the tun. From this content, re- xd to barrels, firkins, and gallons, subtract the *mean* ; and the remainder will be the content at one dry . Again, from this content subtract the same area, you will obtain the content at 2 dry inches, &c. &c. l you arrive at the bottom of the vessel. (See the hod of inching the copper in Problem II.)

Note 1. If we multiply the area by 10, it is evident that the pro- will be the content of 10 inches of the tun; and if this content be acted from the whole content, the remainder will be the content, 10 inches are dry; hence, the *tabling* may be proved at every ches of the depth.

The tun may be tabulated for wet inches, by Prob. IV., of this on.

EXAMPLE.

he length A B of a guile-tun in the form of a paral- ipedon, measures 94.6 inches, its breadth A E 83.4 es, and its depth B C 72 inches; it is required to ate the tun for *dry* inches, as practised in the Excise.



BY THE PEN.

To find the area and content.

Here 94.6 × 83.4 ÷ 282 = 7889.64 ÷ 282 = 27.977 *gallons* = 3 *firkins and* 0.977 *gallons, the mean area; and* 27.977 × 72 = 2014.344 *gallons* = 55 *barrels,* 3 *firkins, and* 7.344 *gallons, the whole content of the vessel.*

DIMENSION BOOK.

A. B.'s Square Guile Tun, No. 1, gauged Aug. 23, 1821.											
Depth.	Length.	Breadth.	Area in Gallons.	Content in Gallons.	Area in			Content in		
					B.	F.	G.	B.	F.	G.
72	94.6	83.4	27.977	2014.344	0	3	0.977	55	3	7.344

LE SHEWING THE METHOD OF INCHING THE FOREGOING GUILE-TUN.

Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.
55	3	7.344	4	52	3	3.436	8	49	2	8.528
0	3	0.977		0	3	0.977		0	3	0.977
55	0	6.367	5	52	0	2.459	9	48	3	7.551
0	3	0.977		0	3	0.977		0	3	0.977
54	1	5.390	6	51	1	1.482	10	48	0	6.574
0	3	0.977		0	3	0.977		0	3	0.977
53	2	4.413	7	50	2	0.505	11	47	1	5.597
0	3	0.977		0	3	0.977		0	3	0.977

:his manner the content may be obtained at every ıch; and the Learner is required to continue the process, and form a Table Book. (See the Key 3 Work.)

: 1. In taking a gauge of liquor in a Common Brewer's guile-ıe depth is always taken to the nearest half-inch: Thus, if the be under 10.3 inches, it is called 10 inches; if it be 10.3 inches, ıder 10.8 inches, it is called 10.5 inches; and if the depth be ıches, or above, it is called 11 inches.

When the depth of a gauge contains a *half-inch*, the content is by taking half the sum of the contents corresponding to the two st depths, in whole numbers, one of which is greater and the less than the depth in question: Thus, if the depth were 10.5 s, we add 48b. 0f. 7g. the content at 10 inches, taken from the : Table, and 47b. 1f. 6g. the content at 11 inches, together; and ɔtain 95b. 2f. 4g. Then this sum being divided by 2, gives 47b. g. the content at 10.5 inches. These observations are applicable ıth *wet* and *dry* inches.

Remarks.

. Guile Tuns in general go under the common deno-ıations of *Rounds* and *Squares*. Thus, if the base of uile-tun be a circle or an ellipse, the vessel is called a

Round; and if the base be a parallelogram, or any other angular figure, the vessel is denominated a Square.

2. Common Brewer's large guile-tuns are generally fixed in an oblique position; and in such a manner, as to be immoveable. Some are raised on pillars, some are placed upon the floor, and others sunk into the ground. In some brew-houses, however, where small quantities are brewed at a time, and consequently large utensils are not required; they use tuns that can be moved to any part of the brew-house at pleasure.

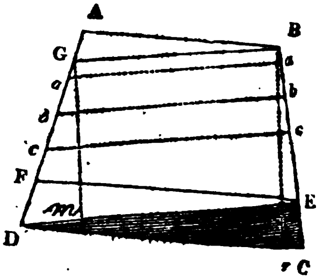
3. Guile-tuns may be tabulated either for *wet* or *dry* inches; it is however most convenient, in Practice, to table large tuns for dry, and small tuns for wet inches. By way of illustration, the vessels given in the following Problems, are tabulated for both *wet* and *dry* inches. Thus the Learner will have an opportunity of becoming fully acquainted with both methods. The method of making an allowance for the *fall* or *drip*, occasioned by the inclination of the tun, is also clearly exemplified.

PROBLEM IX.

To gauge and inch a guile-tun in the form of the frustum of a cone, and make an allowance for the fall or drip.

Directions for finding the content of the drip.

Large guile-tuns, as before observed, are generally fixed in an inclining position, in order that the liquor may drain off quickly; and this inclination is called the *fall* or *drip* of the tun; and the quantity of liquor that is necessary to cover the bottom of the tun, when in this oblique position, is called the content of the drip.



Let A B C D represent the perpendicular section of a guile-tun, in the form of the frustum of a cone standing upon its greater end, in an oblique position; and if E F be parallel to the bottom of the

essel, and D E parallel to the horizon; then the quanty of liquor that will just cover the bottom of the vessel, hen in this position, will be equal to the content of the mical ungula D C E.

The content of this ungula may easily be obtained by rob. XXVII., Part V.; and when the vessel is placed upon s less base, the content of the ungula then formed, may e found by Prob. XXVIII, of the same Part; but as the perendicular height of the ungula seldom exceeds 2 or 3 ches, we may reduce the frustum D C E F to a cylinr, by taking half the sum of the diameters C D and F for a mean diameter; hence the content may be und by Prob. IV., Part V.; half of which may be taken r the content of the *drip*, or ungula D C E.

The most practical method, however, and that which generally adopted by Officers of the Excise, is to cover e bottom of the vessel with water, from a known meare; and thus the content of the ungula is obtained th the greatest accuracy, whatever may be its form.

Note 1. When a vessel in the form of a frustum of a cone, or a amid, is placed upon its greater end, in an oblique position, it is dent that the content of the ungula D C E is greater than half the tent of the frustum D C E F; but if the vessel be placed upon its end, the content of the drip or ungula, will be less than half the tent of the frustum of the same base and altitude.

2. If a vessel in the form of a parallelopipedon, or a cylinder, be ed in an oblique position, the content of the ungula will evidently equal to half the content of the parallelopipedon or cylinder, having same base and altitude.

Directions for taking the dimensions, &c.

As the line D E is parallel to the horizon, it is evident t when the liquor rises to the top of the vessel at B, surface will be parallel to D E; and may be repreted by the line G B; hence, it appears that the vessel l not be full of liquor by the content of the *dry* hoof ungula A B G.

Having caused water to be poured into the vessel, m a known measure, until its bottom is just covered, fall a plumb-line from B, to the surface of the water ; and you will have the perpendicular depth of tha

part of the tun which is to be *tabulated*. With the plumb-line, transfer this perpendicular from the surface of the water at *m*, to the side of the vessel at G; at which point make a mark; and then measure the diameter G B.

Quarter the tun as before directed; and parallel to the diameter G B, measure cross diameters in the middle of every 6, 8, or 10 inches, as at *a a*, *b b*, *c c*, &c. &c.; from these cross diameters find mean diameters; then determine the areas of the several sections; and the whole content of the figure D E B G, to which add the content of the ungula D C E; and the sum will be the quantity of liquor that the vessel will contain, when thus placed in an oblique position.

Note 1. When any circular vessel is placed in an inclining position, all the sections that are taken parallel to the horizontal surface of the liquor, required to cover the bottom, will be ellipses; but as the difference of their diameters will always be small, they may be reduced to circles, by taking half the sum of the diameters for a mean diameter. (See Definitions 17, 18, 20, and 21, Part V.)

2. It is scarcely necessary to observe that after the perpendicular depths of B *n*, and G *m* have been determined, the water that was poured into the vessel, to cover its bottom, must be drawn off, before you can, with convenience, measure the cross diameters.

3. The method of tabling the tun for *dry* inches, is shewn in Problem II.; and for *wet* inches, in Problem III, of this Section.

4. When it is intended to table the tun for wet inches, cross diameters must be taken in the middle of every 6, 8, or 10 inches from the diameter D E, towards the top of the vessel. (See Prob. VI., of the first Section.)

Observations on finding a dipping-place.

As the lines B *n* and G *m*, are both perpendicular to the line D E, supposed to be formed by the surface of the liquor required to cover the bottom of the vessel, it is evident that the *dipping-place* may be either at B or G; but as the perpendicular G *m* is intercepted by the side of the vessel, at G; it is obvious that the depth, in either wet or dry inches, may be most conveniently measured from the top of the tun, in the perpendicular direction of B *n*.

If the tun be *tabulated* for wet inches, the gauging rod, taking the depth, will come in contact with the bottom the vessel, at *r*; consequently, the depth *n r*, of the p, must be ascertained, which let us suppose to be 2 hes; then the number of gallons required to cover the tom, will be the content at 2 inches deep; and this tent added to the area of the first section, will be the tent at 3 inches deep; and thus we may obtain the tent at every inch of the perpendicular depth B *n*.

EXAMPLES.

EXAM. 1.

'he dimensions of the guile-tun A B C D, in the form he frustum of a cone, standing upon its greater base, contained in the following Note Book; it is required abulate the vessel for *dry* inches, as practised in the ise.

NOTE BOOK.

A.B.'s Round Guile Tun, No. 1, gauged Aug. 24, 1821.

Divisions in Inches.	Depths from the Top.	Cross Diameters.		Sum of Ditto.	Mean Diameters.
10	5	92.4	92.2	184.6	92.3
10	15	93.8	93.6	187.4	93.7
10	25	96.3	95.5	191.8	95.9
10	35	98.6	98.4	197.0	98.5
10	45	102.5	102.3	204.8	102.4
10	55	105.7	105.5	211.2	105.6
2	Height of the Drip.	Content of the Drip by measure.			16.5 gallons.

DIMENSION BOOK.

Having found the areas of the several sections, and the contents of the different divisions, as before directed; we hence form the Dimension Book, as below.

A. B.'s Round Guile Tun, No. 1, gauged Aug. 24, 1821.									
Depths in Inches.	Mean Diameters.	Areas in Gallons.	Contents in Gallons.	Areas in B.	F.	G.	Contents in B.	F.	G.
10	92.3	23.7271	237.271	0	2	5.7271	6	2	3.271
10	93.7	24.4520	244.523	0	2	6.4523	6	3	1.523
10	95.9	25.6140	256.140	0	2	7.6140	7	0	4.140
10	98.5	27.0217	270.217	0	3	0.0217	7	2	0.217
10	102.4	29.2039	292.039	0	3	2.2039	8	0	4.039
10	105.6	31.0577	310.577	0	3	4.0577	8	2	4.577
2	Drip.	Drip.	16.500	Per measure.			0	1	7.500
62	Whole Content.		1627.267	Whole content.			45	0	7.267

THOD OF TABULATING THE FOREGOING GUILE-TUN FOR DRY INCHES.

Contents in F.	G.	Dry Inches.	B.	F.	G.	Dry Inches.	B.	F.	G.
0	7.2670	4	42	2	2.3586	8	39	3	6.4502
2	5.7271		0	2	5.7271		0	2	5.7271
2	1.5399	5	41	3	5.6315	9	39	1	0.7231
2	5.7271		0	2	5.7271		0	2	5.7271
3	4.8128	6	41	0	8.9044	10	38	2	3.9960
2	5.7271		0	2	5.7271	*	0	2	6.4523
0	8.0857	7	40	2	3.1773	11	37	3	6.5437
2	5.7271		0	2	5.7271		0	2	6.4523

manner the content, at every *dry* inch, may be and the Learner is required to continue the ocess; and form a Table Book. (See the Key ork.)

EXAM. 2.

ing the dimensions of the foregoing guile-tun n from the bottom upwards; it is required to he vessel for *wet* inches; as practised in the

DIMENSION BOOK.

Round Guile Tun, No. 1, gauged Aug. 24, 1821.

Bottom.	Mean Diameters.	Areas in Gallons.	Contents in Gallons.	Areas in B.	F.	G.	Contents in B.	F.	G.
5	92.3	23.7271	237.271	0	2	5.7271	6	2	3.271
5	93.7	24.4523	244.523	0	2	6.4523	6	3	1.523
5	95.9	25.6140	256.140	0	2	7.6140	7	0	4.140
5	98.5	27.0217	270.217	0	3	0.0217	7	2	0.217
5	102.4	29.2039	292.039	0	3	2.2039	8	0	4.039
5	105.6	31.0577	310.577	0	3	4.0577	8	2	4.577
ip	Drip by Measure.		16.500	....			0	1	7.500
th.	Whole Content.		1627.267	whole content.			45	0	7.267

THE METHOD OF TABULATING THE FOREGOING GUILE-TUN FOR WET INCHES.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
Drip	0	1	7.5000	6	3	3	5.7308	10	7	1	3.9616
	0	3	4.0577		0	3	4.0577		0	3	4.0577
3	1	1	2.5577	7	4	3	0.7885	11	8	0	8.0193
	0	3	4.0577		0	3	4.0577		0	3	4.0577
4	2	0	6.6154	8	5	2	4.8462	12	9	0	3.0770
	0	3	4.0577		0	3	4.0577	*	0	3	2.2039
5	3	0	1.6731	9	6	1	8.9039	13	9	3	5.2809
	0	3	4.0577		0	3	4.0577		0	3	2.2039

In the same manner the content may be obtained at at every *wet* inch; and the Learner is required to continue the process, and form a Table Book.

Note. As the depth of the *drip* is 2 inches, it is evident that the content of the drip added to the area of the first Section, will be the content of the tun at 3 inches, &c. &c. as in the above Table. (See the observations on finding a dipping-place.)

PROBLEM X.

To gauge and inch a circular guile-tun with curved sides, and make an allowance for the drip or fall.

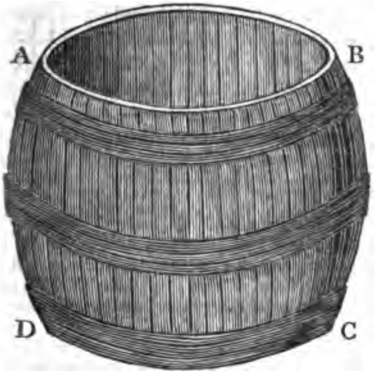
Directions for taking the dimensions, &c. &c.

Pour in water to cover the bottom, as directed in the last Problem; then quarter the tun, and measure cross diameters in the middle of every 6 or 8 inches. From these cross diameters find mean diameters; then determine the areas of the several sections, the contents of the different divisions; and tabulate the vessel, either for *wet* or *dry* inches, as directed in the foregoing Problems. (See Problem VII. of the first Section.)

EXAMPLES

EXAM. 1.

imensions of the guile-tun A B C D, the areas of
ral sections, and the contents of the different
, are contained in the following Dimension Book;
ired to tabulate the tun for wet inches, as prac-
he Excise.



DIMENSION BOOK.

Round Guile Tun, No. 2, gauged Aug. 28, 1821.

Depths rom the Bottom.	Mean Dia-meters.	Areas in Gallons.	Contents in Gallons.	Areas in B.	F.	G.	Contents in B.	F.	G.
52	68.2	12.954	103.632	0	1	3.954	2	3	4.632
44	69.5	13.453	107.624	0	1	4 453	2	3	8.624
36	70.4	13.803	110.424	0	1	4.803	3	0	2.424
28	72.3	14.558	116.464	0	1	5.558	3	0	8.464
20	68.7	13.145	105.160	0	1	4.145	2	3	6.160
12	63.8	11.336	90.688	0	1	2.336	2	2	0.688
4	60.2	10.093	80.744	0	1	1.093	2	0	8.744
Drip.	Drip by Measure		5.250	....			0	0	5.250
Depth.	Whole Content.		719.986	WholeCont.			19	3	8.986

THE METHOD OF TABULATING THE FOREGOING GUILE-TUN, FOR WET-INCHES.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
Drip	0	0	5.250	5	1	1	0.622	9	2	1	4.944
	0	1	1.093		0	1	1.093	*	0	1	2.336
2	0	1	6.343	6	1	2	1.715	10	2	2	7.330
	0	1	1.093		0	1	1.093		0	1	2.336
3	0	2	7.436	7	1	3	2.808	11	3	0	0.666
	0	1	1.093		0	1	1.093		0	1	2.336
4	0	3	8.529	8	2	0	3.901	12	3	1	3.002
	0	1	1.093		0	1	1.093		0	1	2.336

In the same manner the content may be obtained at every *wet* inch; and by way of practice, the Learner is required to continue the process, and form a Table Book.

Note. In the foregoing Example, the depth of the drip is *one* inch; consequently, the content of the drip added to the area of the first Section, gives the content of the tun at 2 inches, as shown in the above Table.

EXAM. 2.

Supposing the dimensions of the foregoing guile-tun to be taken from the top towards the bottom; it is required to tabulate the vessel for *dry* inches, as practised in the Excise.

DIMENSION BOOK.

A. B.'s Round Guile Tun, No. 2, gauged Aug. 28, 1821.

Divisions in Inches.	Depths from the Top.	Mean Diameters.	Areas in Gallons.	Contents in Gallons.	Areas in B.	F.	G.	Contents in B.	F.	G.
8	4	68.2	12.954	103.632	0	1	3.954	2	3	4.632
8	12	69.5	13.453	107.624	0	1	4.453	2	3	8.624
8	20	70.4	13.803	110.424	0	1	4.803	3	0	2.424
8	28	72.3	14.558	116.464	0	1	5.558	3	0	8.464
8	36	68.7	13.145	105.160	0	1	4.145	2	3	6.160
8	44	63.8	11.336	90.688	0	1	2.336	2	2	0.688
8	52	60.2	10.093	80.744	0	1	1.093	2	0	8.744
1	Drip.	Drip by Meas.		5.250	....			0	0	5.250
57	Depth.	Whole Content		719.986	... .			19	3	8.986

THE METHOD OF TABULATING THE FOREGOING GUILE-TUN, FOR DRY INCHES.

Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.	Dry Inches	Contents in B.	F.	G.
Full	19	3	8.986	4	18	2	2.170	8	17	0	4.354
	0	1	3.954		0	1	3.954	*	0	1	4.453
1	19	2	5.032	5	18	0	7.216	9	16	2	8.901
	0	1	3.954		0	1	3.954		0	1	4.453
2	19	1	1.078	6	17	3	3.262	10	16	1	4.448
	0	1	3.954		0	1	3.954		0	1	4.453
3	18	3	6.124	7	17	1	8.308	11	15	3	8.995
	0	1	3.954		0	1	3.954		0	1	4.453

By proceeding as above, the content at every *dry* inch may be obtained; and the Learner is required to continue the process, and form a Table Book.

PROBLEM XI.

To gauge and inch an elliptical guile-tun, and make an allowance for the drip or fall.

Directions for taking the dimensions, &c. &c.

Pour in water to cover the bottom; then quarter the tun; and measure transverse and conjugate diameters in the middle of every 10 inches. Multiply the transverse diameter of each section, by the conjugate diameter; divide the products by 359.05; and the quotients will be the areas of the respective sections. Multiply each area by its corresponding depth; and you will obtain the content of the respective divisions. Then tabulate the tun, either for *wet* or *dry* inches, as directed in the foregoing Problems. (See Prob. VIII. of the first Section.)

EXAMPLES.

EXAM. 1.

The dimensions of an elliptical guile-tun, the areas of the several sections, and the contents of the different

divisions, are contained in the following Dimension Book; it is required to tabulate the vessel for *dry* inches, as practised in the Excise.

DIMENSION BOOK.

A. B.'s Round Guile Tun, No. 3, gauged Sept. 5, 1821.

Divisions in Inches.	Depths from the Top.	Transverse Diameters	Conjugate Diameters.	Areas in Gallons.	Contents in Gallons.	Areas in B.	F.	G.	Contents in B.	F.	G.
10	5	64.8	54.2	9.782	97.82	0	1	0.782	2	2	7.82
10	15	67.7	56.8	10.710	107.10	0	1	1.710	2	3	8.10
10	25	70.4	59.7	11.706	117.06	0	1	2.706	3	1	0.06
10	35	73.5	62.6	12.815	128.15	0	1	3.815	3	3	2.15
9	44.5	76.2	65.3	13.858	124.722	0	1	4.858	3	3	7.722
3	Drip.	Drip by measure.			21.75	....			0	2	3.75
52	Depth.	Whole Content.			596.602	....			16	2	2.602

Note. The areas of the several Sections, and the contents of the different divisions are found in the Key to this Work.

THE METHOD OF TABULATING THE FOREGOING GUILE-TUN FOR DRY INCHES.

Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.
Full	16	2	2.602	4	15	1	8.474	8	14	1	5.346
	0	1	0.782		0	1	0.782		0	1	0.782
1	16	1	1.820	5	15	0	7.692	9	14	0	4.564
	0	1	0.782		0	1	0.782		0	1	0.782
2	16	0	1.038	6	14	3	6.910	10	13	3	3.782
	0	1	0.782		0	1	0.782	*	0	1	1.710
3	15	3	0.256	7	14	2	6.128	11	13	2	2.072
	0	1	0.782		0	1	0.782		0	1	1.710

In this manner may the content be obtained at every *dry* inch; and the Learner is required to finish the process, and form a Table Book.

EXAM. 2.

.et the dimensions, areas, and contents remain the ıe as in the last example; it is required to tabulate the le-tun for *wet* inches.

E METHOD OF TABULATING THE FOREGOING GUILE-TUN FOR WET INCHES.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
Drip	0	2	3.750	7	2	0	5.182	11	3	2	6.614
	0	1	4.858		0	1	4.858		0	1	4.858
4	0	3	8.608	8	2	2	1.040	12	4	0	2.472
	0	1	4.858		0	1	4.858	*	0	1	3.815
5	1	1	4.466	9	2	3	5.898	13	4	1	6.287
	0	1	4.858		0	1	4.858		0	1	3.815
6	1	3	0.324	10	3	1	1.756	14	4	3	1.102
	0	1	4.858		0	1	4.858		0	1	3.815

Thus may the content be obtained at every *wet* inch; l the Learner is required to continue the process, and n a Table Book.

Note 1. As the depth of the drip is 3 inches, it is evident that the of the bottom Section added to the content of the drip, gives content of the tun at 4 inches.

In consequence of the tun being tabulated for *wet* inches, from the e dimensions that were taken for *dry* inches, the nine-inch division* at the bottom of the vessel, in both cases. This division would, ever, have fallen at the top, if the dimensions had been taken with sign to tabulate the tun for *wet* inches only.

REMARKS.

.. When an oval guile-tun is not truly elliptical, it will necessary, for the sake of accuracy, to find the areas of horizontal sections, by the method of equidistant ordi- es, described in Problem XX. Part IV. This method vell applied in Prob. III. of the fifth Section, in *gauging* l *inching* a Distiller's Wash Back.

2. Here it may not be improper to inform the young Learner, that mash-tuns are used for the purpose of mixing malt and hot water together in brewing, in order to extract the saccharine substance from the malt. From the mash-tun the liquor is received into the under-back; from which it is conveyed into the copper, for the purpose of boiling it with hops. When the wort and hops have been boiled together a sufficient time, they are let out of the copper into the hop-back; and thence the liquor is conveyed to the cooler, leaving the hops behind. When the liquor has remained in the cooler, until the heat has evaporated, it is then conveyed into the guile-tun, in order to be fermented with yeast or barm. After it has been in a state of fermentation a proper time, it is put into close casks, which operation is called "tunning" or "cleansing." Thus have we given a short account of the process of brewing, in order to show the purposes to which the vessels are applied, that are *gauged* and *fixed* in the foregoing Problems.

Note. In some brew-houses, no under-back is used; the liquor being conveyed from the mash-tun into the copper. When this is the case, it is evident that one copper must be placed above the mash tun for boiling the water, and another below it, to receive and boil the wort; and it may here be remarked, that much labour may be saved in pumping, when the vessels can be placed in such a manner, that the liquor can be conveyed from one to another by means of pipes. This may be most easily done, by building brew-houses upon inclined planes.

PROBLEM XII.

To deduct the heat from Common Brewers' warm wort.

RULE.

Divide the warm gauge by 10; and the quotient will be the deduction that must be made for the heat. Subtract this deduction from the warm gauge; and the remainder will be the net gauge of the liquor, in barrels, firkins, and gallons.

Note. The method of deducting the heat from victuallers' warm wort, is given in Problem XIII. of the last Section.

EXAMPLE.

. Suppose the content of a warm gauge to be 28 barrels, rkins, and 6 gallons; what is the net gauge, after the t has been deducted?

	B.	F.	G.	
10)	28	3	6	*warm gauge.*
	2	3	5	*deduction.*
	26	0	1	*net gauge.*

'ote 1. In dividing the above warm gauge by 10, we obtain a quotient barrels, and 8 over. Then, 8×4+3=32+3=35 firkins; and this g divided by 10 also, gives a quotient of 3 firkins, and 5 over. Lastly, +6=45+6=51 gallons; and by dividing 51 by 10, we obtain a ient of 5 gallons; hence the whole quotient is 2 barrels, 3 firkins, 5 gallons, the deduction required.

In deducting the heat from common Brewer's warm wort, it is ›mary to reject the decimal part of a gallon, when it is under .3; f it be .3 or upwards to .7, it is called half a gallon; and when it ›eds .7, it is called a whole gallon.

. The content of a warm gauge is 72 barrels, 1 firkin, 2 gallons; what is the content when the heat is de- ted. *Ans.* 65 *bar.* 0 *fir.* 2.5 *gal.*

. If the content of a warm gauge be 29 barrels, 3 ins, and 8 gallons; upon how much liquor must the y be charged? *Ans.* 26 *bar.* 3 *fir.* 8 *gal.*

SECTION III.

THE METHOD OF GAUGING AND ULLAGING CASKS, AS PRACTISED IN THE EXCISE.

CASK GAUGING.

Preliminary Observations.

CASK GAUGING is the most difficult that occurs in Practice. The true content of any *open* vessel may be easily determined; but to ascertain the true content of a *close* cask, by mathematical rules, requires no ordinary share of experience, judgment, and skill. The difficulty arises from casks assuming such a variety of forms, with regard to the curvature of their staves; and the impossibility of giving any *practical* rules, by which the true nature of these curves can be determined.

There are commonly reckoned four forms or *varieties* of casks, viz.;

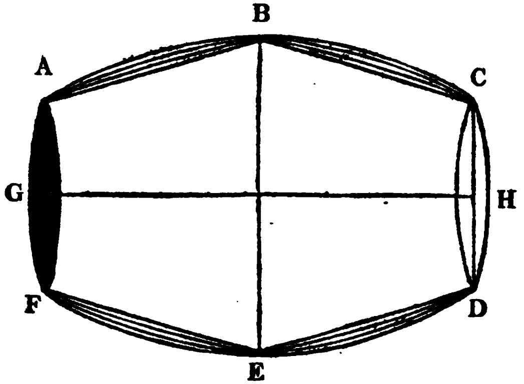
1. The middle frustum of a *spheroid.*
2. The middle frustum of a *parabolic spindle.*
3. The lower frustums of two equal *paraboloids.*
4. The lower frustums of two equal *cones.*

But as a spheroid is formed by the revolution of an ellipse; a parabolic spindle by the revolution of the segment of a parabola; a paraboloid by the revolution of a parabola; and a cone by the revolution of a right-angled triangle; it is more than probable that there never was a cask made, that *exactly* corresponded with any of these varieties; for *experience* proves that few casks are to be met with, that will contain so much as the first form, or so little as the third or fourth; hence it may be concluded that casks in general approach to the second variety; or more probably to a variety between the second and third, which will be the middle frustum of a hyperbolic spindle.

Notwithstanding, Mathematicians affirm that it is almost impossible to make a cask to agree exactly with any of the four varieties before mentioned; yet Excise Officers find it expedient, for the sake of order, accuracy, and expedition, to consider all casks as belonging to some of those varie-

ties. Hence it becomes of the utmost importance to be able to discover to which variety a cask makes the nearest approach, in order that we may apply proper rules in computing its content.

It will readily be perceived, by inspecting the following figure, that the head diameter, the bung diameter, and the length of one cask, may measure exactly the same as those of another; and yet the contents of those casks may differ from each other by several gallons.



For let A B C D E F represent all the four varieties of casks, each cask having the same head diameter A F or C D, the same bung diameter B E, and the same length G H; then it is manifest the cask whose curvature is represented by the external lines A B C, and D E F, will contain more than any of those casks, whose curvatures are denoted by the internal lines A B C, and D E F. It is also evident, that several varieties may be formed between the first and the fourth; hence the impossibility of giving any general rules, by which the *true* contents of all casks can be determined. However, to approach as near to the truth as the nature of the subject will admit; the general method adopted in the Excise, is to examine, very carefully, the casks that are about to be *gauged* and *fixed;* and compare one cask with another. Thus, when it is found that the staves of a cask are very much curved, it is supposed to belong to the *first variety;* when the staves

are not quite so much curved, the cask is considered as belonging to the *second variety*; when the curvature of the staves is very little, the cask is said to be of the *third variety*; and lastly, when the staves are straight from the bung to the head, the cask is denominated as being of the *fourth variety*.

After the variety of the cask has been determined, three dimensions are then measured, in inches and tenths, with the greatest accuracy; namely, the head diameter, the bung diameter, and the length. The cask is then reduced to a cylinder, by using proper multipliers; and hence its content is estimated with ease and expedition, as directed in Problem IV., Part V.

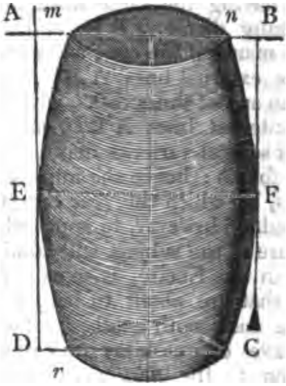
Note. Victuallers' casks are always *gauged* and *fixed* when they are empty; consequently, their dimensions may be easily obtained; but wine pipes, rum puncheons, &c. &c. are generally full of liquor, when they are gauged; and hence their dimensions cannot be ascertained with the same ease, accuracy, and expedition.

PROBLEM I.

To take the dimensions of a standing cask, when it is full of liquor.

EXAMPLE.

Let the following figure represent a standing cask, full of liquor; it is required to determine the head diameter, the bung diameter, and the length.



To find the head diameter.

Measure the distance between the inside of the chimb, close to the head, and the outermost sloped edge of the opposite staff; and this will generally be the head diameter, within the cask, *very nearly*.

Note. When the staves are very thick, the distance from the inside of the chimb to the middle of the opposite staff will generally be equal to the internal head diameter; and sometimes the staves are so strong, that it is only necessary to include one-third of the thickness of the staff, in taking this dimension. In some cases, however, the staves are so slender, that it is necessary to measure from the inside of the chimb to the outside of the opposite staff, for the head diameter.

To find the bung diameter.

Lay a straight rod A B, across the centre of the head; and perpendicular to it, place another straight rod A D, so as to touch the bulge of the cask at E; measure the distance between the outer edge of each chimb, at *m* and *n*; also, measure A *m*, which should be equal to D *r*; then twice A *m* added to *m n*, will give the bung diameter E F, including the thickness of the staff on each side of the cask. From this take twice the thickness of the staff at the bulge, as nearly as your judgment directs, having regard to the size of the cask; and you will obtain the internal bung diameter.

Or, from the rod A B, suspend a plumb-line B C, by a noose or loop at one end; and slide it backward and forward, on the rod, until the line just touches the bulge of the cask at F; then twice B *n* added to *m n*, will be the external bung diameter E F, as before.

Note 1. The external bung diameter of a standing cask may also be found by dividing the circumference by 3.1416. (See Prob. XI., Part IV.; also Note 3, Prob. IX. of this Section.)

2. The staves of casks in general, are thicker at the bulge than at the head; London-made casks, however, have their staves commonly much thicker at the head than at the bulge. The best method of forming a correct judgment, is to examine empty casks of the same size and make, as those you are about to gauge. By this means, you will come to tolerably correct conclusions relating to the deductions necessary to be made for the thickness of the heads, staves, &c. (See the next Problem.)

To find the length.

If there be a hole in the upper head, introduce a rod, and take the external length, from which deduct the

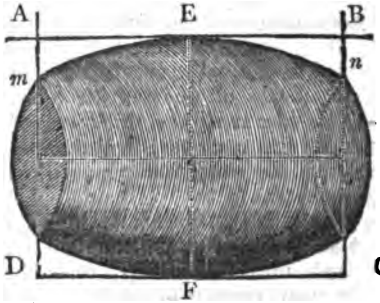
thickness of the upper head; and the remainder will be th internal length of the cask. If this cannot be done measure the external length A D, from which subtrac twice the depth of the upper chimb, together with the thickness of the two heads, as nearly as you can judge and the remainder will be the internal length of the cask

PROBLEM II.

To take the dimensions of a lying cask, when it is full of liquor.

EXAMPLE.

Let the following figure represent a lying cask full of liquor; it is required to determine the head diameter, the bung diameter, and the length.



To find the head and bung diameters.

Measure the head diameter in the same manner as directed for a standing cask; then introduce a rod into the cask at E, and measure the bung diameter E F, from which make a deduction for the thickness of the staff, at the bung-hole; and you will have the internal head and bung diameters of the cask.

To find the length.

The length may be most expeditiously found by a pair of long callipers, taking care to make proper allowance for the thickness of the heads; but as it cannot be expected that every person concerned in gauging, is in pos-

session of this instrument, the length of a cask may be obtained in the following manner. Apply a straight rod, A B, to the bulge of the cask; and at right angles to it, place two others, A D and B C, touching the chimbs at each end, and making A *m* equal to B *n*; then measure the distance from A to B, where the three rods intersect each other; from this subtract the depth of the chimbs, together with the thickness of both the heads, as nearly as you can judge; and the remainder will be the internal length of the cask.

Note 1. In taking the dimensions of a cask, the Gauger ought carefully to observe that the bung-hole be in the middle; that the bung-staff be regular and even within; and that the staff opposite the bung-hole be neither thicker nor thinner than the rest, which he may easily ascertain by the gauging rod; and if any impropriety be discovered, a proper allowance must be made for it in the dimensions.

2. It is also necessary to observe that the heads of the cask be equal, and truly circular; if not take cross diameters of each head, and divide their sum by their number for a mean diameter.

PROBLEM III.

To gauge and fix a cask of the first variety, as practised in the Excise.

RULE I.

Multiply the difference between the head and bung diameters, when it is 6 inches or less, by .68; but if the difference between these diameters exceed 6 inches, multiply it by .70; add the product to the head diameter; and the sum will be the mean diameter of the cask. Multiply the square of the mean diameter by the length; divide the product by 359.05, and 294.12; and the respective quotients will be the content in ale and wine gallons.

RULE II.

Find the mean diameter as directed in the last Rule; then enter the Tables of Ale and Wine areas, and take out the areas corresponding to the mean diameter. Multiply these areas by the length of the cask; and the respective products will be the content in ale and wine gallons. (See Prob. XIII., Part IV.; and also Prob. IV., Part V.)

BY THE SLIDING RULE.

Find the difference between the bung and the head diameters, on the inside of the slide marked C; and opposite to it, on the line marked *spheroid*, is a number, which being added to the head, will give the mean diameter. Then, as the gauge-point on D, is to the length of the cask on C; so is the mean diameter on D, to the content on C.

Note. Neither of the foregoing methods of finding a mean diameter, is mathematically correct; but in consequence of their simplicity, they are generally adopted by Officers of the Excise.

EXAMPLES.

1. The length of a cask is 45, the bung diameter 36, and the head diameter 27 inches; what is its content in ale and wine gallons?

BY RULE I.

Bung Diameter.........	36 *inches.*
Head Diameter.........	27 *inches.*
Difference...............	9 *inches.*
Multiplier...............	.7
Product	6.3
Head Diameter	27.0
Mean Diameter	33.3 *inches.*
Ditto	33.3 *inches.*

$$\begin{array}{r} 999 \\ 999 \\ 999 \\ \hline 1108.89 \; \textit{square.} \\ 45 \; \textit{length.} \\ \hline 554445 \\ 443556 \\ \hline \end{array}$$

Divisor 359.05)49900.05(138.977 *ale gallons.*

Also, 49900.05÷294.12=169.658, *the content in wine gallons.*

BY RULE II.

Here the mean diameter is 33.3 inches; and having entered the Tables of ale and wine areas, in Part VII., we find the areas corresponding to 33.3 inches, are 3.0884 *ale, and* 3.7702 *wine gallons; then,* 3.0884×45=138.978, *the content in ale gallons; and* 3.7702×45=169.659, *the content in wine gallons.*

BY THE SLIDING RULE.

ıe difference between the head and bung diameters is hes; against this number, on the line of inches, we 6.33 on the line marked *spheroid*, which being added ', the head diameter, gives 33.33 inches, for the diameter. Then,

On D.		On C.		On D.		On C.
{18.95 / 17.15}	:	45	::	33.33	:	{139.0 *ale gallons.* / 169.7 *wine gallons.*

The head diameter of a cask measures 24.6, the bung ɜter 30.9, and the length 46.7 inches; what is the nt in ale and wine gallons?

s. The content is 109.460 *ale, and* 133.625 *wine ıs.*

The length of a cask measures 32.5, the bung diameter 26.2, and the head diameter 21.4 inches; what is ontent in ale and wine gallons?

s. The content is 55.220 ale, and 67.414 wine gallons.

The head diameter of a cask measures 19.6, the bung ɜter 23.4, and the length 27.7 inches; required the nt in ale and wine gallons?

s. The content is 38.021 ale, and 46.416 wine gallons.

REMARK.

:he casks given in the foregoing Examples, be con-ɜd as the middle frustums of spheroids, their contents ɔe obtained by Prob. XIV., Part V. The content of the ɡiven in the first Example, is found to be 138.741 ıllons, which differs only two-tenths of a gallon from ɔntent found in this Problem.

PROBLEM IV.

ıuge and fix a cask of the second variety, as practised in the Excise.

RULE I.

ltiply the difference of the diameters, when it is 6 ; or less, by .62; but if it exceed 6 inches, by .64; ıe product to the head diameter; and the sum will be

the mean diameter of the cask. Proceed with this diameter, as directed in the last Problem; and you will obtain the content in ale and wine gallons.

RULE II.

Find the areas in ale and wine gallons, which correspond to the mean diameter; then multiply these areas by the length of the cask, and the respective products will be the content in ale and wine gallons.

BY THE SLIDING RULE.

Find the difference of the diameters, on the inside of the slide marked C; and opposite to it, on the line marked *2nd Variety*, is a number, which being added to the head, will give the mean diameter, with which proceed as directed in the last Problem.

EXAMPLES.

1. The head diameter of a cask is 27, the bung diameter 36, and the length 45 inches; what is the content in ale and wine gallons?

BY RULE I.

Bung diameter...	36 *inches.*
Head diameter...	27 *inches.*
Difference	9 *inches.*
Multiplier	.64
	36
	54
Product.........	5.76
Head diameter	27.00
Mean diameter	32.76 *inches.*
Ditto	32.76 *inches.*
	19656
	22932
	6552
	9828
	1073.2176 *square.*
	45 *length.*
	53660880
	42928704

Divisor 359.05)48294.7920(134.507 *ale gallons.*

lso, 48294.792÷294.12=164.200, the content in wine ons.

BY RULE II.

Here the mean diameter 32.8 inches; the areas answering is diameter, are 2.9963 ale, and 3.6579 wine gallons; 2.9963×45=134.8335, the content in ale gallons; and 579×45=164.6055, the content in wine gallons.

BY THE SLIDING RULE.

'he difference between the head and bung diameters is iches; against this number on the line of inches, we 5.78, on the line marked *2nd variety*, which being ed to 27, the head diameter, gives 32.78 inches, the n diameter. Then,

	On D.		On C.		On D.		On C.
s	18.95 / 17.15	:	45	::	32.78	:	134.5 *ale gallons.* / 164.2 *wine gallons.*

. The head diameter of a cask is 24.6, the bung diaer 30.9, and the length 46.7 inches; what is the con; in ale and wine gallons?

is. The content is 106.626 ale, and 130.165 wine gallons.

. The length of a cask is 32.5, the bung diameter 26.2, the head diameter 21.4 inches; what is the content in and wine gallons?

Ans. The content is 53.888 ale, and 65.786 wine gallons.

. The head diameter, bung diameter, and length of a c are 19.6, 23.4, and 27.7 inches; required the content le and wine gallons?

Ans. The content is 37.339 ale, and 45.583 wine gallons.

REMARK.

f the foregoing casks be considered as the middle frusis of parabolic spindles, their contents may be obtained by b. XVI., Part V. The content of the cask given in the ; Example, is found to be 137.387 ale gallons, which is 3 gallons more than the content found by this Problem. e contents found by this Problem are, however, very rly the same as the *true* contents found by Problem ; hence we may conclude, that the method given in Problem is well adapted for Practical Gauging. (See narks at the end of Problem IX.)

PROBLEM V.

To gauge and fix a cask of the third variety, as practised in the Excise.

RULE.

When the difference of the diameters is 6 inches or under, multiply it by .55; but if it exceed 6 inches, by .57; add the product to the head, and the sum will be the mean diameter, with which proceed as before directed.

Note. The content by the Tables of ale and wine areas, and by the Sliding Rule, may be found as directed in the two last Problems; and as the 3rd and 4th varieties of casks are not placed on any of the slides, the mean diameter found by the multiplier, must be used in casting the content by the Sliding Rule.

EXAMPLES.

1. The length of a cask is 45, the bung diameter 36, and the head diameter 27 inches; required the content in ale and wine gallons?

BY THE PEN.

Bung diameter...	36 *inches.*
Head diameter...	27 *inches.*
Difference	9 *inches.*
Multiplier	.57
	63
	45
Product.........	5.13
Head diameter	27.00
Mean diameter	32.13 *inches.*
Ditto.........	32.13 *inches.*
	9639
	3213
	6426
	9639
	1032.3369 *square.*
	45 *length.*
	51616845
	41293476

Divisor 359.05)46455.1605(129.383 *ale gallons.*

Also, 46455.1605÷294.12=157.946, *the content in wine gallons.*

BY THE TABLES OF ALE AND WINE AREAS.

The areas answering to the mean diameter 32.1 *inches, are* 2.8698 *ale, and* 3.5034 *wine gallons; then* 2.8698×45 =129.141, *the content in ale gallons; and* 3.5034×45= 157.653, *the content in wine gallons.*

BY THE SLIDING RULE.

$$\begin{array}{ccccccc} & \text{On D.} & & \text{On C.} & & \text{On D.} & & \text{On C.} \\ As & \begin{Bmatrix} 18.95 \\ 17.15 \end{Bmatrix} & : & 45 & :: & 32.13 & : & \begin{Bmatrix} 129.4 \text{ ale gallons.} \\ 158.0 \text{ wine gallons.} \end{Bmatrix} \end{array}$$

2. The head diameter of a cask is 24.6, the bung diameter 30.9, and the length 46.7 inches; what is the content in ale and wine gallons?

Ans. The content is 103.367 *ale, and* 126.186 *wine gallons.*

3. The head diameter, bung diameter, and length of a cask, are 21.4, 26.2, and 32.5 inches; required the content in ale and wine gallons.

Ans. The content is 52.136 *ale, and* 63.648 *wine gallons.*

4. The length of a cask is 27.7, the bung diameter 23.4, and the head diameter 19.6 inches; what is the content in ale and wine gallons?

Ans. The content is 36.325 *ale, and* 44.347 *wine gallons.*

REMARK.

If the foregoing casks be supposed to be formed by the frustums of equal paraboloids joined together at their greater ends, their contents may be obtained by Prob. XVIII. Part V. The content of the cask in the first Example, is found to be 126.897 ale gallons, which is 2.486 gallons less than the content found in this Problem.

PROBLEM VI.

To gauge and fix a cask of the fourth variety, as practised in the Excise.

RULE.

If the difference of the diameters be 6 inches or less, multiply it by .50; but if greater than 6 inches, by .52; add the product to the head, and the sum will be the mean diameter, with which proceed as before directed.

Note. The content by the tables of ale and wine areas, and by the sliding rule, may be determined as in the last Problem.

EXAMPLES.

1. What is the content of a cask, in ale and wine gallons, the head diameter of which measures 27, bung diameter 36, and length 45 inches?

BY THE PEN.

Bung diameter...	36	*inches.*
Head diameter...	27	*inches.*
Difference.........	9	*inches.*
Multiplier	.52	
	18	
	45	
Product.........	4.68	
Head diameter	27.00	
Mean diameter	31.68	*inches.*
Ditto.........	31.68	*inches.*
	25344	
	19008	
	3168	
	9504	
	1003.6224	*square.*
	45	*length.*
	50181120	
	40144896	

Divisor 359.05)45163.0080(125.784 *ale gallons.*

Also, 45163.0080 ÷ 294.12 = 153.552, *the content in wine gallons.*

BY THE TABLES OF ALE AND WINE AREAS.

The areas answering to the mean diameter 31.7 *inches, are* 2.7987 *ale, and* 3.4166 *wine gallons; then* 2.7987 × 45 = 125.9415, *the content in ale gallons; and* 3.4166 × 45 = 153.747, *the content in wine gallons.*

BY THE SLIDING RULE.

$$\begin{array}{ccccccc} & \text{On D.} & & \text{On C.} & & \text{On D.} & & \text{On C.} \\ As & \left\{\begin{array}{l}18.95\\17.15\end{array}\right\} & : & 45 & :: & 31.68 & : & \left\{\begin{array}{l}126.9 \textit{ ale gallons.}\\153.7 \textit{ wine gallons.}\end{array}\right. \end{array}$$

2. The head diameter, bung diameter, and length of a cask are 24.6, 30.9, and 46.7 inches respectively; what is the content in ale and wine gallons?

Ans. The content is 107.070 *ale, and* 123.382 *wine gallons.*

3. The length of a cask is 32.5, the bung diameter 26.2, and the head diameter 21.4 inches; what is the content in ale and wine gallons?

Ans. The content is 51.268 *ale, and* 62.591 *wine gallons.*

4. What is the content of a cask, in ale and wine gallons, the head diameter of which measures 19.6, the bung diameter 23.4, and the length 27.7 inches?

Ans. The content is 35,661 *ale, and* 43.536 *wine gallons.*

REMARK.

If the casks in the foregoing Examples be considered as formed by the frustums of equal cones, joined together at their greater ends, their contents may be obtained by Prob. VIII., Part V. The content of the cask given in the first Example, is found to be 125.205 ale gallons, which differs only .579 of a gallon, from the content found by this Problem.

Combination of the Rules given in the four last Problems.

As it is more convenient, in Practice, to have all the Rules placed under the eye at once, than to consult different Problems for them, they are exhibited on the following page, in as concise a manner as possible.

The side which contains the line of inches and tenths, also contains a line of circular ale areas, which shews the area of the base of any cylindrical vessel, when the Rule is applied to its diameter; hence, the content may be found by multiplying this area by the depth of the vessel.

The other two faces of the Rule contain the same lines that are on the Sliding Rule described in Part II. By unfolding Brannan's Rule to 24 inches in length, and placing the two parts back to back, it will perform the principal offices of the Sliding Rule; namely, those of finding the areas and contents of vessels, in ale and wine gallons, and malt bushels; the contents and ullages of casks, &c. This property of Brannan's Rule forms the chief difference between it and the Diagonal Rod before described.

This rule is also a branch of the dimension cane, noticed in Prob. I., Part VI.

To find the content of a cask by the Diagonal Rod.

RULE.

Put the Diagonal Rod into the bung-hole of the cask, in such a manner that the extremity of the rod may meet the head where it intersects the opposite side of the cask; and the content will be exhibited in ale gallons on one diagonal line, and in wine gallons on the other; reckoning from the end of the rod to the centre of the bung-hole.

Note. As the bung-holes of casks are not always in the middle, it is most correct to take both the diagonals in inches and tenths; and the content corresponding to the mean diagonal, will be the true content of the cask, by the Diagonal Rod.

BY THE SLIDING RULE.

As 21.2 on D, is to the diagonal on C; so is the diagonal on D, to the content in ale gallons on C. Or,

As 19.2 on D, is to the diagonal on C; so is the diagonal on D, to the content in wine gallons on C.

Note 1. Those who have neither a Diagonal Rod nor a Sliding Rule, may nevertheless find the content of a cask in the following manner:—Measure the diagonal of the cask in inches and tenths; then multiply the cube of this diagonal by .002288, and .00272 respectively; and the products will be the content in ale and wine gallons.

2. When the diameters and length of a cask are given, its diagonal may be found by Prob. VI.; and when the diagonal and diameters are given, its length may be determined by Prob. VII., Part I.

EXAMPLES.

1. The diagonal of the cask given in the first Example in the foregoing Problems, is 38.7 inches; required the content of the cask in ale and wine gallons.

BY THE DIAGONAL ROD.

Opposite to 38.7 *inches, the given diagonal, we find* 129.8 *ale gallons, and* 158.3 *wine gallons, the contents required.*

BY THE SLIDING RULE.

$$\begin{array}{ccccccc} & \text{On D.} & & \text{On C.} & & \text{On D.} & & \text{On C.} \\ \text{As} & \left\{\begin{array}{c}21.2\\19.2\end{array}\right\} & : & 38.7 & :: & 38.7 & : & \left\{\begin{array}{l}129.8 \textit{ ale gallons.}\\158.2 \textit{ wine gallons.}\end{array}\right.\end{array}$$

BY NOTE I.

Here $38.7 \times 38.7 \times 38.7 = 1497.69 \times 38.7 = 57960.603$, *the cube of the diagonal; then* $57960.603 \times .002228 = 129.136223484$, *the content in ale gallons; and* $57960.603 \times .00272 = 157.65284016$, *the content in wine gallons.*

2. The diagonal of the cask given in the second Example, is 36.2 inches; what is the content in ale and wine gallons?

Ans. The content is 106.0 *ale, and* 129.2 *wine gallons.*

3. The diagonal of the cask given in the third Example, measures 28.6 inches; what is the content in ale and wine gallons?

Ans. The content is 52.2 *ale, and* 63.8 *wine gallons.*

4. The diagonal of the cask given in the fourth Example, measures 25.5 inches; required the content in ale and wine gallons?

Ans. The content is 37.0 *ale, and* 45.1 *wine gallons.*

REMARK.

If the diagonal lines be applied to the head diameter of a regular made cask, they will generally express nearly half the content of the cask, in ale and wine gallons; but if the cask be remarkably long, and the head diameter small in proportion to the length, as in most wine casks, this method gives the content a great deal too little. For example, the diagonal of a wine cask is 37 inches, consequently, its content by the Diagonal Rod, is 139 wine gal-

lons. The head diameter of the same cask, is 24 inches; and twice the content answering to this, on the Diagonal Rod, is only 75 wine gallons; which is too little by 64 wine gallons.

PROBLEM VIII.

To find the contents of casks in general, from the head diameter, bung diameter, and length.

RULE I.

Add into one sum, 39 times the square of the bung diameter, 25 times the square of the head diameter, and 26 times the product of these diameters; multiply this sum by the length of the cask, and the product by .00034; then divide the last product by 11 and 9 respectively; and the two quotients will be the content of the cask in ale and wine gallons.

Note. This rule is taken from Dr. Hutton's excellent Mathematical and Philosophical Dictionary, Vol. I., Page 578. It gives the contents of casks less than the Rule, for the second variety, and more than that for the third, being nearly in the middle between them; but the Dr. observes that, "it agrees well with the real contents of casks in general; as hath been proved by several casks which have actually been filled with a true gallon measure, after their contents had been computed by this method."

RULE II.

Divide the head diameter by the bung diameter, to two places of decimals, and find the quotient in the first column of the following Table, against which we have two multipliers, one for ale, and another for wine; these being respectively multiplied by the square of the bung diameter, and the product thence arising by the length of the cask will give the content in ale and wine gallons.

Note 1. If the quotient of the head by the bung diameter do not terminate in two places of figures, without a fractional remainder, find the multiplier answering to the first two decimals of the quotient, and subtract it from the next greater multiplier; then if the remainder be multiplied by the fractional part of the quotient, the product will be the corresponding proportional part to be added to the first multiplier.

2. This method ought always to be used when the fractional remainder is great, or when accuracy is required; but as it is sometimes

attended with considerable trouble, in multiplying the *tabular* remainder, by the numerator of the fraction, and then dividing the product by the denominator, the work may be much abridged in the following manner: When the numerator is *nearly* equal to one-fourth, one-half, or three-fourths of the denominator, take ¼, ½, or ¾ of the tabular remainder, and add it to the multiplier corresponding to the first-two decimals of the quotient; and use the sum for the multiplier required.

3. This rule gives the content very nearly the same as Rule I.; and will be found much easier in Practice, as it generally requires fewer figures in the operation; particularly when the fractional remainder is rejected.

BY THE SLIDING RULE.

Divide the head by the bung diameter, as before directed, and find the gauge-points opposite to the quotient; then, as the gauge-point on D, is to the length of cask on C; so is the bung diameter on D, to the content of the cask on C.

Note. This method of finding the content of a cask by the Sliding Rule, is remarkably easy and expeditious.

A TABLE OF MULTIPLIERS, &c.

Quotient of the head divided by the bung.	*ALE GALLONS.* *Multipliers.*	*ALE GALLONS.* *Gauge Points.*	*WINE GALLONS.* *Multipliers.*	*WINE GALLONS.* *Gauge Points.*
.50	.0018026	23.55	.0022006	21.31
.51	.0018184	23.45	.0022199	21.22
.52	.0018345	23.35	.0022395	21.13
.53	.0018507	23.24	.0022592	21.04
.54	.0018670	23.14	.0022791	20.95
.55	.0018835	23.04	.0022992	20.85
.56	.0019001	22.94	.0023196	20.76
.57	.0019170	22.84	.0023401	20.67
.58	.0019337	22.74	.0023608	20.57
.59	.0019509	22.64	.0023817	20.48
.60	.0019681	22.54	.0024027	20.39
.61	.0019855	22.45	.0024239	20.30
.62	.0020031	22.35	.0024454	20.21
.63	.0020207	22.25	.0024670	20.13
.64	.0020386	22.15	.0024888	20.04
.65	.0020567	22.05	.0025108	19.95
.66	.0020932	21.96	.0025330	19.87

TABLE CONTINUED.

Quotient of the head divided by the bung.	ALE GALLONS.		WINE GALLONS.	
	Multipliers.	Gauge Points.	Multipliers.	Gauge Points.
.67	.0020750	21.85	.0025553	19.78
.68	.0021116	21.76	.0025780	19.69
.69	.0021303	21.66	.0026007	19.60
.70	.0021491	21.56	.0026237	19.51
.71	.0021680	21.47	.0026468	19.43
.72	.0021871	21.37	.0026701	19.35
.73	.0022064	21.28	.0026936	19.26
.74	.0022258	21.18	.0027173	19.18
.75	.0022454	21.09	.0027412	19.09
.76	.0022651	21.01	.0027653	19.01
.77	.0022850	20.92	.0027896	18.93
.78	.0023050	20.83	.0028141	18.85
.79	.0023253	20.74	.0028387	18.77
.80	.0023457	20.65	.0028635	18.69
.81	.0023660	20.56	.0028885	18.61
.82	.0023867	20.47	.0029137	18.53
.83	.0024075	20.38	.0029392	18.44
.84	.0024285	20.29	.0029647	18.36
.85	.0024496	20.20	.0029905	18.28
.86	.0024710	20.12	.0030165	18.20
.87	.0024923	20.03	.0030426	18.13
.88	.0025140	19.95	.0030690	18.05
.89	.0025357	19.86	.0030955	17.97
.90	.0025576	19.77	.0031223	17.89
.91	.0025796	19.68	.0031493	17.81
.92	.0026019	19.60	.0031763	17.74
.93	.0026242	19.52	.0032036	17.67
.94	.0026467	19.44	.0032311	17.59
.95	.0026693	19.36	.0032588	17.51
.96	.0026922	19.27	.0032867	17.44
.97	.0027152	19.18	.0033147	17.37
.98	.0027384	19.10	.0033428	17.30
.99	.0027616	19.02	.0033713	17.22
1.00	.0027851	18.95	.0034000	17.15

EXAMPLES.

1. The length of a cask is 45, the bung diameter 36, and the head diameter 27 inches; what is its content in ale and wine gallons?

BY RULE I.

Here $36 \times 36 \times 39 = 1296 \times 39 = 50544$, *thirty-nine times the square of the bung diameter;* $27 \times 27 \times 25 = 729 \times 25 = 18225$, *twenty-five times the square of the head diameter; and* $36 \times 27 \times 26 = 972 \times 26 = 25272$, *twenty-six times the product of the diameters; then* $(50544 + 18225 + 25272) \times 45 \times .00034 = 94041 \times 45 \times .00034 = 4231845 \times .00034 = 1438.8273$; *hence* $\frac{1438.8273}{11} = 130.8024$, *the content in ale gallons; and* $\frac{1438.8273}{9} = 159.8697$, *the content in wine gallons.*

BY RULE II.

Here $\frac{27}{36} = .75$, *the quotient of the head divided by the bung diameter. Opposite to this, in the Table of multipliers, we have* .0022454 *for ale, and* .0027412 *for wine gallons; then* $.0022454 \times 1296 \times 45 = 2.9100384 \times 45 = 130.951728$, *the content in ale gallons; and* $.0027412 \times 1296 \times 45 = 3.5525952 \times 45 = 159.866784$, *the content in wine gallons.*

BY THE SLIDING RULE.

The gauge-points opposite .75, *in the Table, are* 21.09 *and* 19.09, *for ale and wine gallons; then,*

	On D.		On C.		On D.		On C.
As	21.09 / 19.09	:	45	::	36	:	131.0 *ale gallons.* / 159.9 *wine gallons.*

2. The head diameter, bung diameter, and length of a cask, are 24.6, 30.9, and 46.7 inches respectively; what is the content in ale and wine gallons?

Ans. The content is 104.1167 *ale, and* 127.2537 *wine gallons.*

3. The length of a cask is 32.5, the bung diameter 26.2, and the head diameter 21.4 inches; what is the content in ale and wine gallons?

Ans. The content is 53.0961 *ale, and* 64.8219 *wine gallons.*

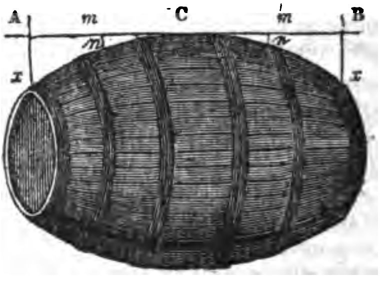
4. What is the content of a cask, in ale and wine gallons, the head diameter of which measures 19.6, the bung diameter 23.4, and the length 27.7 inches?

Ans. The content is 36.7536 *ale, and* 44.8697 *wine gallons.*

PROBLEM IX.

To find the true content of any cask without paying regard to its variety, by means of four dimensions; namely, the length, the bung and head diameters, and the diameter in the middle, between the bung and head.

To find the middle diameter.



Measure the head diameter, bung diameter, and length of the cask, as directed in the first and second Problems. Set off one-fourth of the internal length of the cask both ways, from the middle of the bulge at C, to *n* and *r*.

Lay a straight rod horizontally upon the cask, so as to touch the bulge at C, and in such a manner that the distance A *x*, of the rod from the chimb, may be as nearly equal to the distance B *x* as possible. Measure the two distances *m n* and *m r*; and subtract their sum from the

internal bung diameter; and the remainder will be the internal diameter required.

Note 1. When the rod A B, is not truly horizontal, $m\ n$ will not be equal to $m\ r$; the error however will be rectified by taking their sum from the internal bung diameter. When twice $m\ n$ is taken from the bung diameter, instead of $m\ n + m\ r$, the error may be very considerable, if great care be not taken to have the rod A B parallel to the horizon, or more properly speaking, to the axis of the cask.

2. If the cask be standing on its head, as in the first Problem, set off one-fourth of the internal length, both ways from E towards the ends of the cask; and place the rod A D perpendicular, so that A m may be equal to D r; then measure the distances between the rod A D and the two points made by setting off one-fourth of the cask's length; and subtract their sum from the internal bung diameter; and you will obtain the internal middle diameter of the cask.

3. The external bung, middle, or any other diameter of a cask, may be found to a considerable degree of accuracy, by dividing the circumference of the cask, by 3.1416. Then if twice the thickness of the staff be subtracted from the quotient, the remainder will be the internal diameter of the cask, at that place where the circumference is measured. Thus we have an easy, practical, and mathematical method of finding any diameter of a cask; care, however, must be taken to measure the circumference, and ascertain the thickness of the staves with the greatest accuracy. The latter may be performed by boring them through by an instrument adapted for that purpose, called a "Wood-Guage." (See Prob. XI., Part IV.)

REMARK.

Several writers on Gauging have committed most egregious blunders in directing their Readers to deduct twice $m\ n$ plus twice the thickness of the staff, from the *internal* bung diameter, in order to obtain the *internal* middle diameter. If twice $m\ n$ be taken from the external bung diameter, it is manifest that the remainder will be the external middle diameter; hence, it is evident that twice $m\ n$ plus twice the thickness of the staff ought to be taken from the *external*, not the *internal* bung diameter, in order to obtain the *internal* middle diameter.

To find the content.

RULE I.

To four times the square of the middle diameter, add the sum of the squares of the bung and head diameters; multiply the sum thus obtained by the length; divide

the product by 2154.3, and 1764.72, and the respective quotients will be the content of the cask, in ale and wine gallons. (See Prob. XIX., Part V.)

RULE II.

To four times the area corresponding to the middle diameter, add the areas corresponding to the bung and head diameters; multiply the sum by the length; and one-sixth of the product will be the content in ale or wine gallons, according to the denomination of the areas.

Note 1. The areas may be found either by Prob. XIII., Part IV.; or by the Tables of ale and wine areas, Part VII.

2. If the content of a cask, or any other vessel, in ale gallons, be multiplied by 11, and the product divided by 9, the quotient will be the content in wine gallons, nearly; and *vice versa.* (See the first Rule in the last Problem.)

3. The Rules given in this Problem are the same, in substance, as that given in Problem IX., Part V. (See the Scholium to that Problem.)

BY THE SLIDING RULE.

Set the length on C, to 46.4 for ale, and 42.0 for wine, on D; and find the head, bung, and middle diameters on D, noting the three numbers opposite to them on C; then the sum of the first and second, added to 4 times the third, will give the content in ale or wine gallons.

EXAMPLES.

1. The head diameter of a cask is 27, the middle diameter 33, the bung diameter 36, and the length 45 inches; what is the content in ale and wine gallons?

BY RULE I.

Here $33 \times 33 \times 4 = 1089 \times 4 = 4356$, *four times the square of the middle diameter; and* $36^2 + 27^2 = 1296 + 729 = 2025$, *the sum of the squares of the bung and head diameters; then* $(4356 + 2025) \times 45 = 6381 \times 45 = 287145$; *and* $287145 \div 2154.3 = 133.289$, *the content in ale gallons; also,* $287145 \div 1764.72 = 162.714$, *the content in wine gallons.*

BY RULE II.

Ale Measure.

	Inches.	Area.		Areas.
Middle diameter	33......	3.0330 × 4	=	12.1320
Bung diameter	36......	*area*	=	3.6095
Head diameter	27......	*area*	=	2.0303
Sum of the areas				17.7718
Multiply by the length				45
				888590
				710872
Divide by				6)799.7310
Content in ale gallons				133.2885
By Note 2, multiply by				11
Divide by				9)1466.1735
Content in wine gallons				162.9081

Wine Measure.

	Inches.	Area.		Areas.
Middle diameter	33......	3.7026 × 4	=	14.8104
Bung diameter	36......	*area*	=	4.4064
Head diameter	27......	*area*	=	2.4786
Sum of the areas				21.6954
Multiply by the length				45
				1084770
				867816
Divide by				6)976.2930
Content in wine gallons				162.7155
By Note 2, multiply by				9
Divide by				11)1464.4395
Content in ale gallons				133.1308

BY THE SLIDING RULE.

Ale Measure.

On C.	On D.	On D.	On C.	Ale Gal.
		27.0	15.2......	= 15.2
As 45 :	46.4 ::	36.0 :	27.1......	= 27.1
		33.0	22.7 × 4	= 90.8

*Content in ale gallons......*133.1 *sum.*

Wine Measure.

On C.	On D.	On D.	On C.	Wine Gal.
		27.0	18.5......	= 18.5
As 45 :	42.0 ::	36.0 :	33.1......	= 33.1
		33.0	27.8 × 4	= 111.2

*Content in wine gallons......*162.8 *sum.*

2. The length of a cask is 46.7, the bung diameter 30.9, the middle diameter 29.0, and the head diameter 24.6 inches; what is the content in ale and wine gallons?

Ans. The content is 106.739 *ale gallons, and* 130.303 *wine gallons.*

3. The head diameter of a cask is 21.4, the middle diameter 24.8, the bung diameter 26.2, and the length 32.5 inches; what is the content in ale and wine gallons?

Ans. The content is 54.378 *ale, and* 66.383 *wine gallons.*

4. The length of a cask is 27.7, the bung diameter 23.4, the middle diameter 22.1, and the head diameter 19.6 inches; what is the content in ale and wine gallons?

Ans. The content is 37.098 *ale, and* 45.290 *wine gallons.*

A Table, shewing the Contents of Casks, in Ale Gallons, as found in the foregoing Problems.

Example.	Prob. III. 1st Var.	Prob. IV. 2nd Var.	Prob. V. 3rd Var.	Prob. VI. 4th Var.	Prob VII Dia. Rod.	Prob. VIII Gen. Rule.	Prob. IX. True Rule
1	138.97	134.50	129.38	125.78	129.80	130.80	133.28
2	109.46	106.62	103.36	101.07	106.00	104.11	106.73
3	55.22	53.88	52.13	51.26	52.20	53.09	54.37
4	38.02	37.33	36.32	35.66	37.00	36.75	37.09

Note. If the casks given in the foregoing Examples be considered as the middle frustums of spheroids, the true content of the first, found by Problem XIV., Part V., is 138.741; of the second, 109.028; of the third, 55.240; and of the fourth, 38.041 ale gallons; which contents very nearly agree with those found by Problem III., of this Section. If the casks be considered as the middle frustums of parabolic spindles, the true content of the first, found by Problem XVI., Part V., is 137.387; of the second, 108.340; of the third, 54.962; and of the fourth, 37.892 ale gallons; which contents do not differ much from those found by Problem III., and considerably exceed those found by Problem IV., of this Section.

REMARKS, COMPARISONS, AND OBSERVATIONS ON CASK GAUGING.

By examining the foregoing Table, and comparing the contents in the different columns with each other, we readily perceive that the contents found by Problem III., exceed those found by any of the other Problems; that those obtained by Problems IV., and IX., are nearly equal to each other; that those given by Problems V., and VI., are less than those given by any of the other Problems; and that those found in Problems VII., and VIII., by the diagonal rod, and by Dr. Hutton's general rule, are nearly the same; but both less than those given by Problems IV., and IX.

Now, as the casks in the foregoing Problems, were gauged under very advantageous circumstances; each cask having one head *out*, which enabled us to take all the internal dimensions with the utmost accuracy; and as the method given in Problem IX., is *mathematically* correct, for all kinds of casks; we are inclined to conclude, from these and other premises obtained by experiment, observation, and comparison, that very few casks will contain so much as the *second variety*, considered as the middle frustum of a parabolic spindle.

We also coincide in opinion with Mr. Ward, Mr. Turner, Mr. Fletcher, Dr. Hutton, and many other eminent Gaugers and Mathematicians, that there is no such thing in existence as casks of the *first* and *fourth varieties*, considered as the middle frustums of spheroids, and the lower frustums of equal cones; and that the only true method of gauging casks, is by means of four dimensions, as directed in Problem IX.

We are of opinion that the apparent difficulty of obtaining the middle diameter, has prevented this method from being generally adopted; but this difficulty is only imaginary. The method we have given is extremely simple and easy; and if the operations be performed with care, will always give a true result. However, if a square be used in taking the middle diameter, instead of straight rods, every difficulty will vanish. This instrument may be made by any joiner or cabinet-maker. It consists of two pieces of mahogany, each about 38 or 40 inches in length, $2\frac{1}{2}$ or 3 inches in breadth, and half an inch in thickness. The arms are laid one upon the other, and joined together near one end by means of a pin, which passes through them both; so that when the instrument is *closed* it is very portable, and when it is *open* it forms a square; the two arms being kept at right angles to each other by means of a pin or screw.

In using this instrument, one limb is laid diametrically across the end of the cask, in contact with the chimbs on each side; while the other is made to touch the bulge of the cask in the middle. In this position of the instrument, the middle diameter, or any other diameter between the head and bung may be easily obtained; marks having been previously made upon the sides of the cask, as directed in Problem IX. (See Note 3, of that Problem.)

Note 1. The only real difficulty which arises in taking external diameters, is from the wooden hoops with which some casks are bound. When a wooden hoop happens to be situated where the middle diameter falls, its thickness must be ascertained, and added to the distance $m\,n$ or $m\,r$; as in this case n and r will fall at the external part of the hoop; consequently, its thickness must be added in order to obtain the true distance between the horizontal rod, and the external part of the staff. (See the last Fig.) Or, to prevent further trouble, the hoop where the middle diameter falls, may be struck off the cask.

2. The deductions made by Port Gaugers, in order to reduce casks to the first variety, is another proof of the impropriety of the practice

ised by some Officers of the Excise; namely, that of gauging all, as belonging to the first variety. (See the method of gauging by Callipers.)

Remarks, Comparisons, and Observations on Cask Gauging,

By Dr. Charles Hutton.

Dr. Hutton appears to have bestowed much pains on subject of Cask Gauging; and his remarks and experiments appear to be so judicious, that we think it our duty to lay them before the public, as given in his Mensuration, 8vo., price eighteen shillings in boards. After adding to the Rule given in Problem IX., he says, "The above Rule has been proved to be accurately true, not only for all the four different varieties of casks, but also for all casks and solids generated from any conic section whatever; and although the cask be not precisely in the form of any such curve, the Rule will give the content very near the truth; so that whatever be the form of the cask, we may in all cases be pretty sure of the content to within $\frac{2}{10}$ of a gallon, or perhaps less, supposing the dimensions to be truly taken. So that, more perfect than this, both with respect to truth and expedition, nothing can be expected, or indeed wished for, in Gauging: which makes me hope that one day this method will come into general use with the Practitioners in the Excise; and till then, I am fully persuaded that much of their Practice must be mere guess-work. The pretended difficulty of taking the middle diameter may perhaps deter some from using this method; but there cannot be any real difficulty in taking this diameter, except when a wooden hoop may happen to be at the part where the diameter ought to be taken; but that will very rarely happen."

"The following collection of casks happened in real practice; and their dimensions were carefully taken; but the contents were computed by a Sliding Rule, and so may not all be precisely true."

From hence it appears, that a *spheroidal* cask is a mere imaginary thing, the contents of real casks being less than is assigned to them by that form; as indeed they

ought, from the nature of the curves; for a spheroidal cask would be least curved in the middle, and the most at the ends: whereas a real cask is the least curved at the ends, if it be any thing curved there at all; and indeed there is reason to think it is not, as it appears from observation that all casks are in the form of the frustum of a cone near their ends; and that the middle part approaches nearer to the middle frustum of a parabolic spindle, than any other figure."

Casks gauged by four Dimensions.

Number.	Length.	Head diam.	Bung diam.	Middle diam.	Ale gallons.	Diff. less than sphr.
1	28.3	23.2	27.7	26.3	53.6	1.0
2	29.8	22.2	26.0	24.8	50.2	1.1
3	30.8	23.2	27.5	26.1	57.7	1.1
4	32.2	24.5	30.1	28.4	70.6	1.3
5	30.0	24.7	29.2	27.6	62.6	1.8
6	32.5	23.8	28.2	26.8	63.6	1.9
7	34.3	26.3	33.5	31.1	90.4	2.9
8	34.5	26.4	33.0	30.7	89.0	3.0
9	41.0	26.3	32.2	30.2	102.2	3.0
10	37.0	26.1	31.8	29.9	90.3	3.1
11	44.5	34.4	40.8	38.8	183.8	3.2
12	47.0	26.3	33.8	31.4	126.3	3.5
13	34.2	27.2	33.8	31.4	92.3	3.8
14	47.0	25.3	32.0	29.7	113.1	4.3
15	45.5	30.7	38.0	35.5	157.0	4.7
16	44.6	24.7	32.2	29.6	106.6	4.7
17	48.6	24.2	32.1	29.4	114.4	4.9
18	46.0	25.7	34.7	31.7	125.3	5.5
19	48.8	24.2	32.1	29.4	114.8	5.5
20	51.2	23.3	31.0	28.2	111.3	5.8
21	49.3	23.8	32.6	29.5	117.0	6.1
22	48.0	28.2	33.8	31.4	137.3	6.1
23	45.2	26.6	33.2	30.4	115.6	6.5
24	51.6	36.6	41.6	39.6	223.3	6.7

Casks gauged by four Dimensions.

Number.	Length.	Head diam.	Bung diam.	Mid. diam.	Ale Gall.	Diff. less than sphr
25	44.2	28.1	36.4	33.3	134.6	6.8
26	57.0	32.7	42.0	39.1	236.6	6.8
27	51.0	33.1	38.1	35.7	181.0	8.0
28	51.5	33.3	40.0	37.2	197.0	8.7
29	54.0	34.8	44.8	41.5	253.5	8.8
30	50.0	34.3	40.5	37.7	197.6	9.4
31	49.0	29.5	36.0	33.0	148.1	9.4
32	51.0	33.5	39.2	36.4	189.6	9.7
33	51.0	33.4	39.8	37.0	194.0	9.8
34	55.5	30.6	40.6	37.0	207.2	10.2
35	45.6	28.0	34.6	32.4	134.8	12.0
36	55.0	35.8	48.0	43.2	282.2	17.8

Note. If the figures opposite to each other, in the two last columns, be added together, their sum will be the contents of the casks, when gauged as spheroids; thus, 53.6 + 1.0 = 54.6 gallons, the content of No. 1, when gauged as a spheroid.

ULLAGING CASKS.

THE liquor contained in a cask when it is not full, is called the WET ULLAGE; the vacuity or space not occupied by the liquor, is termed the DRY ULLAGE; and the method of finding the content of the liquor, is called *ullaging* a cask. Ullaging naturally divides itself into two cases; *first,* when the cask is in a horizontal position, or lying on its bulge; and *secondly,* when the cask is in a perpendicular position, or standing upon its head. Ullages are chiefly confined to the wine and spirit trades. They arise either from the leakage of casks, after they have been filled with liquor; or from its having been drawn off in sale.

The contents of ullages may be found either by the Pen, the Sliding Rule, or the Ullage Rule.

DESCRIPTION OF THE ULLAGE RULE.

This is a flat Rule, either nine or twelve inches in length; and has all the lines upon it that are upon the Sliding Rule, described in Part II.; and consequently

may be used in the same manner, in finding the areas and contents of vessels, casting malt-gauges, &c. &c.

The two slides marked B, work together; and are used with the two lines of segments, in casting the ullages of casks.

This instrument has a considerable advantage over the common Sliding Rule, as by it we can find the ullages of both standing and lying casks, without changing the slides; consequently it is very expeditious in casting the stocks of Wine Merchants, Spirit Merchants, &c. &c. where the standing and lying ullages are frequently numerous.

PROBLEM X.

To ullage a lying cask, having the bung diameter, wet inches, and content.

RULE.

BY THE PEN.

Divide the wet inches by the bung diameter, to three places of decimals; and if the quotient exceed .500, add to the said quotient one-fourth of the excess; but if it be less than .500, subtract one-quarter of the deficiency; then multiply the whole content of the cask by the sum in the first case; or the remainder in the second, and the product will be the ullage required.

BY THE SLIDING RULE.

Set the bung diameter on C to 100 on the line marked *Seg. Ly.* or *S. L.*; then against the wet inches on C, you will have a *segment* on the line *S. L.*, which call the fourth number.

Again, set the content of the cask on the line B, to 100 on the line A; then against the fourth number on A, is the ullage of the cask on B.

Or, set the fourth number on B, to 100 on A; then against the content of the cask on A, is the ullage on B.

Note. The latter method has a considerable advantage, in Practice, over the former; as the fourth number may be so very easily transferred from the line of segments to the line B; while, by the former method, there is some probability of the fourth number escaping the memory, before the content on the line B, can be placed to 100 on the line A.

BY THE ULLAGE RULE.

Set the bung diameter on B, to 100 on the line marked *Seg. Ly.* or *S. L.*; then against the wet inches on B, you will have the fourth number on the line *S. L.*

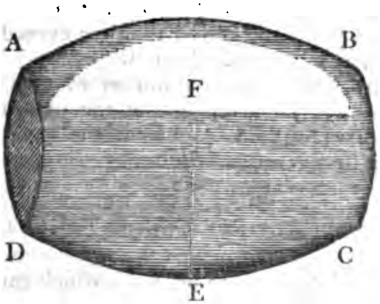
Again, set the content of the cask on B, to 100 on A; then against the fourth number on A, is the ullage of the cask on B.

Or, set the fourth number on B, to 100 on A; then against the content of the cask on A, is the ullage on B.

Note. When the content of a cask is not known, it must be found from proper dimensions, before the above Rules can be applied.

EXAMPLES.

1. Let A B C D represent a lying cask of the second variety, the head diameter of which measures 24, the bung diameter 32, and the length 40 inches; required the ullage of the cask in wine gallons; the *wet* inches E F being 21.6 inches.



To find the content of the cask.

BY PROB. IV.

Here $32 - 24 = 8$ *inches, the difference of the diameters; and* $.64 \times 8 = 5.12$; *then* $24 + 5.12 = 29.12$ *inches, the mean diameter; hence we have* $29.12 \times 29.12 \times 40 = 847.9744 \times 40 = 33918.976$; *and* $33918.976 \div 294.12 = 115.323$, *the content in wine gallons.*

To find the content of the ullage.

BY THE PEN.

Here 21.6 ÷ 32 = .675, *which exceeds* .500 *by* .175, *one-fourth part of which is* = .04375; *then* .675 + .04375 = .71875, *the multiplier; hence* 115.823 × .71875 = 82.88840625 *wine gallons, the ullage required.*

BY THE SLIDING RULE.

On C. On S. L. On C. On S. L.
As 32 : 100 :: 21.6 : 73.7 *the fourth number.*
And,
On A. On B. On A. On B.
As 100 : 115.3 :: 73.7 : 85.0 *wine gallons.*
Or,
As 100 : 73.7 :: 115.3 : 85.0 *the ullage required,*

BY THE ULLAGE RULE.

On B. On S. L. On B. On S. L.
As 32 : 100 :: 21.6 : 73.7 *the fourth number.*
And,
On A. On B. On A. On B.
As 100 : 115.3 :: 73.7 : 85.0 *wine gallons.*
Or,
As 100 : 73.7 :: 115.3 : 85.0 *the ullage required.*

Note. If the whole content of the cask be multiplied by the fourth number, and two decimals more than those in the content and segment, be cut off; the result will be the content of the ullage, without using the lines A and B; thus 115.3 × 73.7 = 84.9761 wine gallons, the content of the ullage, as before.

2. Required the ullage of the foregoing cask, when the wet inches are 10.4 inches.

Ans. 32.434 *wine gallons, by the Pen; and* 30.0 *by the Sliding Rule.*

REMARK.

In the first Example the content by the Sliding Rule exceeds the content by the Pen; but the contrary is the case in the second Example; however, the sum of the two ullages either by the Pen or by the Rule, is equal to the whole content of the cask. The ullage found by the

Sliding Rule is undoubtedly more correct than that found by the Pen; and no other method is practised in the Excise; the Rule by the Pen is, however, very simple; and may be useful to those who are not in possession of a Sliding Rule.

PROBLEM XI.

To ullage a standing cask, having the length, the wet inches, and the content.

RULE.

BY THE PEN.

Divide the wet inches by the length of the cask, to three places of decimals; and if the quotient exceed .500, add to the said quotient one-tenth of the excess; but if it be less than .500, subtract one-tenth of the deficiency; then the whole content of the cask being multiplied by the sum in the first case, or the remainder in the second, will give the ullage required.

BY THE SLIDING RULE.

Set the length of the cask on C, to 100 on the line marked *Seg. St.* or *S. S.*; then against the wet inches on C, you will have a segment on the line *S. S.*, which call a fourth number.

Again, set the content of the cask on B, to 100 on A; then against the fourth number on A, is the ullage of the cask on B.

Or, set the fourth number on B, to 100 on A; then against the content of the cask on A, is the ullage on B.

BY THE ULLAGE RULE.

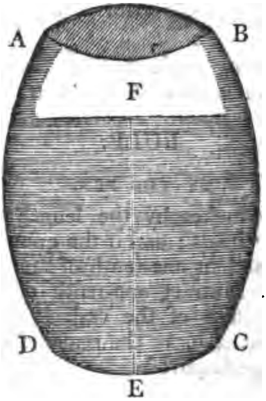
Set the length of the cask on C, to 100 on the line marked *Seg. St. S. S.*; then against the wet inches on C, you will have the fourth number on *S. S.*

Again, set the content of the cask on C, to 100 on A; then against the fourth number on A, is the ullage on C.

Or, set the fourth number on C, to 100 on A; then against the content of the cask on A, is the ullage on C.

EXAMPLES.

1. Let A B C D represent a standing cask of the second variety; required the ullage in wine gallons; the wet inches E F being 31.2 inches, the length 40 inches, and the content 115.323 wine gallons.



BY THE PEN.

Here 31.2 ÷ 40 = .780, *which exceeds* .500, *by* .280, *one-tenth part of which is* = .028; *then* .780 + .028 = .808, *the multiplier; hence* 115.323 × .808 = 93.180984 *wine gallons, the ullage required.*

BY THE SLIDING RULE.

On C. On S. S. On C. On S. S.
As 40 : 100 :: 31.2 : 80.2 *the fourth number.*

And,

On A. On B. On A. On B.
As 100 : 115.3 :: 80.2 : 92.4 *wine gallons.*

Or,

As 100 : 80.2 :: 115.3 : 92.4 *the ullage required.*

BY THE ULLAGE RULE.

On C. On S. S. On C. On S. S.
As 40 : 100 :: 31.2 : 80.2 *the fourth number.*

And,

On A.	On C.	On A.	On C.
As 100 :	115.8 ::	80.2 :	92.4 *wine gallons.*

Or,

As 100 : 80.2 :: 115.8 : 92.4 *the ullage required.*

2. What is the ullage of the foregoing cask, when the wet inches are 8.8 inches?

Ans. 22.142 *wine gallons, by the Pen; and* 28.0 *by the Sliding Rule.*

PROBLEM XVI.

To inch a standing cask of the second or third variety.

RULE.

For the second Variety.

1. From the area of the bung diameter, subtract the area of the head diameter; and multiply the remainder by 3.5. Divide the product thus obtained by 9 times the square of half the length of the cask; and the quotient will be a *common multiplier* for a cask of the second variety.

2. Multiply the square of the proposed distance from the bung, by the *common multiplier;* and subtract the product from the area of the bung diameter. Then multiply this remainder by the proposed distance from the bung; and the product will be the content of the cask from the bung to the extremity of the proposed distance.

3. Having thus found the content of the cask at 1, 2, 3, and 4 inches from the bung, the content at each of the remaining inches may easily be determined by the method of differences; for if we subtract each content from the following one, we shall obtain a set of *first differences.* Again, if these *differences* be subtracted from each other, we shall obtain a set of *second differences;* and lastly, by subtracting the second differences from each other, we shall obtain a set of *third differences.*

4. Now, as the third differences will be all equal to each other; it follows that by adding this common third difference to the first second difference, we shall obtain the

next second difference; and if to this difference we again add the third difference, the sum will be the next second difference, &c. &c. until we have obtained all the *second differences.*

5. By subtracting any *second difference* from the corresponding *first difference,* we shall obtain the next first difference; and by adding any first difference to the corresponding content, we may obtain the next content; and thus we may determine the content at every inch from the bung to the head of the cask, either in ale or wine gallons.

6. Now, as the last of the first differences is considered to express the area, or *nearly* the area corresponding to the head diameter; the next greater difference, the area corresponding to the diameter at the second inch from the head; &c. &c., it follows that we may obtain the content at every inch from the head to the bung, by the successive addition of the first differences; and then by adding the same differences, in a reversed order; we may continue the tabulation from the bung to the upper head; and thus obtain the content of the cask at every inch of its length; as exhibited in the two following Tables.

EXAMPLE.

The bung diameter of a cask of the second variety, is 26, the head diameter 22, and the length 30 inches; it is required to find the content, in wine gallons, at every inch of its length, when it is standing upon its head.

OPERATION.

	Inches.	*Area.*	
Bung diameter	26	2.2984	*wine gallons.*
Head diameter	22	1.6456	*wine gallons.*
Difference		.6528	
Multiplier		3.5	
		32640	
		19584	

Divisor $15 \times 15 \times 9 = 2025$)2.28480(.001128 *quotient.*

For the first inch from the bung.

From the area of the bung diameter	2.298400
Subtract $.001128 \times 1 \times 1 =$	0.001128
Content of one inch from the bung	2.297272

For the second inch from the bung.

From the area of the bung diameter ...	2.298400
Subtract .001128 × 2 × 2 = ...	0.004512
Difference	2.293888
Distance from the bung	2
Content of two inches from the bung ...	4.587776

For the third inch from the bung.

From the area of the bung diameter ...	2.298400
Subtract .001128 × 3 × 3 = ...	0.010152
Difference	2.288248
Distance from the bung	3
Content of three inches from the bung	6.864744

For the fourth inch from the bung.

From the area of the bung diameter ...	2.298400
Subtract .001128 × 4 × 4 = ...	0.018048
Difference	2.280352
Distance from the bung	4
Content of four inches from the bung	9.121408

A TABLE showing the Method of obtaining the three series of differences; and the content of the cask at every inch from the bung to the head.

Inches from the bung.	Contents.	First differences.	Second differences.	Third differences.
1	2.2978	2.2905	.0136	.0066
2	4.5878	2.2769	.0202	.0066
3	6.8647	2.2567	.0268	.0066
4	9.1214	2.2299	.0334	.0066
5	11.3513	2.1965	.0400	.0066
6	13.5478	2.1565	.0466	.0066
7	15.7043	2.1099	.0532	.0066
8	17.8142	2.0567	.0598	.0066
9	19.8709	1.9969	.0664	.0066
10	21.8678	1.9305	.0730	.0066
11	23.7983	1.8575	.0796	.0066
12	25.6558	1.7779	.0862	.0066
13	27.4337	1.6917	.0928	......
14	29.1254	1.5989	....	......
15	30.7243	....	....	......

ELUCIDATION.

Having found the content of the cask, at 1, 2, 3, and 4 inches from the bung, as directed in Nos. 1 and 2 of the Rule; then by No. 3, we have $4.5878 - 2.2973 = 2.2905$; also, $6.8647 - 4.5878 = 2.2769$; and $9.1214 - 6.8647 = 2.2567$, the first three differences in the third column.

Again, we have $2.2905 - 2.2769 = .0136$; also, $2.2769 - 2.2567 = .0202$, the first two differences in the fourth column.

And, $.0202 - .0136 = .0066$, the common difference in the fifth column; then by No. 4 of the Rule, we have $.0136 + .0066 = .0202$; also $.0202 + .0066 = .0268$; &c. &c. until we obtain all the differences in the fourth column.

By No. 5 of the Rule, we have $2.2905 - .0136 = 2.2769$; also, $2.2769 - .0202 = 2.2567$; &c. &c. until we obtain all the first differences in the third column. Also, by the same No. of the Rule, we have $2.2973 + 2.2905 = 4.5878$; also, $4.5878 + 2.2769 = 6.8647$; &c. &c. until we obtain all the contents in the second column. Thus having found the content at every inch from the bung to the head, we may then find the content at every inch from the head to the bung, by Subtraction, as follows: $30.7243 - 29.1254 = 1.5989$, the content at one inch from the head; also $30.7243 - 27.4337 = 3.2906$, the content at two inches from the head; &c. &c. until we have arrived at the bung, where the content is 30.7243; then by Addition, we have $30.7243 + 2.2973 = 33.0216$, the content at 16 inches from the head; also, $30.7243 + 4.5878 = 35.3121$, the content at 17 inches from the head, &c. &c. until we have obtained the content at every inch of the length. The cask may, however, be more easily tabled by Addition, as directed in No. 6 of the Rule; and exhibited in the following Table.

A TABLE

Shewing the Method of Inching the Cask given in the foregoing Example.

Wet Inches.	Contents.	Wet Inches.	Contents.	Wet Inches.	Contents.	Wet Inches.	Contents.
1	1.5989	8	15.0198	15	30.7242	22	46.4286
	1.6917		2.1565		2.2974		2.1099
2	3.2906	9	17.1763	16	33.0216	23	48.5385
	1.7779		2.1965		2.2905		2.0567
3	5.0685	10	19.3728	17	35.3121	24	50.5952
	1.8575		2.2299		2.2769		1.9967
4	6.9260	11	21.6027	18	37.5890	25	52.5919
	1.9305		2.2567		2.2567		1.9305
5	8.8565	12	23.8594	19	39.8457	26	54.5224
	1.9967		2.2769		2.2299		1.8575
6	10.8532	13	26.1363	20	42.0756	27	56.3799
	2.0567		2.2905		2.1965		1.7779
7	12.9099	14	28.4268	21	44.2721	28	58.1578
	2.1099		2.2974		2.1565		1.6917
8	15.0198	15	30.7242	22	46.4286	29	59.8495
							1.5989
						30	61.4484

Note 1. The above Table is formed according to the directions given in No. 6 of the foregoing Rule; and the Table Book may be made as directed in Prob. I. of the last Section.

2. The content of the foregoing cask, found by Prob. IV., is 61.12 wine gallons.

3. We have not given the method of *inching* the first variety of casks, as we consider that this variety is never met with in practice.

RULE.

For the Third Variety.

Divide $\frac{1}{3}$ of the difference of the areas of the head and bung diameters, by the square of half the length of the cask; and the quotient will be a *common multiplier* for a cask of the third variety, with which proceed precisely as directed in the last Rule.

EXAMPLE.

The head diameter of a cask of the third variety, is 24, the bung diameter 30, and the length 40 inches; it is required to find the content, in wine gallons, at every inch of the depth, when it is standing upon its head.

Ans. The content of one inch from the bung, is 3.0588; *of two inches,* 6.1102; *of three inches,* 9.1470; *of four inches,* 12.1617; *and the whole content, found by inching the cask, is* 102.5426 *wine gallons. (See the Key to this Work.)*

REMARK.

The foregoing method of *inching* casks is attended with considerable labour; notwithstanding this, Officers had much better *inch* all Spirit Merchants' casks that are fixed for general use, even by this tedious method, than have several ullages to cast every time they survey such traders. If diameters could be taken in the middle of every 8 or 10 inches, from the head to the bung, any cask might then easily be inched in the same manner as a guile-tun; and we certainly conceive it possible to obtain these diameters to a considerable degree of accuracy, by some of the methods laid down in the *ninth* Problem of this Section, and the subjoined Remarks.

Note. Large casks used by Distillers and Rectifiers, as store-casks, have man-holes in their upper heads; consequently, the Gauger may descend into the casks, and take cross diameters in the middle of every 6, 8, or 10 inches; and hence the casks may be tabulated with as much ease and accuracy as circular guile-tuns.

CASK GAUGING
BY THE CALLIPERS.

The Method of Gauging Casks by the Callipers, Bung Rod, and Head Rod, as practised in the Ports of Great Britain and Ireland, by the Port Gaugers of the Excise and Customs.

DESCRIPTION OF THE INSTRUMENTS.

The instruments used by the Port Gaugers of the Excise and Customs, are four in number; namely, the Long Callipers, the Cross Callipers, the Bung Rod, and the Head Rod.

The Long and Cross Callipers.

The Long Callipers are used in taking the length of a cask; and the Cross Callipers in taking the horizontal bung diameter. Each of these instruments consists of a stock, which is divided into inches and tenths; and two legs fixed at the ends of the stock, and forming with it two right angles.

The stock is divided into two parts which slide by each other; the legs are brought in contact with the heads, or with the bulge of the cask; and the length, or the horizontal bung diameter, is expressed by the inches and tenths upon the stock.

The Long Callipers are constructed in such a manner as to allow an inch for the thickness of each head; consequently, when the heads are exactly of that thickness, the Callipers will exhibit the internal length of the cask. If the heads exceed an inch in thickness, the excess must be deducted from the apparent length, as given by the Callipers; but if they be less than an inch in thickness, the deficiency must be added to the apparent length; and you will thus obtain the true internal length of the cask.

The Callipers are so simple, that further explanation is unnecessary. They may be completely understood by a few minutes inspection.

Note. The lower part of the legs of the Long Callipers are turned towards each other, in a direction parallel to the stock, in order that the chimbs may not prevent them from coming in contact with the heads of the cask. (See the next Fig.)

The Bung Rod.

This instrument is a slender, square rod, 48 inches in length; and divided into inches and tenths on two of its opposite sides. Another side contains the diagonal line for finding the contents of casks, in wine gallons, as directed in Problem VII.; and on the opposite side is a line of circular wine areas, for finding the contents of cylinders. This line exhibits the area of the base of any cylindrical vessel, in wine gallons, when the Rod is applied to the diameter; consequently, if this area be multiplied by the perpendicular depth, the product will be the content of the vessel, in wine gallons.

A brass sliding plate is fitted to the Bung Rod, large enough to cover the bung-hole of the cask; and on the under side of this plate are two projecting slips of brass, each one inch in length; and divided into four equal parts by notches, denoting $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of an inch.

This instrument is chiefly used to take the perpendicular bung diameters of casks, and also the wet inches; the horizontal bung diameter being taken by the Cross Callipers; hence the mean bung diameter is obtained by taking half the sum of the perpendicular and horizontal diameters.

Note 1. The Brannan's Rule contains a line of circular ale areas, by which the contents of cylinders may be found in ale gallons, in the same manner as described above, for wine gallons. (See Problem VII., of this Section.)

2. These lines of areas are part of the Tables of ale and wine areas, in Part VII., transferred to the Diagonal Rod, Brannan's Rule, and Bung Rod.

The Head Rod.

This instrument is a Sliding Rule, 45 inches in length; having only one slide in the middle of the stock.

The lower part of the stock, on one side of the Rule, is divided from right to left, into inches and tenths, by which the head diameters of casks are taken. On the same side of the Rule, at the right end of the stock, is fixed a crooked piece of brass, which projects perpendicularly from the face of the Rule, about two inches; and then turns to the right; and upon the middle of the Slide is fixed another projecting piece of brass, that serves as an index to point out the head diameters of casks upon the lower part of the stock.

The upper parts of the stock and Slide, on the same side of the Rule, contain two lines marked C and D, which are used to find the contents of vessels, in the same manner as those on the common Sliding Rule, described in Part II.; and the lower part of the Slide contains a line of inches and tenths; adjoining those on the lower part of the stock. On the same side of the Slide, towards the left, is a line marked *spheroid*, which is used to find the mean diameters of casks, in order to reduce them to cylinders of equal contents.

The other side of the Rule contains three lines marked A, B, and N, which are all similar to each other; and are used in the same manner as the lines A, B, and C, on the common Sliding Rule. The first of these lines is on the upper part of the stock; the other two are on the Slide; and below them, on the lower part of the stock, are the lines of *segments* for ullaging lying and standing casks, as directed in Problems X., and XI., of this Section.

(See the description of the Sliding Rule, Part II.)

To take the Dimensions of Casks by the Bung Rod, Head Rod, and Callipers.

PRELIMINARY OBSERVATIONS.

The Port Gaugers of the Excise and Customs do not finish the process of taking the dimensions of one cask, before they begin with those of another, as the Officers of the Excise do in gauging the casks of Victuallers. Wine and spirit casks are placed upon the Quays, in regular rows, with their bung-holes uppermost, as they are landed from the vessels; and at such a distance from each other as to allow room for the play of the Cross Callipers, and to admit the Officers to pass freely between them. All the horizontal bung diameters of the first row, are then taken, by the Cross Callipers; and put down upon the right bulges of the casks, with chalk. Cross diameters of the back heads are next taken, with the Head Rod; and their means entered upon the left bulges of the casks.

The Bung Rod is next used to determine the perpendicular bung diameters and wet inches. From the cross bung diameters the mean bung diameters are determined; and both them and the wet inches are put down upon the front heads of the respective casks; the wet inches being generally placed undermost.

Three lengths of each cask are then taken with the Long Callipers; namely, one over the bung-hole, and one on each side, at one-third of the circumference of the cask, from the top or middle of the bung-staff; and from these three lengths the mean lengths are obtained, which are also entered upon the front heads; leaving room for the mean head diameters to be placed below them.

Lastly, cross diameters of the front heads are taken; their means determined; and from the mean diameters of the two heads, the true mean head diameter of each cask is obtained. These mean diameters are put down immediately below the lengths; thus we have on each front head, the length, the head diameter, the bung diameter, and the wet inches of each cask.

The Officers then copy all the dimensions from the heads of the casks into their books; and to prevent mistakes, they compare the entries in their books with those upon the casks; and this completes the process of taking the dimensions.

Note 1. The greatest accuracy is observed in taking the dimensions of wine and spirit casks; and in order to prevent mistakes, and promote truth, the dimensions of the first Officer are *checked* by those of other Officers that follow him, and take the same dimensions, by similar instruments.

2. Sometimes the staff opposite the bung-hole is made much thicker than any of the other staves; at other times a piece of wood is *nicely* fixed upon it, in order to lessen the bung diameter; such frauds, however, may easily be discovered by drawing the Bung Rod two or three times over the bulge of the cask, in different directions; or by comparing the perpendicular with the horizontal bung diameter, after the thickness of the staves has been deducted. Or, if you bung the cask, and turn it one-fourth over, you may take the external bung diameter, by the Cross Callipers; and hence the internal bung diameter may be obtained by deducting twice the thickness of the staff at the bung-hole. (See the next article.)

3. The foregoing order of taking the dimensions of casks is not observed in all ports. Some Officers take the length first; then the head diameter; and lastly, the bung diameter and wet inches. (See the third Example.)

To take the Bung Diameter.

Lay the Cross Callipers over the bung-hole of the cask with their legs downwards, and contract them until they touch the bulge of the cask on each side; and you will have the external horizontal bung diameter on the stock of the Callipers, from which deduct twice the thickness of the staff at the bung; and the remainder will be the internal bung diameter. Next take the perpendicular bung diameter, with the Bung Rod; and the mean between these two diameters, will be the true diameter of the cask. At the same time, by observing the upper ex-

tremity of the wet part of the Rod, you will obtain the *wet* inches, when the cask is not full, which is very often the case. The difference between the bung diameter and the wet inches will show the dry inches; hence either the *wet* or the *dry* ullage may be obtained. The dry ullage taken from the whole content of the cask, will leave the wet ullage.

Note 1. The thickness of the staves or heads of any cask may be accurately obtained by boring them through with an instrument adapted for that purpose, called a Wood-Gauge.

2. Suppose the horizontal bung diameter of a cask, as shown by the Callipers, to be 34.8 inches, and the thickness of both the staves 1.6 inches; then $34.8 - 1.6 = 33.2$ inches, the internal bung diameter; and if the perpendicular diameter, by the Bung Rod, be 32.6 inches, we have $\frac{33.2 + 32.6}{2} = \frac{65.8}{2} = 32.9$ inches, the true bung diameter of the cask.

3. Here it must be observed, that when a cask is nearly full, and the perpendicular bung diameter is less than the true diameter, the difference must be added to the wet inches. Let the wet inches be 30.7; then as the perpendicular bung diameter, in the above Example is .3 less than the true diameter; we have $30.7 + .3 = 31.0$ for the true wet inches. If the perpendicular bung diameter had exceeded the true diameter by .3; then $30.7 - .3 = 30.4$, would have been the true wet inches; for, according to general custom, the wet inches should always be increased or decreased in the same proportion as the perpendicular bung diameter is increased or decreased in finding the true diameter. However, in Practice, the same number is generally used for both purposes.

4. When casks are regularly made, and want several inches of being full; the foregoing additions and deductions are not always attended to in taking the wet inches.

To take the Head Diameter.

Place the lower edge of the crooked brass that is on the end of the Head Rod, within the chimb of the cask, close to the head; then move the slide until the brass index intersects the middle of the opposite chimb; and you will have the internal head diameter of the cask, on the lower line of the stock, directly opposite to the index on the slide.

Note. The distance between the inside of the chimb and the middle of the opposite staff or chimb, will generally be equal to the internal head diameter. However, when the staves are remarkably strong, it will only be necessary to include one-third of the thickness of the chimb in taking the head diameter. (See a Note in Problem I., of this Section.)

To take the Length.

Lay the Long Callipers over the bung-hole, with their legs downwards, and contract them until their ends come in contact with the heads of the cask; and you will have the length of the cask, on the stock, which allows an inch for the thickness of each head.

Again, measure other two lengths, one on each side of the bung, so that three lengths thus taken may be at equal distances from each other; divide the sum of these lengths by their number; and the quotient will be the mean length of the cask.

If the two heads exceed two inches in thickness, a deduction must be made; but if they be less than two inches, an addition must be made for the difference, in taking the length by the Callipers; and thus you will obtain the true internal length of the cask.

Note 1. Suppose the length of a cask, as taken by the Callipers, to be 48.6 inches; and let the thickness of the two heads be 2.4 inches; then $48.6 - .4 = 48.2$ inches, the true internal length of the cask; but if the thickness of the heads be only 1.8 inches; then we have $48.6 + .2 = 48.8$ inches, for the true internal length.

2. Before the length of a cask is put down upon the front head, as directed in the preliminary observations, a proper deduction should be made to reduce the cask to the first variety. Suppose the deduction to be six-tenths of an inch; then in the first part of the above Example, instead of four-tenths we must deduct one inch; hence the true length will be 47.6 inches. In the latter part of the Example, instead of adding two-tenths we must deduct the difference between two-tenths and six-tenths, which is four-tenths; hence the true length will be 48.2 inches. (See the next article for deductions.)

Allowances to be made in the lengths of casks, in order to reduce them to the first variety.

It has already been observed that very few, if any casks will contain so much as the middle frustum of a spheroid; and as the lines on the Head Rod, for casting the contents of casks, are calculated for that variety only, it becomes necessary to make such deductions in the lengths of casks as will, in the judgment of the Officers, reduce them to the first variety, before their contents are found. Now, as casks imported from different countries vary very much in their form and make, it is necessary that those deductions should vary accordingly; hence such Tables of allowances

ave been made by experienced Officers, and sanctioned y the Commissioners of the Customs, as will, in their pinion, effect the purpose for which they are intended. he following are the allowances mostly in use, in the fferent Ports of the United Kingdom; these however, are metimes varied at the discretion of the Port Gaugers, on whose skill and experience, much of the accuracy of sk Gauging will always depend.

TABLE OF DEDUCTIONS.

Names of Casks.	*Inches.*
Port pipes, about 6, 7, or	0.8
Lisbon pipes	1.3
Madeira pipes	1.3
Vidonia pipes	0.8
Mountain butts	0.4
Nice butts	2.3
Sherry butts	0.7
Rhenish half vats, or four aulms	0.7
Brandy puncheons about 4, 5, or	0.6
Rum puncheons 3, 4, or	0.5
Geneva puncheons, 5, 6, or	0.7
Gallipoli or Olive oil casks from 0 to	1.0

1. In several Tables of allowances, 1.5 inches are put down t pipes; but experience proves this to be too much.
eductions are seldom made for Port, Lisbon, Madeira, Vidonia, herry, and other hogsheads; as their heads are not often more ht or nine-tenths of an inch in thickness; consequently no ns are necessary, because the Callipers allow an inch for each

Vhen casks are regularly formed, there are seldom any deduc- de from the true wet inches of wine and oil casks. Three or hs are generally deducted from the true wet inches of spirit casks; deductions are entirely at the discretion of the Gauger. (See s and Notes for taking the Bung Diameter.)

T THE CONTENTS AND ULLAGES OF CASKS BY THE HEAD ROD.

e a cask to a cylinder, by finding a mean diameter.

RULE.

e left end of the brass index which is on the middle de, to the head diameter on the lower line of the

stock; and opposite the bung diameter on the same line, note the number on the line marked *spheroid*. Then this number being added to the head, will give the mean diameter; or, opposite to this number on the lower line of the slide, we have the mean diameter on the lower line of the stock; and thus the cask is reduced to a cylinder.

To find the content from the length and mean diameter.

RULE.

Set the brass mounting, or gauge-point that is at the left end of the slide D, to the length of the cask on the upper line of the stock C; then against the mean diameter on the line D, is the content of the cask, in wine gallons, on the line C.

Note. The contents of all casks that hold less than 60 gallons, are taken by the Diagonal Rod; and such deductions are made as the forms of the casks appear to warrant. The mean contents must always be found as directed in Problem VII.; and at the same time the bung diameters and wet inches must be taken, in order to cast the ullages.

To find the ullage, from the content, bung diameter, and wet inches.

RULE.

Set the bung diameter on N, to 100 on the line of segments S. L.; then against the wet inches on N, you will have the fourth number on S. L.

Again, set the fourth number on B, to 100 on A; then against the content of the cask on A, is the content of the ullage on B. (See the Rules in Prob. X.)

REMARK.

The Head Rod being 45 inches in length, is generally thought inconvenient for casting the contents and ullages of casks. To remedy this inconvenience, a similar Rod or Sliding Rule, 24 inches in length, is generally used for these purposes. This Rule contains all the lines that are on the Head Rod; but in an abridged form. The line of

inches on the lower part of the stock commences at 20, and proceeds to 44; that on the slide, adjoining the line *spheroid*, is only 12 inches in length; the line C, on the upper part of the stock, commences at 25; and the first number on the adjoining line D, on the upper part of the slide, is 18. The lines on the other side of the Rule are all perfect. This instrument is generally called a Jerkin.

EXAMPLES.

EXAM. 1.

Let A B C D represent a Port pipe, on the Quay, with the Long Callipers, in the act of taking the length of the cask, over the bung-hole; and admit the dimensions to be taken in the order laid down in the Preliminary Observations, and to be as follow; namely, the horizontal bung diameter, including the thickness of the staves at the bulge, 34.0 inches; the back head diameters 23.2 and 23.4; the perpendicular bung diameter 32.6; the wet inches 28.6; three lengths 49.2, 48.8, and 48.7; the front head diameters 23.6 and 23.4; the thickness of each head 0.8; the thickness of the staff at the bung hole 0.9; and the deduction to be made in the length, in order to reduce the cask to the first variety, 0.6 inches; required the true dimensions of the cask, and also its content and ullage, in wine gallons.

To find the true dimensions.

	Inches.
External horizontal bung diameter ...	34.0
Twice the thickness of the staff	1.8
Internal horizontal bung diameter	32.2
Internal perpendicular bung ditto	32.6
Divide by	2)64.8
True bung diameter	32.4
Wet inches	28.6
Bung diameters 32.6 – 32.4 *deduction*	0.2
Corrected wet inches	28.4
Back head diameter	23.2
Ditto	23.4
Divide by	2)46.6
Mean back head diameter	23.3
Front head diameter	23.6
Ditto	23.4
Divide by	2)47.0
Mean front head diameter	23.5
Mean back head ditto	23.3
Divide by	2)46.8
True head diameter	23.4
Length	49.2
Ditto	48.8
Ditto	48.7
Divide by	3)146.7
Mean length	48.9
Addition for the heads 2.0 – 1.6......	0.4
Sum	49.3
Deduction for the variety	0.6
True length	48.7

Note. In Practice, the true dimensions are found mentally, which considerably shortens the operation. (See the Method of taking the Dimensions.)

True Dimensions.

	Inches.
Length...............	48.7
Head diameter ...	23.4
Bung diameter ...	32.4
Wet inches	28.4

To find the Mean Diameter.

Set the brass index on the slide, to the head diameter 23.4 *on the lower line of the stock; then against the bung diameter* 32.4 *on the same line of the stock, we have* 6.3 *nearly, on the line marked spheroid; and opposite this number on the lower line of the slide, we have* 29.7, *the mean diameter, on the lower line of the stock.*

To find the content.

Set the gauge-point on D, to the length 48.7 *on C; then against the mean diameter* 29.7 *on D, is* 146 *very nearly, on C, the content in wine gallons.*

To find the ullage.

Set the bung diameter 32.4 *on N, to* 100 *on S. L.; then against the wet inches* 28.4 *on N, we have* 94, *the fourth number, on S. L.*

Again, set 94 *on B, to* 100 *on A; then against* 146, *the content of the cask on A, is* 137, *the content of the ullage on B. (See Prob. X.)*

Note. Here it may not be improper to observe that the Head Rod will only cast the content and ullages of casks in wine gallons; ale gallons being seldom or never wanted in Port Gauging.

EXAM. 2.

The following are the dimensions of a Jamaica rum butt, as taken by the Callipers, Bung Rod, and Head Rod; namely, the external horizontal bung diameter 36.8 inches; the back head diameters 28.5 and 28.1; the perpendicular bung diameter 34.2; the wet inches 30.5; three lengths 36.4, 36.3, and 36.5; the front head diameters

28.6 and 28.4; the thickness of each head 1.2; the thickness of the staff at the bung-hole 1.0; and the deduction in the length, to reduce the cask to the first variety, 0.8 inches; required the true dimensions of the cask, and also its content and ullage, in wine gallons.

Ans. The true length is 35.2, *the head diameter* 28.4, *the bung diameter* 34.5, *the wet inches* 30.8, *the content* 127, *and the ullage* 121 *wine gallons.*

EXAM. 3.

The following Table contains the true dimensions of six wine pipes, from Madeira; being part of the cargo of the Mary Ann Jones, of Newcastle-upon-Tyne. The dimensions, contents, and ullages, are given as found by the Port Gaugers of the Excise and Customs; and, by way of Practice, the Learner is required to cast the content and ullage of each cask by the Head Rod.

TABLE OF DIMENSIONS, CONTENTS, AND ULLAGES.

No.	Length.	Head Dia.	Bung Dia.	Wet Inches.	Content.	Ullage.
1	47.5	22.4	27.3	24.7	107	103
2	46.8	21.3	27.9	25.9	106	103
3	48.4	21.7	27.8	26.0	110	108
4	47.2	22.3	27.8	25.7	109	106
5	47.1	22.3	27.5	25.9	107	105
6	48.3	22.3	27.5	25.7	110	108

Note. The mean diameter of No. one, is found to be 25.8; of No. two, 25.9; of No. three, 25.9; of No. four, 26.1; of No. five, 25.9; and of No. six, 25.9 inches; hence the Contents and Ullages are found as in the two last columns of the above Table. (See the Key to this Work.)

The Method of finding the Wet Ullages of Casks by subtracting the Dry Ullages from the whole Content.

It has already been observed that the wet ullages of casks may be obtained by subtracting the dry ullages from the whole content. Now, it may be seen, by inspecting the preceding Table, that the dimensions and contents of the same species of casks seldom differ much from each other. Consequently, if we fix upon a regular made cask, suppose a Port pipe; and find the contents of a number of dry ullages, beginning at 1 dry inch, and proceeding to 3 or 4 inches; we may form such a Table from the results, as will express the contents of the dry ullages of Port pipes in general. The process of ullaging may then be performed with ease and expedition, by subtracting the ullages corresponding to the dry inches, from the whole contents of the casks.

EXAMPLE.

Let the length of a regular made Port pipe be 50, the bung diameter 32, and the head diameter 23 inches; it is required to find the content of all ullages from 1 to 4 dry inches.

By the Head Rod, the mean diameter of the cask is found to be 29.3 inches, and its content 146 *gallons; then by continually varying the dry inches, we find the fourth number corresponding to each segment; and hence the content of the ullages, as in the following*

TABLE.

Dry Inches.	Ullages.	Dry Inches.	Ullages.	Dry Inches.	Ullages.
1.0	0.7	2.1	2.8	3.1	5.8
1.1	0.8	2.2	3.1	3.2	6.1
1.2	0.9	2.3	3.4	3.3	6.4
1.3	1.0	2.4	3.7	3.4	6.7
1.4	1.2	2.5	4.0	3.5	7.0
1.5	1.4	2.6	4.3	3.6	7.3
1.6	1.6	2.7	4.6	3.7	7.6
1.7	1.8	2.8	4.9	3.8	7.9
1.8	2.0	2.9	5.2	3.9	8.3
1.9	2.2	3.0	5.5	4.0	8.7
2.0	2.5	....	....	....	....

In Port Gauging the contents of casks are always found in whole numbers; therefore, from the preceding Table we form the following one, which is more convenient for Practical purposes.

A TABLE OF DRY ULLAGES FOR PORT PIPES.

Dry Inches.	Ullages.	Dry Inches.	Ullages.
From 1.0 to 1.4	1	From 3.0 to 3.3	6
From 1.5 to 1.9	2	From 3.4 to 3.6	7
From 2.0 to 2.3	3	From 3.7 to 3.9	8
From 2.4 to 2.6	4	From 4.0 to 4.2	9
From 2.7 to 2.9	5	....	... .

EXAMPLE.

The bung diameter of a Port pipe is 32.4, the wet inches 28.4, and the content 146 gallons; required the content of the ullage by the preceding Table.

Here 32.4 — 28.4 = 4.0, *the dry inches; opposite to this number in the last Table, we find* 9 *gallons; then,* 146 — 9 = 137 *gallons, the content of the ullage, the same as found by the Head Rod. (See the first Example.)*

Note 1. The last Table will answer very well for Brandy and Gin puncheons, and Madeira pipes. It gives the true ullages of all the casks contained in the third Example.

2. Similar Tables may be easily calculated for all other kinds of casks; taking care to make choice of regular made casks from which to make the estimations.

3. If the Tables be committed to memory, the process of ullaging may be performed mentally; and thus the business of Gauging will be greatly facilitated.

DIRECTIONS

For Gauging Casks of an irregular Form.

Irregularities in the forms of casks, should be particularly attended to, in taking the dimensions; and such additions or deductions must be made, as will, in the opinion of the Gauger, reduce such casks to regular shapes, in order that their contents may be obtained with as much

accuracy as possible. If the middle diameter between the bung and head, be nearly equal to the bung diameter, the cask is said to be *high-quartered*. When this is the case, two or three tenths, &c. must be added to the true length, in order to increase the content. On the contrary, if a cask appears to be not sufficiently curved in the quarters; then two or three tenths, &c. at the discretion of the Officer, must be deducted from the true length, in order to decrease the content; as it is evident, in the first case, the cask will contain more, and in the second less, than if it were regularly formed.

If a cask be full-quartered, and flattened at the bung, so that the perpendicular is considerably less than the horizontal bung diameter; add a few tenths to the perpendicular bung diameter, in order to reduce the cask to a regular form. The general method is to add *two-thirds* of the difference; thus, if the horizontal diameter exceeds the perpendicular diameter, suppose .9; then .6 must be added to the perpendicular diameter; and also to the *wet* inches; and the sums thus obtained must be considered as the true bung diameter and *wet* inches of the cask.

If the perpendicular exceed the horizontal bung diameter; then a few tenths must be deducted from the perpendicular bung diameter. The general custom is to deduct *one-half* of the difference; thus, if the perpendicular diameter exceeds the horizontal diameter, suppose .8; then .4 must be deducted from the perpendicular diameter and *wet* inches, in order to obtain the true bung diameter and *wet* inches of the cask.

Every other irregularity that may be met with in casks, must be taken into consideration; and a proper allowance made in some of the dimensions. No general Rules can, however, be given that will apply in every case; consequently, much will always depend upon the skill and experience of the Officers.

Note. Every *unfairly made* cask should be rolled over, before it is gauged, in order that the Officer may obtain a proper idea of its shape and curvature.

SECTION IV.

THE METHOD OF GAUGING AND FIXING MALTSTERS' UTENSILS, AS PRACTISED IN THE EXCISE.

MALT GAUGING.

PRELIMINARY OBSERVATIONS.

NOTWITHSTANDING the methods of finding the areas and contents of all kinds of figures, in malt bushels, have already been given in Parts IV. and V. of this Work; yet it is deemed necessary to shew the methods of gauging and fixing the Utensils of Maltsters, as practised in the Excise.

By Act of Parliament, Officers are to take account of all grain making into malt, by gauge only; and estimate the quantity by the Winchester bushel of 2150.42 cubic inches.

It is also enacted that barley must be under water, in the cistern, 40 hours. In this time it is found by experiment to swell or increase to about *one-fourth* more than its original bulk; consequently, *four* bushels in *twenty* are allowed for this increase.

From the cistern the barley is removed to the couch; and after having lain there 26 hours, it is deemed a floor. The same allowance is made in the gauge of the couch as in that of the cistern; but when the corn has been thrown out of the couch into the floor, and permitted to grow according to the regular custom, in making malt, it is supposed to increase about *one-half;* hence an allowance is made of *ten* bushels in *twenty*.

It may also be observed that barley is first gauged in the cistern, then in the couch, next on the floor, and lastly on the kiln.

PROBLEM I.

To gauge and fix a Maltster's cistern, in the form of a parallelopipedon.

To take the dimensions.

Measure three lengths near the top, and three near the bottom, at equal horizontal distances from each other; and divide their sum by their number, for the mean length. Find the mean breadth in the same manner; also, measure the perpendicular depth; and you will have all the necessary dimensions.

Note. In the real Practice of Gauging, the mean depth of the grain in a cistern is obtained by taking several depths, and dividing their sum by their number for a mean depth. The most convenient number of depths is ten; as the mean depth is then obtained by removing the decimal dot one place towards the left. Five depths, however, are generally thought sufficient.

To find the area and content.

BY THE PEN.

RULE.

Multiply the mean length by the mean breadth; divide the product by 2150.42, and the quotient will be the area in malt bushels. Multiply this area by the depth of the cistern, and the product will be the content. (See Tables IX. and X. of Dry Measure, and the following Remarks, Part IV.)

Note. It is scarcely necessary to observe that the content of any quantity of grain in a cistern or couch, may be obtained by multiplying the area by the mean depth.

BY THE SLIDING RULE.

To find the area.

Set 2150.42 on A, to the length on B; then against the breadth on A, is the area on B.

To find the content.

Set unity on A, to the area on B; then against the depth on A, is the content on B.

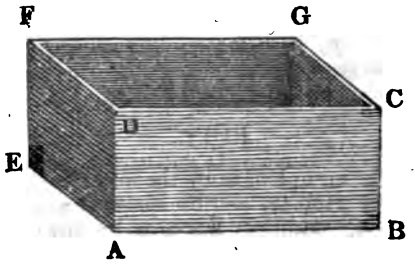
Or,

The content may be found by means of the line M. D. on the Sliding Rule, without knowing the area, thus; set the length on B, to the depth on the line M. D.; then against the breadth on A, is the content on B.

EXAMPLES.

1. The length A B, of a cistern in the form of a parallelopipedon, is 96.8, the breadth A E 63.6, and the depth B C 45.3 inches; what is the area and content in malt bushels?

Note. Here A B and A E are considered to be the mean length and breadth of the cistern, found as before directed.



BY THE PEN.

To find the area.

Inches.

```
      96.8 length.
      63.6 breadth.
     -----
     5808
    2904
   5808
```

Divisor 2150.42)6156.48(2.862 *bushels, the area.*

BY THE SLIDING RULE.

	M. B.		*Length.*		*Breadth.*		*Area.*
As	2150.42 *on* A	:	96.8 *on* B	::	63.6 *on* A	:	2.86 *on* B.

BY THE PEN.

To find the content.

Bushels.
2.86 *area.*
45.3 *depth.*
―――
858
1430
1144
―――
129.558 *content.*

BY THE SLIDING RULE.

	Unity.		*Area.*		*Depth.*		*Content.*
As	1 *on* A	:	2.86 *on* B	::	45.3 *on* A	:	129.6 *on* B.

Or,

	Length.		*Depth.*		*Breadth.*		*Content.*
As	96.8 *on* B	:	45.3 *on* M.D.	::	63.6 *on* A	:	129.6 *on* B.

To inch or tenth the foregoing cistern.

Having found the area of the cistern, in malt bushels, we may hence easily form a Table by continual addition, which will exhibit the content at every inch of the depth; thus $2.86 + 2.86 = 5.72$, the content at 2 inches; $5.72 + 2.86 = 8.58$, the content at 3 inches, &c. &c.

A Table, however, still more useful in the Practice of Gauging, may be formed by *tenthing* the cistern; and as cisterns are generally more than half full of grain, when gauges are taken, it will only be necessary to commence the *tenthing* at half the perpendicular depth, and continue it *nearly* to the top. Thus $2.86 \times 20 = 57.2$, the content at 20 inches of the depth; and $2.86 \div 10 = .286$, the content at one-tenth of an inch; hence, $57.2 + .286 = 57.486$, the content at 20.1 inches of the depth; $57.486 + .286 = 57.772$, the content at 20.2 inches; $57.772 +$

.286 = 58.058, the content at 20.3 inches, &c. &c. until we have continued the process as far as we think necessary.—See the Key to this Work, where the above operation is continued to 25 inches; and hence we form the following Table Book.

A. B.'s CISTERN, No. 1, TABULATED.

Depth in Inches.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
20	57.2	57.5	57.8	58.1	58.3	58.6	58.9	59.2	59.5	59.8
21	60.1	60.3	60.6	60.9	61.2	51.5	61.8	62.1	62.3	62.6
22	62.9	63.2	63.5	63.8	64.1	64.4	64.6	64.9	65.2	65.5
23	65.8	66.1	66.4	66.6	66.9	67.2	67.5	67.8	68.1	68.4
24	68.6	68.9	69.2	69.5	69.8	70.1	70.4	70.6	70.9	71.2
25	71.5	.....	......	......	......	......	......	......	......	......

In this manner the Table may be continued to the top of the cistern; and hence the content at any depth contained in the Table, may be found by inspection.

EXAM. 2.

The mean length of a Maltster's cistern is 128.6, the mean breadth 85.4, and the depth 52.8 inches; what is the area and content in malt bushels?

Ans. The area is 5.107, *and the content* 269.6496 *malt bushels.*

EXAM. 3.

How many bushels are contained in the cistern, the dimensions of which are given in the last Example, when the mean depth of the grain is 43.7 inches?

Ans. 223.1759 *bushels.*

Remark.

By Chap. 128, Sec. I., of a Statute made in the 52d year of the reign of George III., it is enacted that Maltsters shall construct their cisterns in such a manner that the Officers of the Excise shall be able conveniently to gauge, in any part of two sides, the corn or grain contained in such cisterns; and if the length or breadth of any cistern shall exceed 9 feet, then the Maltster shall provide a ladder and plank of sufficient length and strength, so that by laying the plank across the cistern, the Officer may be enabled to take gauges in any part thereof; and if any maker of malt shall neglect or refuse to comply with the provisions of this Act, he shall forfeit 200£.

PROBLEM II.

To gauge a couch of Malt contained in a rectangular frame.

Directions for taking the dimensions, &c.

As a couch of malt in a rectangular frame, forms precisely the same figure of which we have treated in the last Problem, it is manifest that the dimensions, area, and content must be obtained in the same manner. It must be observed, however, that only the length, breadth, and area of a couch-frame appear in the Officer's Malt Book; but in gauging and fixing a cistern, both the length, breadth, area, and depth are always entered.

Note 1. Notwithstanding, a couch-frame may be said to be fixed, when the length, breadth, and area are entered at the top of the Malt Book; yet as the Traders sometimes make alterations, the dimensions of couch-frames ought frequently to be examined.

2. The readiest method of obtaining the mean depth of grain in a cistern, couch, or floor, is to choose some integral number, as a *supposed* mean depth, after having ascertained *nearly* what the mean depth will be, from your first or second depth; then, add together the number of *tenths* which each depth exceeds the *chosen number*; divide the sum by the number of depths; and the quotient being added to the *chosen number*, will give the true depth of the grain. Thus, let the *chosen number* or supposed mean depth of a couch of malt be 24 inches; and let five depths, taken in different places, be 24.3, 24.4, 24.7, 24.9, and

24.2; then we have $(3 + 4 + 7 + 9 + 2) \div 5 = 25 \div 5 = .5$; hence $24 + .5 = 24.5$ inches, the mean depth of the grain. The whole of this process may easily be performed mentally, as you take the respective depths; thus, 3 and 4 are 7; 7 and 7 are 14; 14 and 9 are 23; 23 and 2 are 25; then 5 in 25, five times, and nothing over; hence 24.5 inches is the depth required.

3. If any of the depths should be less than the *chosen number*, then the deficiency must be deducted from the sum of the tenths already obtained; thus, if the third depth had been 23.8, then 2 tenths must have been taken from 7 tenths; and the difference would have been 5 tenths, which must have been added to the 9 tenths in the fourth depth, &c. &c.

4. If the difference between the *chosen number* and any of the depths be an inch or upwards, it must be reduced into tenths; thus, suppose the difference to be 1.2; then this must be called 12 tenths, &c.

EXAMPLES.

1. The mean length of a couch-frame is 126, the mean breadth 112, and the mean depth of the grain 20.2 inches; required the content of the couch, in malt bushels.

BY THE PEN.

To find the area and content.

```
                    Inches.
                      126  length.
                      112  breadth.
                     ----
                      252
                     126
                    126
                   -----
Divisor 2150.42)14112(6.562  area.

                    Bushels.
                      6.56  area.
                      20.2  depth.
                     -----
                     1312
                    1312
                   -------
                   132.512  content.
```

BY THE SLIDING RULE.

	M. B.		*Length.*		*Breadth,*		*Area.*
As	2150.42 *on* A	:	126 *on* B	::	112 *on* A	:	6.56 *on* B.

And,

Unity.	*Area.*	*Depth.*	*Content.*
As 1 *on* A :	6.56 *on* B ::	20.2 *on* A :	132.5 *on* B.

Or,

Length.	*Depth.*	*Breadth.*	*Content.*
As 126 *on* B :	20.2 *on* M. D. ::	112 on A :	132.5 on B.

Note. The length and breadth of cisterns are always taken in inches and tenths; but the length and breadth of couch-frames, floors, and kilns, are only taken to the nearest whole inch. The depth of the grain, however, in all cases, is taken in inches and tenths.

2. The mean length of a couch-frame is 148, the mean breadth 125, and the mean depth of the grain 26.8 inches; required the area and content, in malt bushels.

Ans. The area is 8.602, *and the content* 230.5336 *malt bushels.*

3. How many bushels are contained in the couch-frame, the dimensions of which are given in the last Example, when the mean depth of the grain is 18.7 inches?

Ans. 160.8574 *bushels.*

PROBLEM III.

To gauge a couch of Malt, not in a frame, but laid upon the floor, in a square or rectangular form, with one or more of its sides slanting.

To take the dimensions, &c.

Measure several lengths, from which find the mean length, as before directed. Find the mean breadth, and the mean depth in the same manner; then from these dimensions find the area and content as in the foregoing Problems.

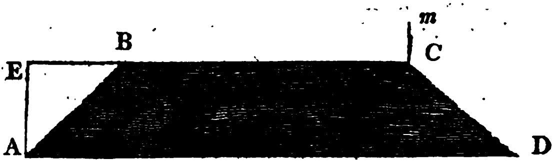
Note 1. If both ends and sides of a couch of malt be slanted, the lengths and breadths must be taken in the following manner: Fix the tape at the innermost sloping edge of one end, by putting your dipping piece through the ring; then extend the tape, in a horizontal direction, to the outermost sloping edge of the opposite end of the couch, which may be found by dropping a few grains of corn; and you will thus have one length of the couch; for what is lost in the length at one end, will be obtained by extending the tape to the sloping extremity of the

other end. In the same manner, find several other lengths; taking care to fix the end of the tape alternately at the opposite ends of the couch, by which means, the necessary corrections will be made, when one end slants more than the other; and from these lengths determine the mean length as before directed.

2. If one end or one side only be sloped, and the other laid against an upright wall or partition, fix the end of the tape close to the wall by the assistance of the dipping piece, as before directed; and extend the tape, in a horizontal direction, over the surface of the grain, to the extremity of the opposite sloping edge; then return the tape to the inner extremity of the sloping edge; and the doubling of the tape will shew the length or breadth required.

EXAMPLES.

The mean length of a couch of malt with sloping ends and sides, is found to be 96, the mean breadth 84, and the mean depth 22.8 inches; required the area and content in malt bushels.



Note. The above perpendicular section shows the two sloping ends A B and C D of the couch. The line C *m* denotes the rod or dipping piece placed at the inner extremity of the slanting end C D; the line C E represents the tape extended from C to the outer extremity of the slanting end A B; consequently, C E denotes one length of the couch, obtained in the manner directed in the first Note.

BY THE PEN.

To find the area and content.

Here $96 \times 84 \div 2150.42 = 8064 \div 2150.42 = 3.749$, *the area of the couch; and* $3.749 \times 22.8 = 85.4772$, *the content in malt bushels.*

BY THE SLIDING RULE.

Length.	*Depth.*	*Breadth.*	*Content.*
As 96 *on* B :	22.8 *on* M. D. ::	84 *on* A :	85.47 *on* B.

2. The mean length of a couch of malt with one end and one side slanting, (the other being placed against two upright walls,) is found to be 138, the mean breadth 113, and the mean depth 28.6 inches; required the area and content in malt bushels.

Ans. The area is 7.251, and the content 207.3786 malt bushels.

Note 1. The above perpendicular section exhibits one slanting and one perpendicular end of the couch. The line C F represents the tape extended from C to the outer extremity of the slanting end A B; B E denotes the tape returned or doubled back from F to B; hence the line C E represents one length of the couch, found as directed in the second Note.

2. The foregoing method of gauging malt, may be well applied in taking malt-stocks, when an increase or decrease of duty takes place.

PROBLEM IV.

To gauge a couch of Malt either in the form of a cone, or a conical frustum.

To find the content of a cone of Malt.

RULE.

BY THE PEN.

Multiply the square of the mean diameter of the base by the perpendicular height; divide ⅓ of the product by 2738; and the quotient will be the content in malt bushels.

BY THE SLIDING RULE.

As 52.32 on D, is to one-third of the perpendicular altitude on C; so is the mean diameter on D, to the content on C.

To find the content of a conical frustum of Malt.

RULE.

BY THE PEN.

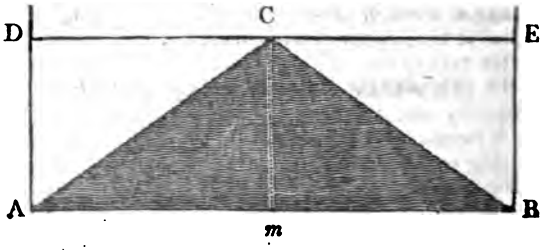
The content of the frustum of a cone may be found by Problem VIII., Part V.; but as this process is thought too tedious in the Practice of Malt Gauging, the general method is to reduce the frustum to a cylinder, by adding the two diameters together, and taking half their sum for the mean diameter; then, if the square of this mean diameter be multiplied by the perpendicular height of the frustum, and the product divided by 2738, the quotient will be the content in malt bushels. (See Prob. IV., Part V.)

BY THE SLIDING RULE.

As 52.32 on D, is to the altitude on C; so is the mean diameter on D, to the content on C.

EXAMPLES.

1. The mean diameter of the base of a couch of malt in the form of a cone, is 138, and its perpendicular altitude 47.3 inches; required the content in malt bushels.



Note. The above figure A B C represents a perpendicular section of the cone; and A D and B E two perpendicular staves used in taking the dimensions. The line D E denotes the tape stretched horizontally from one staff to the other; and hence we have the diameter A B. Another diameter at right angles to A B, being obtained in the same way; we thence find the mean diameter of the cone's base. The lines A D

and B E are equal to each other, and also equal to the perpendicular height C m; and in this manner, the dimensions of any conical heap of grain may be obtained.

BY THE PEN.

To find the content.

Here $(138 \times 138 \times 47.3) \div 3 = (19044 \times 47.3) \div 3 = 900781.2 \div 3 = 300260.4$; *and* $300260.4 \div 2738 = 109.664$, *the content in malt bushels.*

BY THE SLIDING RULE.

	On D.		On C.		On D.		On C.
As	52.32	:	15.8	::	138	:	109.66 *content.*

2. The mean diameter of the base of a conical frustum f malt, is 144, the mean diameter of the top 64, and the erpendicular height 28.2 inches; required the content in ıalt bushels, when the figure is reduced to a cylinder.

Ans. 111.399 *bushels.*

REMARK.

In order to have a check upon the method of gauging alt when in the form of a cone or a conical frustum, we st gauged a couch of malt in a rectangular frame; we en had it thrown out, and formed it into a conical shape; d after taking the dimensions and casting the content, th the greatest accuracy, we found that the gauge was *little* by about 6 bushels. This being the case, we nd it necessary to increase the dimensions a little, by asuring to the very extremities of the base and top; l the dimensions thus taken are given in the first ample.

Ve then formed the heap of grain into a conical frustum levelling its top. This operation of course increased the e diameter a little; but notwithstanding this, the con- : obtained from the dimensions of the frustum reduced ı cylinder, was some bushels too little; hence we ıd *it necessary* again to increase the dimensions. The eased dimensions are given in the second Example;

and the content obtained from these dimensions is precisely the same as that found when the grain was gauged in a couch-frame.

The reason why the contents found from the *true* dimensions of the cone and the conical frustum, were less than the content found from the dimensions taken in the couch frame, appeared evident; for, notwithstanding we took considerable pains in shaping the cone and frustum, yet their sides always formed curved lines, instead of straight lines, in consequence of the grains of corn adhering to each other; hence it is manifest that the contents of cones and conical frustums of malt will always be too little, when found from the true dimensions; because those dimensions will only give the true contents, when the sides of the figures are straight from the top to the bottom. (See the Definitions of the Cylinder and Cone, Part V.)

Note. From the foregoing observations, it appears evident that couch-frames ought always to be used, in order to obtain the true content of grain making into malt; for all gauges taken of grain not inclosed in frames, are more or less liable to error.

PROBLEM V.

To gauge a rectangular floor of Malt.

Directions for taking the dimensions, &c.

Find the mean length, breadth, and depth, as directed in Problem III.; and from these dimensions, find the area and content, as directed in the first Problem of this Section. Or, multiply the length, breadth, and depth continually together; divide the last product by 2150.42, and the quotient will be the content in malt bushels.

BY THE SLIDING RULE.

As the length on B, is to the depth on M D; so is the breadth on A, to the content on B. Or, as the breadth on B, is to the depth on M D; so is the length on A, to the content on B.

Note 1. In Practice, the contents of small floors of malt may always be determined by the *Sliding Rule;* but when a floor exceeds 200 bushels, then the *Pen* ought invariably to be used; because the tenths from 2 to 5, on the lines A and B, are only divided into two parts each; but those from 1 to 2, are each divided into five parts; hence the content of a floor may be pretty correctly estimated by the Sliding Rule, when it is under 200 bushels. It may also be observed that *floor-charges* must always be cast by the Pen, whatever be their contents.

2. The breadths of malt-floors, are generally fixed dimensions; and should be placed upon some conspicuous part of the malt-house.

EXAMPLES.

1. The mean length of a floor of malt is 329, the mean breadth 242, and the mean depth of the grain 5.6 inches; required the content in malt bushels.

BY THE PEN.

To find the content.

Inches.	
329	*length.*
242	*breadth.*
658	
1316	
658	
79618	*product.*
5.6	*depth.*
477708	
398090	
Divisor 2150.42)445860.8(207.33	*content.*

BY THE SLIDING RULE.

Length.	*Depth.*	*Breadth.*	*Content.*
As 329 *on* B :	5.6 *on* M. D : :	242 *on* A :	207.3 *on* B.

Or,

Breadth.	*Depth.*	*Length.*	*Content.*
As 242 *on* B :	5.6 *on* M. D : :	329 *on* A :	207.3 *on* B.

Note. Some experienced Officers think the latter method of casting the content by the Sliding Rule is more convenient than the former; it is best, however, to cast the contents of all floors by both methods; be-

cause one method then serves as a proof for the other; consequently, any error committed in the first operation will be detected in the second.

2. The mean length of a floor of malt is 435, the mean breadth 218, and the mean depth of the grain 5.2 inches; required the content in malt bushels.

Ans. 229.311 *bushels.*

3. The length, breadth, and depth of a floor of malt measure 485, 224, and 3.4 inches respectively; what is the content in malt bushels? *Ans.* 171.769 *bushels.*

PROBLEM VI.

To find the content of a rectangular floor of Malt by another method.

RULE.

Multiply the length of the floor by half the breadth, or the breadth by half the length; and point off three figures as decimals. Then multiply the number thus obtained by the depth; and this product by .93; and you will obtain the content of the floor, in malt bushels.

Note 1. If the length of a floor be multiplied by half the breadth, or the breadth by half the length, and this product by the depth; the last product will give the content of the floor 7 bushels too much, in every hundred; care however must be taken to point off three figures, as decimals, before you multiply by the depth. Thus it appears that the *true* content may also be obtained by subtraction; for, if the *false* content be multiplied by 7; the product divided by 100; and the quotient subtracted from the *false* content, the remainder will be the *true* content of the floor.

2. If the malt divisor had been 2000, the length multiplied by half the breadth, and three figures pointed off, as decimals, would have given the true area of the floor; for this process is, in effect, the same as dividing the product of the length and breadth by 2000; but as the malt divisor is 2150.42, it is evident that the above method will always give the area too much; hence the necessity of the deduction mentioned in the first Note.

3. If either the length or breadth of a floor be 215 inches, then the other dimension will be the area of the floor, when *one* figure is pointed off as a decimal; consequently, if this area be multiplied by the depth, the product will be the *true* content of the floor, in malt bushels.

4. When half the length or half the breadth of a floor of malt, is 120, 110, 100, 90, 80, &c. &c. the product of the other dimension and any of these numbers may easily be obtained *mentally*; then three figures being cut off, as decimals, the remainder may also be multiplied by the depth; and thus you will obtain the *false* content of the floor. Lastly, multiply this content by 7; cut off two figures for decimals; subtract the remainder from the *false* content; and you will obtain the *true* content of the floor, without using either the Pen or the Sliding Rule.

EXAMPLES.

1. The length of a rectangular floor of malt is 329, the breadth 242, and the depth 5.6 inches; required the content in malt bushels.

BY THE RULE.

Inches.

```
     329      length.
     121      half the breadth.
    ----
     329
    658
   329
   ------
   39.809     product.
      5.6     depth.
   -------
   238854
  199045
  --------
  222.9304    product.
       .93    factor.
  --------
   6687912
 20063736
 ----------
 207.325272   content.
```

BY THE FIRST NOTE.

Here $329 \times 121 \times 5.6 = 222.930$ *the false content.*

and $\dfrac{222.93 \times 7}{100} = \dfrac{1560.51}{100} = 15.605$, *the deduction.*

207.325 *the true content.*

2. The length of a floor of malt is 435, the breadth 218, and the depth 5.2 inches; what is the content in malt bushels? *Ans.* 229.298 *bushels.*

3. The length of a floor of malt is 215, the breadth 186, and the depth 6.7 inches; required the content in malt bushels. *Ans.* 124.62 *bushels.*

4. The length of a floor of malt is 250, its breadth 100, and its depth 6 inches; it is required to determine its content *mentally.* *Ans.* 69.75 *bushels, nearly.*

Remark.

The methods given in the last Problem, for finding the content of a floor of malt, are extremely simple; and may be of considerable service to young Officers, as a *check*, in making compares with preceding gauges, when a floor is found very irregular in its dimensions. This compare ought to be made before the dimensions of the floor are entered in the Malt Book; and if any error has been committed, it must be immediately rectified, by re-taking the dimensions with greater accuracy.

PROBLEM VII.

To gauge and fix a rectangular Malt-kiln.

Directions for taking the dimensions, &c.

Find the mean length and breadth of the kiln, as before directed; then, multiply the length by the breadth; divide the product by 2150.42; and the quotient will be the area in malt bushels. Multiply this area by the depth of the grain, and the product will be the content.

BY THE SLIDING RULE.

As 2150.42 on A, is to the length on B; so is the breadth on A, to the area on B; and as unity on A, is to the area on B; so is the depth on A, to the content on B.

Note 1. The entrance to a malt-kiln is generally from an adjoining room; sometimes, however, the well-hole of the stairs which lead to

the kiln, forms an opening or aperture in a corner of the kiln-floor; in this case, either the area of the well-hole must be deducted; or which is better, divide the area of the well-hole by the breadth of the kiln; deduct the quotient from the length; and the remainder will be the true length required. This Rule is formed on the well-known principle, that the area of a rectangle divided by one of its dimensions, will give the other dimension. One of the dimensions of the copper-back, in Prob. IV., Sect. II., might have been reduced in this manner.

2. It is scarcely necessary to observe that malt should be in a raw state, or nearly so, when gauged on the kiln, or the content cannot be expected to compare well with the preceding gauges.

3. The lengths and breadths of malt-kiln floors, are fixed dimensions; and should be placed in some conspicuous part of malt-kilns. The dimensions and area must also be entered at the top of the malt book.

EXAMPLES.

1. The length of a rectangular malt-kiln is 212, and its breadth 203 inches; required the area in malt bushels.

BY THE PEN.

To find the area.

Inches.	
212	*length,*
203	*breadth.*
636	
424	*Area.*

Divisor 2150.42)43036.00(20.012 *bushels.*

BY THE SLIDING RULE.

	M. B.		*Length.*		*Breadth.*		*Area.*	
As	2150.42	*on* A :	212	*on* B ::	203	*on* A :	20.01	*on* B.

2. If the depth of the grain on the kiln, the dimensions of which are given in the above Example, be 4.7 inches; required the content in malt bushels.

Ans. 94.0564 *bushels.*

3. The length of a rectangular malt-kiln is 236, and its breadth 212 inches; the length of the well-hole for the stairs is 50, and its breadth 30 inches; required the true

length of the kiln, so as to make a deduction for the well-hole; and also the area in malt bushels.

Ans. The true length is 229 inches; and the area 22.576 malt bushels.

Remark.

The areas or contents of any figures not treated of in the preceding Problems, may be found by the respective Problems in Parts IV., or V.; and should a cistern be met with in the form of the frustum of a cone or pyramid, it must be gauged as directed in the first Section of Part VI.

PROBLEM VIII.

To find whether the duty will arise from the cistern, the couch, or the floor.

RULE.

The duty is always charged upon the *best gauge* of the cistern, couch, or floor; and in order to find from which of these the charge will arise, without reducing them to *net* bushels, proceed thus: Multiply the best gauge of the cistern or couch by 1.6; and if the product exceed the floor bushels, the charge must be made from the cistern or couch; but if not, the charge must be made from the floor.

Note 1. If cistern or couch bushels be multiplied by .8, the product will be net bushels; but floor bushels must be multiplied by .5, in order to reduce them to net bushels. (See Preliminary Observations.)

2. If floor bushels be multiplied by .625, the product will be cistern or couch bushels.

3. The above factors are obtained by the following Proportions:

As
- 20 : 16 :: 1 : .8, for reducing cistern or couch bushels to net bushels.
- 20 : 10 :: 1 : .5 for reducing floor bushels to net bushels.
- .5 : .8 :: 1 : 1.6 for reducing cistern or couch bushels to floor bushels.
- .8 : .5 :: 1 : .625 for reducing floor bushels to cistern or couch bushels.

4. When barley is much damaged, it loses part of its vegetative quality. Under such a circumstance, the best floor-gauge will some-

times not advance much more than one-fifth above the dry barley. When this is the case, the reason ought to be assigned in the Malt Book.

EXAMPLES.

1. The best cistern-gauge is 129.6, the best couch-gauge 132.5, and the best floor-gauge 207.3 bushels; from which of these gauges will the charge of the duty arise; and what are the number of net bushels?

Bushels.	
132.5	*couch-gauge.*
1.6	*multiplier.*
7950	
1325	
212.00	*product.*

Here the number of bushels obtained by multiplying the couch-gauge by 1.6, exceeds the number of floor-bushels; hence the charge will arise from the couch.

Bushels.	
132.5	*couch bushels.*
.8	*multiplier.*
86.00	*product.*

Here the net bushels are 86, the number upon which the duty must be charged.

2. If the best cistern-gauge be 68.4, the best couch-gauge 69.8, and the best floor-gauge 112.6 bushels; from which will the charge of the duty arise; and what are the number of net bushels?

Ans. The charge of the duty will arise from the floor; and 56.3 are the net bushels upon which the duty must be charged.

Note. Here it may be proper to observe that the malt-duty is always charged upon the net bushels and tenths of a bushel; and when the second place of decimals is five or above, the first is called one more; but when the second place is under five it is rejected.

A SPECIMEN OF A DIMENSION BOOK,

Containing the Dimensions and Contents of all the Casks given in Problems III., IV., V., and VI., of Cask Gauging.

No.	V.	H.	B.	L.	Con.	No.	V.	H.	B.	L.	Con.
1	1	27.0	36.0	45.0	139	9	3	27.0	36.0	45.0	129
2	1	24.6	30.9	46.7	109	10	3	24.6	30.9	46.7	103
3	1	21.4	26.2	32.5	55	11	3	21.4	26.2	32.5	52
4	1	19.6	23.4	27.7	38	12	3	19.6	23.4	27.7	36
5	2	27.0	36.0	45.0	135	13	4	27.0	36.0	45.0	126
6	2	24.6	30.9	46.7	107	14	4	24.6	30 9	46.7	101
7	2	21.4	26.2	32.5	54	15	4	21.4	26.2	32.5	51
8	2	19.6	23.4	27.7	37	16	4	19.6	23.4	27.7	36

Note. Those Officers who are in the habit of gauging Victuallers' Casks as being of the first variety, invariably find that the gauge of the tunnage, in the casks, exceeds the gauge of the wort in the guile-tuns. This is another proof that casks are always less than the first variety.— (See Remarks on Cask Gauging.)

A SPECIMEN OF A DIMENSION BOOK,

Containing the Dimensions and Areas of the Malt Cisterns and Kilns given in Problems I., and VII., of Malt Gauging.

Malt Cisterns.					Malt Kilns.			
No.	D.	L.	B.	Areas.	No.	L.	B.	Areas.
1	45.3	96.8	63.6	2.86	1	212	203	20.01
2	52.8	128.6	85.4	5.11	2	229	212	22 .58

SECTION V.

THE METHOD OF GAUGING AND INCHING DISTILLERS' UTENSILS, AS PRACTISED IN THE EXCISE.

DISTILLERY.

PRELIMINARY OBSERVATIONS.

It being a duty incumbent upon Officers of the Excise to do the utmost justice between the Revenue and the Trader, they ought, on all occasions, to take the dimensions, and find the areas and contents of the vessels they gauge, with the greatest accuracy. If this be necessary in the ordinary course of business, it is still more so, when an Officer has to gauge and inch the Utensils of Distillers; because the duty on a gallon of Spirits, is high when compared with that on a gallon of Ale; hence, small errors in the dimensions, areas, or contents of Stills, Wash-backs, Jack-backs, &c. will, by a constant repetition, lead to very serious results.

Some Writers on Gauging affect to divide Stills into different mathematical figures, and find their contents accordingly; thus, the upper part of a Still they suppose to form the frustum of a sphere; the middle part the frustum of a spheroid; and the bottom they take as the segment of a sphere; but it is more than probable that there never was any Still made, the parts of which exactly corresponded to these figures; consequently, this method of Gauging must be subject to considerable inaccuracy.

We have no hesitation in saying that the same method we have adopted in Gauging Coppers, may be equally well applied in Gauging Stills; and will be found much more *accurate* and *practical* than the *abstruse* method of dividing them into different Parts, without any possibility of proving the existence of those Parts, either by comparison or by Mathematical Rules.

Still-heads are also divided, by the same Writers, into various mathematical figures; namely, the upper and lower parts are considered to form spherical frustums, and the middle part is taken as the middle frustum of a spheroid; but we object to this method of division, on the same grounds as we do to that of the Still. The content of a Still-head, at every inch of its perpendicular depth is never wanted, but merely the content of the whole head; hence, we are of opinion that a more *accurate* and *practical* Rule cannot be devised than that which we have given in Prob. XX., Part V.

We also conceive that the method of Gauging oval Wash-backs, and Jack-backs, by means of equidistant ordinates, is very imperfectly treated by most Writers on Gauging; as they have neither elucidated the Rules, nor applied them to Practice, in such a manner as to be comprehended by the generality of Readers. We trust that the Rules, Remarks, and Examples we have given in Prob. XX., Part IV., together with their Practical Application in this Section, will give general satisfaction, not only to Masters of Seminaries, but also to Officers of the Excise.

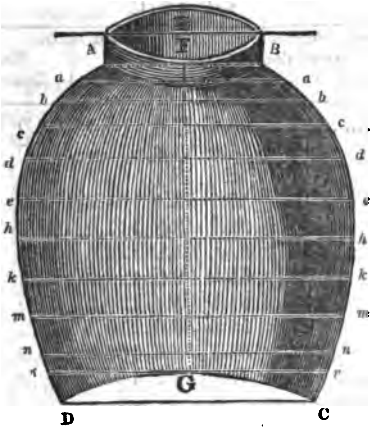
PROBLEM I.

To gauge and inch a Wash-Still.

EXAMPLES.

EXAM. 1.

Let the following figure A B C D represent a Wash-Still; it is required to gauge and inch it, for dry inches, as practised in the Excise.



To take the dimensions.

Lay a straight rod diametrically across the top of the Still; let fall a plumb-line from the middle of the rod at E, to the centre of the crown at G; measure E G = 61; also measure E F = 5; then 61 – 5 = 56 inches = F G, the depth to be tabulated.

Next, measure the diameter A B = 28; quarter the top of the Still; place a lighted candle upon the centre of the crown at G; suspend a plumb-line successively from the four quartering points; darken the top of the Still; and trace the quartering lines down its sides by the shadow of the plumb-line.

Lastly, measure cross diameters in the middle of every 4, 6, or 8 inches; from these cross diameters find mean diameters; and enter all the dimensions in a Note Book, as directed for the copper, in Prob. II., Sect. II., Part VI.

NOTE BOOK.

A. B.'s Wash-Still, No. 1, gauged Dec. 8th, 1821.							
Divisions in Inches.	Depths from the Top.	Cross Diameters.		Sum of Ditto.	Mean Diameters.	
....	....	28.2	27.8	56.0	28.0	*A B*
4	2	38.2	37.8	76.0	38.0	*a a*
4	6	46.6	46.2	92.8	46.4	*b b*
4	10	53.8	53.4	107.2	53.6	*c c*
8	16	58.7	58.3	117.0	58.5	*d d*
8	24	61.6	61.2	122.8	61.4	*e e*
8	32	63.0	62.6	125.6	62.8	*h h*
8	40	61.8	61.4	123.2	61.6	*k k*
6	47	58.4	58.0	116.4	58.2	*m m*
6	53	54.2	53.8	108.0	54.0	*n n*
....	56	52.6	52.2	104.8	52.4	*r r*
Height of the Crown.				5.0	46.0	*C D*

To find the area and content.

RULE.

Find the area corresponding to each mean diameter, in the Table of *Wine Areas*, Part VII.; then multiply each area by its respective depth; and the sum of the products will be the whole content of the Still.

Note. The content of the liquor to cover the crown, may be found by any of the Rules given in Prob. II., Sect. II., Part VI.

DIMENSION BOOK.

Having found the areas of the several Sections, and the contents of the different Divisions, we hence form the Dimension Book as follows.

A. B.'s Wash-Still, No. 1, gauged Dec. 8th, 1821.				
Divisions in Inches.	Depths from the Top.	Mean Diameters.	Areas in Gallons.	Contents in Gallons.
4	2	38.0	4.910	19.640
4	6	46.4	7.320	29.280
4	10	53.6	9.768	39.072
8	16	58.5	11.636	93.088
8	24	61.4	12.818	102.544
8	32	62.8	13.409	107.272
8	40	61.6	12.902	103.216
6	47	58.2	11.517	69.102
6	53	54.0	9.914	59.484
56	Depth.	Crown per Meas.		22.000
Whole Content.				644.698

To tabulate the foregoing Wash-Still for Dry Inches.

RULE.

From the whole content in wine gallons, subtract the area of the first Section; and the remainder will be the content, when one inch is dry. From this remainder, subtract the same area, and you will obtain the content when two inches are dry. Proceed in this manner until you have tabulated the whole Still; taking care to change the area when you arrive at 4 inches, at 8 inches, at 12 inches, at 20 inches, &c. of the perpendicular depth of the Still, as in the following Table.

A TABLE,

Shewing the Method of Inching the foregoing Wash-Still.

Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.
Full	644.698 4.910	6	610.418 7.320	12 *	556.706 11.636	18	486.890 11.636
1	639.788 4.910	7	603.098 7.320	13	545.070 11.636	19	475.254 11.636
2	634.878 4.910	8 *	595.778 9.768	14	533.434 11.636	20 *	463.618 12.818
3	629.968 4.910	9	586.010 9.768	15	521.798 11.636	21	450.800 12.818
4 *	625.058 7.320	10	576.242 9.768	16	510.162 11.636	22	437.982 12.818
5	617.738 7.320	11	566.474 9.768	17	498.526 11.636	23	425.164 12.818

In this manner we may obtain the content of the Still at every *dry* inch of its perpendicular depth; and, by way of Practice, the Learner is required to continue the above process. (See the Key to this Work.)

To form a Distiller's Table Book, &c.

The gauge of Wash is taken through a cylindrical tube fixed in the breast of the Still, and rising 3 or 4 inches above the base of the collar, where the tabulating commences; consequently, in forming the Table Book, we must not begin at the base of the collar, as in the foregoing Table, but as many inches above it as the top of the tube exceeds it in height. Thus, if the top of the tube rises 4 inches above the base of the collar, or diameter A B; then the Still will be full when the gauge exhibits 4 dry inches, &c. &c. as in the following Table Book.

A DISTILLER'S TABLE BOOK.

A. B.'s Wash-Still, No. 1.

Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.
4	645	10	610	16	557	22	487
5	640	11	603	17	545	23	475
6	635	12	596	18	533	24	464
7	630	13	586	19	522	25	451
8	625	14	576	20	510	26	438
9	618	15	566	21	499	27	425

In this manner the Table Book may be completed; and by way of Practice, the Learner is required to continue the above process.

Note. In forming the above Table Book, the first decimal place is rejected when it is under five-tenths; but when it is five-tenths or above, it is called one gallon.

REMARK.

If we consider the upper part $AB\,cc$, of the foregoing Still, to represent the frustum of a sphere, then we have the diameter $AB = 28$, the diameter $cc = 53.6$, and the perpendicular height $= 10$ inches; hence, by Prob. XII., Part V., we find the content to be 64.434 wine gallons.

Again, taking the middle part $cc\ rr$ as the frustum of a spheroid, we have the diameter $cc = 53.6$, and the diameter $rr = 52.4$; consequently $\frac{53.6 + 52.4}{2} = \frac{106}{2} = 53$, the *mean* end diameter. The perpendicular altitude is 46; and the middle diameter is found to be 62.5 inches; hence, by Prob. XIV., Part V., we find the content to be 553.729 wine gallons.

The quantity of liquor necessary to cover the crown, is found by measure, to be 22 gallons; hence, we have $553.729 + 64.434 + 22.0 = 640.163$ gallons, the whole

content, which is 4.535 gallons less than the content given in the foregoing Dimension Book. This difference arises chiefly in consequence of the upper part of the Still being rather larger than the frustum of a sphere; hence it appears that the foregoing method of *tabulating* a Still is more accurate than the following one; which is likewise so very tedious and complex, that it is almost impracticable. (See Prob. XV., Sect. I., Part VI.)

Note. By Rule 2, Prob. II., Sect. II., Part VI., the content of the liquor necessary to cover the crown, is found to be 24½ gallons, which is too much by 2½ gallons.

The Method of Tabulating the foregoing Still, when the upper part is considered as the frustum of a Sphere.

In order to make the foregoing dimensions answer our purpose, we shall take the altitude of the globular part = 12 inches; then we have the top diameter = 28 inches, and the bottom diameter = 55.6 inches; hence, by Prob. XV., Sect. I., Part VI., we find the first intermediate diameter to be 31.9, the second 35 3, the third 38.3, the fourth 41.0, the fifth 43.4, the sixth 45.6, the seventh 47.6, the eighth 49.5, the ninth 51.2, the tenth 52.8, and the eleventh 54.2 inches. (See Exam. 3, of the Prob. to which we last referred.)

Now, in order to find the mean diameters in the middle of every inch, we must divide the sum of the diameters at the extremities of each inch, by two; and the quotients will be the diameters by which the globular part of the Still must be *tabulated.* These diameters and the areas corresponding to them, are exhibited in the next page.

In.	In.	In.	Diam.	Areas.
$\frac{28.0 + 31.9}{2} = \frac{59.9}{2} =$			29.9	......3.040
$\frac{31.9 + 35.3}{2} = \frac{67.2}{2} =$			33.6	......3.838
$\frac{35.3 + 38.3}{2} = \frac{73.6}{2} =$			36.8	......4.604
$\frac{38.3 + 41.0}{2} = \frac{79.3}{2} =$			39.6	......5.332
$\frac{41.0 + 43.4}{2} = \frac{84.4}{2} =$			42.2	......6.055
$\frac{43.4 + 45.6}{2} = \frac{89.0}{2} =$			44.5	......6.733
$\frac{45.6 + 47.6}{2} = \frac{93.2}{2} =$			46.6	......7.383
$\frac{47.6 + 49.5}{2} = \frac{97.1}{2} =$			48.5	......7.998
$\frac{49.5 + 51.2}{2} = \frac{100.7}{2} =$			50.3	......8.602
$\frac{51.2 + 52.8}{2} = \frac{104.0}{2} =$			52.0	......9.194
$\frac{52.8 + 54.2}{2} = \frac{107.0}{2} =$			53.5	......9.732
$\frac{54.2 + 55.6}{2} = \frac{109.8}{2} =$			54.9	......10.248
Whole content of the spherical frustum				82.759

Note 1. In finding the foregoing diameters, the second figure of decimals is neglected; and in taking the areas from the Table, the third figure of decimals is called one more, when the fourth figure is found to be five or upwards.

2. The content of the globular part of the foregoing Still, found by Prob. XII., Part V., is 82.97 wine gallons, which agrees well with the content found from the mean diameters; but if we add together the contents of the first three divisions, in the foregoing Dimension Book, we obtain 87.992 gallons, which proves that the upper part of the Still is greater than a spherical frustum.

To tabulate the Still for dry inches.

RULE.

The whole content of the Still, as given in the Dimension Book, is 644.698, from which take 87.992, the content of the first three divisions; and we obtain 556.706, the content of the remainder of the Still. To this number add 82.759, and we have 639.465 wine gallons, the content of the Still, when the upper part is taken as the frustum of a sphere. Thus having found the areas of the different sections of the globular part, and the whole content, we proceed to tabulate the first 12 inches of the Still. We then take the areas 11.636, 12.818, &c. &c. from the Dimension Book, and continue the tabulation, as in the following Table.

A TABLE

Shewing the Method of Inching the foregoing Still, when the upper part is considered as the frustum of a Sphere.

Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.
Full	639.465 3.040	6	609.863 7.383	12	556.706 11.636	18	486.890 11.636
1	636.425 3.838	7	602.480 7.998	13	545.070 11.636	19	475.254 11.636
2	632.587 4.604	8	594.482 8.602	14	533.434 11.636	20	463.618 12.818
3	627.983 5.332	9	585.880 9.194	15	521.798 11.636	21	450.800
4	622.651 6.055	10	576.686 9.732	16	510.162 11.636		
5	616.596 6.733	11	566.954 10.248	17	498.526 11.636		

In this manner the content of the Still may be found at every *dry* inch of its depth; and the Learner is required to continue the above process. (See the Key to this Work.)

To make deductions for Spindles, Cross-Bars, Braces, Chains, and other Impediments contained in a Still.

A large Still has generally a wooden spindle extending perpendicularly from its mouth to within three or four inches of the top of the crown. This spindle is supported by *cross-bars* and *braces*; chains are attached to it, in different places; and during the time the liquor is boiling, the spindle and chains are made to perform a rotary motion, either by the hand, or by machinery. By this means the *wash* is *roused*, and the sediment prevented from settling to the bottom, where it would become burnt; and thus impart an unpleasant flavour to the spirits.

Suppose a spindle extends from the top of the Still, given in the following Example, to within 8 inches of the crown; passing through an iron bar, placed across the Still, at 32 inches from the top; and let the circumference of the spindle be 9.5 inches; then its diameter will be 3 inches, and its area .03 of a wine gallon. Also, let the length of the cross-bar be 87 inches, its breadth 5 inches, and its depth 4 inches; then $87 \times 5 \div 231 = 435 \div 231 = 1.88$ wine gallons, the area of the cross-bar.

Now, it is evident before we can tabulate the Still correctly, we must deduct the area of the spindle from each area of the Still that is affected by it; and the area of the cross-bar must also be deducted from that part of the Still where it is placed; namely, from the area used in tabulating the Still from 32 to 36 inches of its depth, because the depth of the bar is 4 inches.

The contents of the *chains* must be found by any practical method, and deducted from the quantity of liquor necessary to cover the crown of the Still; the tabulation may then be performed as before directed.

Note. The most correct method of finding the content of any very irregular body, is to immerse it in a cylindrical vessel of water; then take it out, and find the content of that part of the vessel through which the water has descended, by removing the irregular body; and it will be the content required.

EXAM. 2.

The dimensions, areas, and deductions of a Still, are contained in the following Dimension Book; it is required to tabulate the vessel for dry inches, as practised in the Excise.

DIMENSION BOOK.

A. B.'s Wash-Still, No. 2, gauged Dec. 18th, 1821.

Divisions in Inches.	Depths from the Top.	Mean Diameters.	Cross Areas.	Impediments. Depths in Inches.	Impediments. Areas of Deduc.	Net Areas.	Contents in Gallons.
4	2	47.8	7.77	4	.03	7.74	30.96
4	6	55.2	10.36	4	.03	10.33	41.32
4	10	63.3	13.62	4	.03	13.59	54.36
4	14	72.4	17.82	4	.03	17.79	71.16
8	20	78.2	20.79	8	.03	20.76	166.08
8	28	85.4	24.80	8	.03	24.77	198.16
8	36	88.2	26.45	4 4	1.88 0.03	24.57 26.42	98.28 105.68
8	44	88.5	26.63	8	.03	26.60	212.80
8	52	87.3	25.91	8	.03	25.88	207.04
8	60	83.4	23.65	8	.03	23.62	188.96
6	67	77.6	20.47	3 3	.03 .00	20.44 20.47	61.32 61.41
70	Depth.	Crown per Measure.					52.00
		Whole Content.					1549.53

A TABLE

Shewing the Method of Tabulating the Still given in the foregoing Example.

Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.
Full	1549.53 7.74	4	1518.57 10.33	8	1477.25 13.59	12	1422.89 17.79
1	1541.79 7.74	5	1508.24 10.33	9	1463.66 13.59	13	1405.10 17.79
2	1534.05 7.74	6	1497.91 10.33	10	1450.07 13.59	14	1387.31 17.79
3	1526.31 7.74	7	1487.58 10.33	11	1436.48 13.59	15	1369.52 17.79

In this manner the content of the Still may be found at every *dry* inch; and the Learner is required to continue the above process. (See the Key to this Work.)

Note. It may perhaps be necessary to remind the young Reader that in tabling this Still, the depths in the fifth, and the areas in the seventh column of the Dimension Book, are used, in consequence of the deductions that are made for the impediments.

PROBLEM II.

To find the content of a Still-Head.

To take the dimensions.

Measure the depth from the upper part of the head to the bottom of the collar, from which deduct the depth of the collar; and the remainder will be the depth of the head. Find mean diameters at the top, bottom, middle, and at one-fourth, and three-fourths of the depths of the head; and enter all the dimensions in your Note Book.

To find the content.

RULE.

Find the area corresponding to each diameter, in the Table of Wine Areas; then to the sum of the areas of the top and bottom, add twice the area of the middle, and four times the sum of the areas at one-fourth and three-fourths of the depth; multiply this sum by the depth; divide the product by 12, and the quotient will be the content in wine gallons. (See Prob. XX., Part V.)

By Prob. IV., Part V., find the content of the cylindrical collar, which add to the content of the head; and you will obtain the whole content required.

Note. By Act of Parliament, Distillers are not allowed to charge their Stills with a less proportion of wash, at one time, than three-fourths of the whole content of the Still and head; and immediately before the wash is conveyed to the Still, it is always gauged in a circular or an elliptical vessel, kept expressly for that purpose; and called a Jack-Back.

EXAMPLES.

EXAM. 1.

Let the following figure represent the head of the Wash-Still, given in the first Example of the last Problem; and let the depth $m\,n = 34$, the depth of the collar $n\,r = 5$, the diameter $A\,B = 24.6$, $a\,a = 41.2$, $b\,b = 46.0$, $c\,c = 41.6$; and $C\,D = 27.7$ inches; required the content of the head and collar, in wine gallons.

A. B.'s Wash-Still Head, gauged Dec. 8th, 1821.

Inches from the Top.	Mean Diameters.		Areas in Wine Gallons.
0.0	A B	24.6	2.058
8.5	*a a*	41.2	5.771
17.0	*b b*	46.0	7.194
25.5	*c c*	41.6	5.884
34.0	C D	27.7	2.609
Collar's height 5, and diam. 27.7 inches.			

To find the content.

Here 2.058 + 2.609 = 4.667, *the sum of the top and bottom areas; also,* 7.194 × 2 = 14.388, *twice the area of the middle section; and* 5.771 + 5.884 × 4 = 11.655 × 4 = 46.62, *four times the sum of the areas at one-fourth and three-fourths of the depth; then,* (4.667 + 14.388 + 46.62) $\times \frac{34}{12} = \frac{65.675 \times 34}{12} = \frac{2232.950}{12} = 186.079$ *wine gallons, the content of the head.*

Again, 2.609 × 5 = 13.045 *wine gallons, the content of the cylindrical collar; then,* 186.079 + 13.045 = 199.124 *wine gallons, the whole content required.*

Note 1. The content of the Still is 644.698 gallons, to which add 199.124, and we obtain 843.822 gallons, the content of the Still and Head. Three-fourths of this content is 632.866 gallons, the least quantity of wash with which the Still must be charged at one time.

2. Some writers on Gauging consider the parts A B *a a*, and C D *c c*, as the frustums of spheres; and the part *a a c c* as the middle frustum of a spheroid; but this method will never give the true content unless the head is actually composed of these figures, which is perhaps never the case; hence, we recommend the *simple* Rule given in this Problem.

3. If a Still-Head be very large, so that five diameters are considered too few, then the content may be obtained by Problem XXI., Part V.

EXAM. 2.

The dimensions of the Still-Head, belonging to the Still given in the second Example of the last Problem, are contained in the following Dimension Book; required the content of the head and collar, in wine gallons. *(See the last Fig.)*

DIMENSION BOOK.

A. B.'s Wash-Still Head, gauged Dec. 18th, 1821.			
Inches from the Top.	Mean Diameters.		Areas in Wine Gallons.
0.0	A B	27.2	2.515
10.0	*a a*	44.4	6.703
20.0	*b b*	51.0	8.843
30.0	*c c*	48.1	7.866
40.0	C D	35.4	4.261
Collar's height 6.2, diam. 35.4 inches.			

Ans. The content is 302.211 *wine gallons.*

Note. The content of the Still is 1549.53, to which add 302.211, and we obtain 1851.741 gallons, the content of the Still and Head. Three-fourths of this content is 1388.805 gallons, the least quantity of wash with which the Still must be charged at one time.

PROBLEM III.

To gauge and inch a Distiller's oval Wash-Back, by the method of equidistant ordinates.

Directions for taking the dimensions.

By Prob. XX., Part IV., find the transverse and conjugate diameters of the base of the Wash-Back; and also double ordinates, at equal distances from each other; all of which must be struck with a chalk-line.

By Prob. III., Part V., set off, upon the sides of the vessel, at the ends of the double ordinates, the perpendicular distances that the horizontal sections are intended to be taken from each other; then measure the transverse and conjugate diameters, and also the double ordinates of each section; and enter all the dimensions in a Note Book.

Note 1. When a Wash-Back stands upon its greater base, the distance between the two extreme ordinates of the first horizontal section, must be somewhat less than the transverse diameter of the top, in order that all the dimensions of the uppermost section may fall within the vessel.

2. Horizontal sections must be taken in the middle of every 6, 8, or 10 inches, as the case may require.

To find the area and content.

RULE.

By Prob. XX., Part IV., find the area of each horizontal section; then multiply each area by its corresponding depth; and the sum of the products will be the whole content. The vessel must then be tabulated for wet inches, by adding the area of the first section to itself, &c. &c. as before directed. (See Prob. I., Sec. II., Part VI.)

Note 1. The area of the segments may be most easily found by the latter part of Note 3, Prob. XX., Part IV.

2. The content of the drip, if any; and also the dipping-place, must be found as directed in Prob. IX., Sect. II., Part VI.

EXAMPLES.

EXAM. 1.

The dimensions of a Distiller's oval Wash-Back, are contained in the following Dimension Book; it is required to tabulate the vessel for wet inches, as practised in the Excise.

DIMENSION BOOK.

A. B.'s Wash-Back, No. 1, gauged January 28th, 1822.													
9 Ordinates, 12 inches equidistant = 96 inches.													
Divisions in Inches.	Depths from the Bottom.	Trans-verse Diam.	1	2	3	4	Conj. 5 Diam.	6	7	8	9	Sum of the Segs.	Total Areas.	Contents in Gallons.
10	35	100.0	20.4	53.0	67.0	74.1	76.4	74.1	67.0	53.0	20.4	4.0	25.835	258.35
10	25	102.2	27.0	56.1	69.7	76.8	79.0	76.8	69.7	56.1	27.0	6.2	27.392	273.92
10	15	105.4	33.6	59.6	72.6	79.4	81.6	79.4	72.6	59.6	33.6	9.4	29.185	291.85
10	5	108.0	38.6	62.8	75.4	82.1	84.2	82.1	75.4	62.8	38.6	12.0	30.884	308.84
1	Drip.		Drip per Measure.											15.00
41	Whole Depth.		Whole Content.											1147.96

To find the area of the first Section.

Here 38.6 + 38.6 = 77.2, *the sum of the extreme ordinates;* (62.8 + 82.1 + 82.1 + 62.8) × 4 = 289.8 × 4 = 1159.2, *four times the sum of all the even ordinates; and* (75.4 + 84.2 + 75.4) × 2 = 235.0 × 2 = 470.0, *twice the sum of all the odd ordinates; then,* (77.2 + 1159.2 + 470.0) × 4 = 1706.4 × 4 = 6825.6 *square inches, the area of that part of the section circumscribed by the sides of the vessel and the two extreme ordinates.*

Again, $\frac{77.2 \times 12}{3} = \frac{926.4}{3} = 308.8$ *square inches, the area of the two segments; and* 6825.6 + 308.8 = 7134.4 *square inches, the area of the whole section; then* 7134.4 ÷ 231 = 30.884, *the area in wine gallons.*

In the same manner the areas of the other sections must be found by the Learner; and both the areas and contents entered as in the foregoing Dimension Book. (See the Key to this Work, in which the process of finding the areas of the other sections is given.)

Note. The contents in the foregoing Table, are found by removing the decimal points in the areas one place towards the right hand.

A TABLE

Shewing the Method of Inching the foregoing Wash-Back.

Wet Inches.	Contents in Gallons.	Wet Inches.	Contents in Gallons.	Wet Inches.	Contents in Gallons.	Wet Inches.	Contents in Gallons.
Drip	15.000 30.884	5	138.536 30.884	9	262.072 30.884	13	382.210 29.185
2	45.884 30.884	6	169.420 30.884	10	292.956 30.884	14	411.395 29.185
3	76.768 30.884	7	200.304 30.884	11 *	323.840 29.185	15	440.580 29.185
4	107.652 30.884	8	231.188 30.884	12	353.025 29.185	16	469.765 29.185

In this manner the content at every *wet* inch may be obtained; and the Learner is required to continue the above process. (See the Key to this Work.)

Note 1. The Table Book may be formed precisely in the same manner as directed in Prob. I., of this Section.

2. Any Wash-Back may be lenthed in the same manner as directed in Prob. VII., Sect. II., Part VI.

EXAM. 2.

The dimensions and areas of a Distiller's Wash-Back, are contained in the following Dimension Book; it is required to tabulate the vessel for wet inches, as practised in the Excise.

DIMENSION BOOK.

A. B.'s Wash-Back, No. 2, gauged Jan. 28, 1822.													
9 Ordinates, 12 inches equidistant = 96 inches.													
Divisions in Inches.	Depths from the Bottom.	Transverse Diam.	1	2	3	4	Conj. 5 Diam.	6	7	8	9	Sum of the Segs.	Total Areas.	Contents in Gallons.
10	35	100.0	25.8	55.8	69.5	75.8	76.4	75.7	69.5	55.8	25.7	4.0	26.872	268.72
10	25	102.2	32.5	59.1	72.7	78.5	79.0	78.4	77.2	59.2	32.6	6.2	28.698	286.98
10	15	105.4	39.3	62.7	75.1	81.1	81.6	81.0	75.2	62.6	39.2	9.4	30.361	303.61
10	5	108.0	44.2	66.8	77.6	82.2	84.2	82.1	77.5	66.8	44.3	12.0	31.986	319.86
1	Drip.		Drip per Measure.											15.00
41	Whole Depth.		Whole Content.											1194.17

Note. The transverse and conjugate diameters in this, are the same as those given in the first Example; but the areas of the sections and the whole contents differ very materially from each other; hence the necessity of gauging all such vessels by equidistant ordinates. (See the following Remark.)

A TABLE

Shewing the Method of Inching the foregoing Wash-Back.

Wet Inches.	Contents in Gallons.	Wet Inches.	Contents in Gallons.	Wet Inches.	Contents in Gallons.	Wet Inches.	Contents in Gallons.
Drip	15.000 31.986	4	110.958 31.986	7	206.916 31.986	10	302.874 31.986
2	46.986 31.986	5	142.944 31.986	8	238.902 31.986	11	334.860 30.361
3	78.972 31.986	6	174.930 31.986	9	270.888 31.986	12	365.221 30.361

By proceeding in this manner, the content at every *wet* inch may be obtained; and the Learner is required to continue the above process.

Note. In the Key to this Work, the areas of all the sections in the foregoing Dimension Book, are found from the given dimensions; and it is expected that the Learner also determine those areas, before he proceeds to tabulate the vessel.

REMARKS.

1. By the Rule for the ellipse, the area of the first section is 30.918; the second 29.242; the third 27.450; and the fourth 25.976; hence the content is 1150.86 wine gallons. This content is 2.90 gallons more than the content found in the first Example, and 43.31 gallons less than that found in the second; which proves that the vessel is not *truly* elliptical in either case. (See Remarks in Prob. XX., Part IV.)

2. All Coolers, Jack-Backs, and other vessels used by Distillers, whatever may be their form, must be gauged and fixed according to the directions given in the respective Problems of this Work.

SECTION VI.

THE METHOD OF GAUGING AND FIXING THE UTENSILS OF SOAP MAKERS, STARCH MAKERS, AND GLASS MAKERS, AS PRACTISED IN THE EXCISE.

SOAP GAUGING.

PRELIMINARY OBSERVATIONS.

SOAP is chiefly made from leys drawn from potash and lime; and boiled up with tallow or oil. It is much used for washing and bleaching linens; and for various other purposes, by Dyers, Perfumers, Hatters, Fullers, &c. &c.

The principal soaps manufactured in this country, are *hard soap, soft soap,* and *ball soap:* the soft kind being either white or green.

Hard soap is made of leys and tallow; and is most commonly boiled at twice; the first operation being called half-boiling. *White soft soap,* is composed of leys and tallow; and *green soft soap* of leys, tallow, and fish oil. *Ball soap* is made of leys and fine oils; and *yellow soap* of common resin boiled up with tallow.

When *hard soap* has been sufficiently boiled in the copper, it is then put into rectangular frames or boxes, in order to cool and harden. Each of these frames must be at least 45 inches in depth, and none of them must exceed 45 inches in length, or 15 inches in breadth. These frames must all be fairly marked and numbered, for the purpose of preventing mistakes in taking the gauges.

After the process of boiling *soft soap* is completed, it is put into casks, which ought to be barrels, each containing

256 pounds; half-barrels, each containing 128 pounds; firkins, each containing 64 pounds; or half-firkins, each containing 32 pounds avoirdupois, exclusive of the *tare*, or weight of the cask.

The duty on all soaps is charged by the pound avoirdupois; the weight of *hard soap* is ascertained from the gauge; but the weights of soft soap, and ball soap, are always found by scales and weights. Sometimes, however, the *dry* inches of the copper are taken, in order to make compares.

PROBLEM.

To find the area and content of a rectangular hard soap-frame, in pounds avoirdupois.

RULE.

BY THE PEN.

Multiply the mean length by the mean breadth; divide the product by 28.00 for hot, and 27.14 for cold hard soap; and the respective quotients will be the area in pounds. Multiply this area by the depth, and the product will be the content.

Note. If the square of the diameter of a circular vessel, be divided by 35.65 for hot, and 34.56 for cold hard soap; the respective quotients will be the area of the vessel in pounds avoirdupois. (See the Table of Divisors on the 83d page.)

BY THE SLIDING RULE.

To find the area.

RULE.

As the square divisor on A, is to the length on B; so is the breadth on A, to the area in pounds, on B.

To find the content.

RULE.

As one on A, is to the area on B; so is the depth on A, to the content on B.

EXAMPLES.

1. The depth of a rectangular hard soap-frame is 54.0, the length 45, and the breadth 15 inches; required its area and content in pounds, for both hot and cold soap.

BY THE PEN.

Inches.
45 *length.*
15 *breadth.*
225
45

Divisor 28.00)675(24.10 *area in pounds hot.*

Divisor 27.14)675(24.87 *area in pounds cold.*

Then $24.1 \times 54 = 1301.4$, the content in pounds hot; and $24.87 \times 54 = 1342.98$, the content in pounds cold.

BY THE SLIDING RULE.

To find the area.

	On A.		On B.		On A.		On B.
As	28.00 / 27.14	:	45	::	15	:	24.10 *pounds hot.* / 24.87 *pounds cold.*

To find the content.

	On A.		On B.		On A.		On B.
As	1	:	24.10 / 24.87	::	54	:	1301.4 *pounds hot.* / 1342.9 *pounds cold.*

2. If the depth of hot soap, in the foregoing frame, be 48 inches; how many pounds of soap does it contain?
Ans. 1156.8 *pounds.*

3. The depth of cold soap, in the preceding frame, is 46 inches; what is the content of the gauge, in pounds?
Ans. 1144.02 *pounds.*

4. The depth of a soap-frame is 58 inches, its length 43

inches, and its breadth 14 inches; required its area and content in pounds, for both hot and cold soap.

		Areas.	*Contents.*
Ans.	*Hot*	21.50	......1247.00 *pounds.*
	Cold	22.18	......1286.44 *ditto.*

STARCH GAUGING.

PRELIMINARY OBSERVATIONS.

STARCH is generally made from the meal or flour of wheat, which is put into a vat, with a sufficient quantity of water; and suffered to ferment for several days. It is then removed from the vat, into hair sieves, where it is washed, to separate the starch from the bran; and the fine matter which passes through the sieves, is collected in tubs or frames, and is called a *sour water*.

In two or three days, the starch collected at the bottom of the *sour water* is washed again; and the fine matter thus obtained is denominated a *green water;* and the coarse part is called a *slime.*

Sometimes the starch which settles to the bottom of a green water, is also washed as before; and being passed through sieves, and collected in frames, it is denominated a *white water*.

From the *green* or *white water*, the starch is removed to boxes with holes in their bottoms; where it is permitted to drain and harden; it is then taken out of the boxes, in order to be dried on bricks. After it is sufficiently dried, it is put into a stove to crust; the crusted part is then scraped off; and the pure part put into papers, which are then labelled, and stamped. In this state it is again placed in the stove; and when it is completely dry, it is taken out and weighed in the presence of the Officer who surveys the Works.

Note 1. Starch is not only weighed as noticed above, but it is also gauged in the process of making, both in the vats, frames, and boxes.

When the weight arising from the worst gauge in the backs, exceeds the weight obtained by the scales, the charge of the duty is made from the gauge; and *vice versa.*

2. The instruments used in Starch Gauging are the same as those of which we have before treated; with the addition of an iron spit, divided into inches and tenths; and a brass plate to screw to the end of the Dimension Cane. Also a spatula to examine the starch in the materials, during the process of making.

PROBLEM I.

To gauge and fix a starch vat in the form of a parallelopipedon.

Directions for taking the dimensions, &c.

Find the mean length and breadth as before directed; and also the area in square inches. Divide this area by 2828; and the quotient will be the number of bushels of raw materials.

After the materials have been in a state of fermentation, the area in square inches must be divided by 2386, in order to obtain the area in bushels; and if the area in bushels be multiplied by 25, the product will be the area of the vat in pounds avoirdupois, of *green starch*; hence the whole content in pounds, may be obtained by multiplying the area by the depth.

Note 1. It has been ascertained by experience that on charging the vat with raw materials, each bushel of meal resolves itself into 2828 cubic inches; and that after fermentation, it is reduced to 2386 cubic inches; hence the divisors given in the preceding Rule.

2. It has also been found that on an average, every bushel of meal produces 25 pounds of green starch; consequently any number of bushels multiplied by 25, will give the content in pounds.

3. When starch is made from damaged flour, a bushel of such flour will yield more than 25 pounds of starch; and if it be made from bran, the quantity obtained from a bushel, will be less than 25 pounds. In such cases, the Officer must use his discretion in estimating the quantity of green starch in the vat.

TABLE OF DIVISORS AND GAUGE POINTS FOR STARCH BUSHELS.

State of the Materials.	Divisors for Bushels.		Gauge Points for Bushels.	
	Squares.	Circles.	Squares.	Circles.
Before Fermentation	2828	3601	53.17	60.0
After Fermentation	2386	3038	48.80	55.1

EXAMPLES.

1. The mean length of a starch vat, in the form of a parallelopipedon, is 186.6 inches, and its mean breadth 136.8 inches; required its area in bushels, and in pounds, both before and after fermentation.

BY THE PEN.

Inches.
186.6 *length.*
136.8 *breadth.*
14928
11196
5598
1866 *bush.*
Divisor 2828)25526.88(9.026 *area before ferm.*
Divisor 2386)25526.88(10.698 *area after ferm.*

BEFORE FERMENTATION.
Bushels.
9.026 *area.*
25
45130
18052
225.650 *ditto in pounds.*

AFTER FERMENTATION.
Bushels.
10.698 *area.*
25
53490
21396
267.450 *ditto in pounds.*

BY THE SLIDING RULE.

As the square divisor on A, is to the length on B; so is the breadth on A, to the area in bushels, on B.

$$\begin{array}{ccccccc} & \text{On A.} & & \text{On B.} & & \text{On A.} & & \text{On B.} \\ As & \left\{\begin{matrix}2828\\2386\end{matrix}\right\} & : & 186.6 & :: & 136.8 & : & \left\{\begin{matrix}9.0 \textit{ before ferm.}\\10.7 \textit{ after ferm.}\end{matrix}\right. \end{array}$$

2. Required the content in bushels and pounds, of the materials and starch in the foregoing vat, before fermentation; when the depth is 28.6 inches.

Ans. The content is 258.1436 *bushels, and* 6453.590 *pounds.*

3. Required the content in bushels and pounds, of the materials and starch in the preceding vat, after fermentation; when the depth is 26.4 inches.

Ans. The content is 282.4272 *bushels, and* 7060.680 *pounds.*

4. The length of a starch vat is 225.8 inches, and its breadth 156.6 inches; required its area in bushels and in pounds, both before and after fermentation.

Ans. The area before fermentation is 12.503 *bushels, and* 312.575 *pounds; and after fermentation* 14.819 *bushels, and* 370.475 *pounds.*

PROBLEM II.

To gauge and fix a starch vat in the form of the frustum of a cone.

To take the dimensions, &c.

Measure cross diameters in the middle of every 10 inches, from the bottom towards the top of the vessel, as directed in Victuallery; then divide the square of each mean diameter by 3601, and 3038; and the quotients will be the areas of the respective sections, in bushels, before and after fermentation.

Note. The areas in pounds must be found by multiplying by 25, as before.

EXAMPLES.

EXAM. 1.

The mean diameter, taken at 5 inches from the bottom of a starch vat, in the form of the frustum of a cone,

measures 98.2 inches; the diameter at 15 inches from the bottom, 96.4; the diameter at 25 inches from the bottom, 94.3; and that at 36 inches from the bottom measures 92.5 inches; it is required to find the area of each section in bushels, and to form a Dimension Book.

BY THE PEN.

To find the area of the first section.

Here $(98.2 \times 98.2) \div 3601 = 9643.24 \div 3601 =$ 2.677 *bushels, the area before fermentation;* and $9643.24 \div 3038 = 3.174$ *bushels, the area after fermentation.*

BY THE SLIDING RULE.

As the circular gauge-point on D, is to unity on C; so is the diameter on D, to the area, in bushels, upon C.

$$\begin{array}{ccccccccc} & \text{On D.} & & \text{On C.} & & \text{On D.} & & \text{On C.} & \\ As & \left\{\begin{matrix}60.0\\55.1\end{matrix}\right\} & : & 1 & :: & 98.2 & : & \left\{\begin{matrix}2.7 \textit{ before ferm.}\\3.2 \textit{ after ferm.}\end{matrix}\right. \end{array}$$

Note. The areas of the three remaining sections are found in the Key to this Work.

DIMENSION BOOK.

A. B.'s Starch Vat, No. 1, gauged Feb. 14, 1822.

Divisions in Inches.	Depths from the Bottom.	Mean Diameters.	Areas in Bushels. Before Fermen.	After Fermen.
12	36	92.5	2.376	2.816
10	25	94.3	2.469	2.927
10	15	96.4	2.580	3.058
10	5	98.2	2.677	3.174

2. If the depth of the materials in the preceding vat, be 38.6 inches; required the content in bushels and pounds, before fermentation.

Ans. The content is 97.69 *bushels, and* 2442.25 *pounds.*

3. The diameter taken at 5 inches from the bottom of a conical starch vat, measures 124.3; at 15 inches, 121.4; at 25 inches, 118.3; at 35 inches, 115.5; and at 46 inches from the bottom, the diameter measures 112.6 inches; it is required to determine the area of each section, in bushels, and to form a Dimension Book.

Answer.

	Before Fermentation. Areas in Bushels.	*After Fermentation. Areas in Bushels.*
First section	4.290	5.085
Second section	4.109	4.851
Third section	3.886	4.606
Fourth section	3.704	4.391
Fifth section	3.520	4.173

PROBLEM III.

To gauge and fix a water-frame, in the form of a parallelopipedon.

Directions for taking the dimensions, &c.

Take several lengths, in various parts of the frame, from which find a mean length. Find a mean breadth in the same manner; then multiply the mean length by the mean breadth; divide the product by 34.8, and the quotient will be the area of the frame in pounds avoirdupois of green starch. Multiply this area by the depth; and the product will be the content.

Note 1. In the Practice of Gauging, the depth from the top of the frame to the surface of the starch must be taken with a gauging-rod and a float; then if this depth be subtracted from the whole depth of the frame, the remainder will be the depth of the green starch.

2. Any water-frame may be tabulated from the top towards the bottom, either for inches, half inches, or tenths; but as the upper part of a frame is always occupied by water, it will only be requisite to commence the tabulation at the distance of 12 or 15 inches from the top of the frame. This may be most easily performed by subtracting 12 or 15 inches from the whole depth of the frame.

EXAMPLES.

EXAM. 1.

The length of a water-frame, in the form of a parallelopipedon, is 126.5, its breadth 54.7, and its depth 32

Note 1. In order to keep the starch clean, and to prevent it from passing through the holes in the bottom of the box, a cloth is always spread over it previously to putting in the starch. A slider is also sometimes used to divide the box into two parts, in order that the starch may be cut out in smaller pieces. This slider extends from one end of the box to the other; generally dividing the breadth of the box into two equal parts. As these sliders are only used occasionally, it becomes necessary to find two areas of each box; that is, one from dimensions taken in such a manner as to make an allowance for the cloth and slider; and another from dimensions in which an allowance is made for the cloth only. This we conceive may be most correctly done by taking the dimensions of the box without either slider or cloth in it; and then a make a proper deduction for them both.

2. When a cloth only is used, twice its thickness must be deducted from both the length and breadth of the box; but when both a slider and cloth are used, then twice the thickness of the cloth must be deducted from the length, and four times its thickness, together with the thickness of the slider, must be deducted from the breadth of the box, in order to obtain the proper dimensions.

3. The mean depth of green starch, in a box, is found by taking several depths, with a spit; and then dividing their sum by their number for a mean depth.

EXAMPLES.

1. The length of a starch-box is 65.6 inches, and its breadth 12.5 inches; required its area in pounds of green starch; making an allowance of one-tenth of an inch, in both dimensions, for twice the thickness of the cloth.

Inches.
65.5 *reduced length.*
12.4 *reduced breadth.*
2620
1310
655
Divisor 34.8)812.20(23.33 *lbs. area.*

2. If the depth of the starch, in the foregoing box, be 5.4 inches; required the content in pounds.

Ans. 125.982 *pounds.*

3. The dimensions of a starch-box, both with and without a slider, are contained in the following Dimension Book; required the content of the starch in pounds, when the depth is 6.3 inches.

DIMENSION BOOK.

A. B.'s Starch-Box, gauged Feb. 20, 1822.

Mean Length.	Mean Breadth.		Areas in Pounds.	
	With Slider.	Without Slider.	With Slider.	Without Slider.
64.4	11.6	12.2	21.46	22.57

Ans. The content with the slider is 135.198*; and without the slider,* 142.191 *pounds.*

Note 1. Before the Learner proceeds to find the contents, he must find the areas, both with and without the slider.

2. Any starch-box may be tabulated either for inches or tenths, by the continual addition of the whole area, or one-tenth of the area, as before directed. (See Prob. VII., Sect. II., Part VI.)

GLASS GAUGING.

PRELIMINARY OBSERVATIONS.

THE materials and preparations used for making the various sorts of glass; and also the methods of manufacturing them are different. The time required to found, melt, and refine the materials for each species of glass, after they are prepared, mixed, put into the pots, and placed in the furnace, is likewise somewhat uncertain. These pots are made of clay; and when they are properly tempered in the annealing oven, they are capable of bearing a very great degree of heat.

CROWN GLASS materials consist of kelp, barilla, pot-ash, Lynn-sand, cullet, &c.; and require from twenty-four to thirty hours to found and melt them.

PLATE GLASS is made from barilla, salt-petre, Lynn-sand, cullet, &c.

FLINT GLASS materials are salt-petre, red-lead, Lynn-sand, arsenic, &c.

BROAD GLASS is made from kelp, soft-sopers, whitsters, ashes, cullet, and various sorts of fresh ashes and sand.

COMMON BOTTLE GLASS is made from kelp, hard-sopers, ashes, lime, wood-ashes, cullet, and coarse sea or river sand.

The pots in which it is intended to melt materials for making glass, must first be gauged in the pot-chamber; and when one area is not sufficient for the whole depth, dimensions must be taken in the middle of every 6, 8, or 10 inches, from the top downwards. After the pots have been *annealed*, they must be re-gauged; and their numbers and dimensions, in both cases, must be entered in the Dimension Book.

ALLOWANCES.

In calculating the content of each pot, you must make the following allowances for the respective sorts of glass, in consideration of part of the materials being left in the bottom of each pot, which cannot be wrought out; and for other waste in manufacturing; namely, in all pots containing more than one hundred weight, used for making flint, stained, or phial glass, one-fourth part of the materials contained therein, and one inch in the depth of every pot; but in smaller pots, which do not contain one hundred weight, one-fifth only of the materials contained in the same must be allowed. In pots used for making crown, plate, or window glass, one-fourth of the materials must be allowed; and four inches at the bottom; but in pots used for making common bottles, only one-fifth of the materials must be allowed, and three inches at the bottom of each pot.

A TABLE OF DIVISORS AND GAUGE-POINTS FOR DIFFERENT SORTS OF GLASS.

Note. The divisors for squares express the number of cubic inches in a pound of glass.	Divisors for		Gauge Points for
	Squares.	Circles.	Circles.
A pound of flint glass........	8.469	10.77	3.28
A pound of plate glass	9.178	11.68	3.42
A pound of crown and broad glass...	10.516	13.39	3.66
A pound of phial and bottle glass ...	10.178	12.96	3.60
A pound of British cast plate glass...	11.300	14.38	3.79
A pound of German sheet glass......	11.250	14.32	3.78

*** The factors, divisors, and gauge-points for glass, given in the Table on page 83, are not correct. Those given in the preceding Table, are always used in gauging and fixing pots employed in manufacturing glass.

PROBLEM I.

To find the area and content of any circular pot, used in making glass.

RULE.

BY THE PEN.

Divide the square of the diameter by the circular divisor; and the quotient will be the area in pounds avoirdupois. Multiply this area by the depth; and the product will be the content.

Note. All pots used in melting materials for glass, are circular; consequently, their areas are always determined by the circular divisors and gauge-points, given in the foregoing Table.

EXAMPLES.

1. The mean diameter of a cylindrical *glass-pot* is 32.6, and its depth 33.8 inches; required its area and content in flint and plate glass.

BY THE PEN.

To find the area.

Inches.
32.6 *diameter.*
32.6 *ditto.*

1956
652
978

Divisor 10.77)1062.76(98.67 *lbs. the area in flint glass.*
Divisor 11.68)1062.76(90.98 *lbs. the area in plate glass.*

To find the content.

Then 98.67 × 33.8 = 3335.046, *the content in pounds of flint glass; and* 90.98 × 33.8 = 3075.124, *the content in pounds of plate glass.*

BY THE SLIDING RULE.

To find the area.

As the circular divisor on A, is to the diameter on B; so is the diameter on A, to the area on B.

$$\begin{array}{ccccccc} & \text{On A.} & & \text{On B.} & & \text{On A.} & & \text{On B.} \\ \textit{As} & \left\{\begin{array}{l}10.77\\11.68\end{array}\right\} & : & 32.6 & :: & 32.6 & : & \left\{\begin{array}{l}98.67 \textit{ lbs. area.}\\90.98 \textit{ lbs. area.}\end{array}\right. \end{array}$$

To find the content.

As the circular gauge-point on D, is to the depth on C; so is the diameter on D, to the content on C.

$$\begin{array}{ccccccc} & \text{On D.} & & \text{On C.} & & \text{On D.} & & \text{On C.} \\ \textit{As} & \left\{\begin{array}{l}3.28\\3.42\end{array}\right\} & : & 33.8 & :: & 32.6 & : & \left\{\begin{array}{l}3335.0 \textit{ lbs. content.}\\3075.1 \textit{ lbs. content.}\end{array}\right. \end{array}$$

2. The diameter and depth of a cylindrical pot used in melting materials for making glass, are 36.4, and 38.6 inches respectively; required the area and content in crown and bottle glass.

Ans. The area is 98.95, *and the content* 3938.210 *pounds of crown glass; and* 102.23, *and* 4068.754 *pounds of bottle glass.*

3. Required the area and content of a cylindrical pot, in British plate, and German sheet glass; when the diameter and depth are 38.7 and 39.8 inches respectively.

Ans. The area is 104.15, *and the content* 4145.170 *pounds of British plate glass; and* 104.58, *and* 4162.284 *pounds of German sheet glass.*

PROBLEM II.

To gauge and fix a glass-pot in form of a cylinder or the frustum of a cone, as practised in the Excise.

Directions for gauging and fixing a cylindrical pot.

First find the area of the base, as directed in the last Problem; from this area, make the proper deductions for waste of materials; and consider the remainder as the net area of the vessel. Multiply the net area by the whole depth, and the product will be the net content. Reduce the net area and content into hundred weights, quarters, and pounds; and from the content subtract the area, and the remainder will be the content at one *dry* inch. From this content subtract the same area; and the remainder will be the content at two *dry* inches, &c. &c. until you have tabulated the whole vessel.

Note 1. Before you begin to table, you must not only make an allowance of *one-fourth* or *one-fifth* of the area, for waste of materials; but also deduct from the whole content, *once*, *thrice*, or *four times* the area, as the case may require, in order to make a proper allowance for the drossy materials left at the bottom of the pot, which cannot be manufactured. *(See Allowances.)*

2. The deduction at the bottom may be most easily made, by subtracting the allowance from the whole depth of the pot, before you find the content.

Directions for gauging and fixing a pot in the form of the frustum of a cone.

Take diameters in the middle of every 6, 8, or 10 inches, from the top downwards; find the area of each section; and make the proper deductions for waste of materials; then multiply each area by its respective depth, and the sum of the products will be the whole content.

Reduce the areas and content into hundred weights, quarters, and pounds; and proceed to tabulate the vessel for *dry* inches, as before directed.

Note. In order to make the deduction at the bottom, subtract the allowance from the depth corresponding to the bottom section.

EXAMPLES.

EXAM. 1.

The mean diameter of a cylindrical pot is 23.3; and its depth 21.0 inches; it is required to tabulate the pot for flint-glass; making an allowance of one-fourth of the whole content, and one inch at the bottom for waste of materials.

To find the area and content.

Here $23.3 \times 23.3 = 542.89$, the square of the diameter; and $542.89 \div 10.77 = 50.4$ pounds, the gross area. One-fourth of this area is $= 12.6$ pounds; then $50.4 - 12.6 = 37.8$ pounds, the net area of the vessel; and $37.8 \times 20 = 756$, the net content.

Now, 37.8 pounds = 1 qr. 9.8 lbs; and 756 pounds = 6 cwt. 3 qrs. 0 lbs.; hence we proceed to tabulate the vessel, as in the following Table.

A TABLE

Shewing the Method of Inching the Pot given in this Example.

Dry Inches.	Contents in C.	Q.	P.	Dry Inches.	Contents in C.	Q.	P.	Dry Inches.	Contents in C.	Q.	P.
Full	6	3	00.0	3	5	2	26.6	6	4	2	25.2
	0	1	09.8		0	1	09.8		0	1	09.8
1	6	1	18.2	4	5	1	16.8	7	4	1	15.4
	0	1	09.8		0	1	09.8		0	1	09.8
2	6	0	08.4	5	5	0	07.0	8	4	0	05.6
	0	1	09.8		0	1	09.8		0	1	09.8

In this manner the content of the pot may be found at every *dry* inch of its depth; and the Learner is required to continue the process, and form a Table Book.

A GLASS-MAKER'S TABLE BOOK.

Dry Inches.	Contents in C.	Q.	P.	Dry Inches.	Contents in C.	Q.	P.	Dry Inches.	Contents in C.	Q.	P.
Full	6	3	00	3	5	2	27	6	4	2	25
1	6	1	18	4	5	1	17	7	4	1	15
2	6	0	08	5	5	0	07	8	4	0	06

In this manner the whole Table Book may be formed; and the Learner is required to continue the process, from the preceding Table, after it has been completed. (See the Key to this Work.)

Note. In forming a Glass-Maker's Table Book, the decimals are rejected when they are under five-tenths; but when a decimal is five-tenths or above, it is called one pound.

EXAM. 2.

The dimensions of a glass-pot in the form of the frustum of a cone, the areas of the several sections, and the contents of the different divisions, are contained in the following Dimension Book; it is required to tabulate the pot for bottle glass; making an allowance of one-fifth of the whole content, and 3 inches at the bottom, for waste of materials.

DIMENSION BOOK.

Divisions in Inches.	Mean Diams.	Cross Areas.	Net Areas.	Reduced Divisions.	Contents in Pounds.	Areas in C.	Q.	P.	Contents in C.	Q.	P.
6	36.1	100.55	80.44	× 6 =	482.64	0	2	24.44	4	1	6.64
10	32.3	80.50	64.40	× 10 =	644.00	0	2	08.40	5	3	0.00
10	29.5	67.14	53.72	× 10 =	537.20	0	1	25.72	4	3	5.20
10	25.5	50.17	40.14	× 7 =	280.98	0	1	12.14	2	2	0.98
36	depth	Reduced depth		33	1944.82	Content.			17	1	12.82

Note 1. The net areas are found by deducting one-fifth of the gross areas from themselves.

2. Before the Learner proceeds to tabulate the vessel, he ought, by way of Practice, to find all the areas and contents given in the Dimension Book.

A TABLE

Shewing the Method of Inching the Pot given in this Example.

Dry Inches.	Contents in C.	Q.	P.	Dry Inches	Contents in C.	Q.	P.	Dry Inches.	Contents in C.	Q.	P.
Full	17	1	12.82	4	14	1	27.06	8	11	3	17.38
	0	2	24.44		0	2	24.44		0	2	08.40
1	16	2	16.38	5	18	3	02.62	9	11	1	08.98
	0	2	24.44		0	2	24.44		0	2	08.40
2	15	3	19.94	6	13	0	06.18	10	10	3	00.58
	0	2	24.44		0	2	08.40		0	2	08.40
3	15	0	23.50	7	12	1	25.78	11	10	0	20.18
	0	2	24.44		0	2	08.40		0	2	08.40

In this manner the content of the pot may be obtained at every *dry* inch; and the Learner is required to complete the process.

REMARK.

Notwithstanding the areas and contents of glass-pots are estimated in pounds, by the gauge; yet the duty is always charged upon the real weight of the glass, as found by the scales; except plate glass, which cannot be conveniently weighed. For the method of charging the duty on flint and phial glass, we must refer the young Officer to the General Letter of Sept. 4th, 1819.

Note. In the Practice of Gauging, the *dry* inches of glass-pots are taken with an iron gauging-rod, adapted for that purpose.

CANDLES.

THE METHOD OF ESTIMATING THE WEIGHT OF CANDLES, BOTH BY THE PEN AND THE SLIDING RULE.

PRELIMINARY OBSERVATIONS.

No person residing out of the limits of the Head Office of Excise in London, who is not assessed to the Church and Poors' Rate, in the parish where he resides, can, according to law, make or manufacture any candles.

It is also enacted that every maker of candles shall give due notice, in writing, of the time when he intends to begin the process of making candles; and specify in such notice, the number of rods of which the making is intended to consist; likewise, the true size and number of candles on each rod; and when it is intended to manufacture mould candles, the exact size and number of the moulds must be specified.

PROBLEM.

Given the number of Candles in one pound, the number on one rod, and the number of rods, to find the whole weight in pounds.

RULE.

BY THE PEN.

Multiply the number of candles on one rod, by the number of rods; divide the product by the number of candles in one pound, and the quotient will be the number of pounds required.

EXAMPLES.

1. It is required to reduce 6.5 gallons of strong beer, at 9s. 2d. per barrel, to table beer, at 1s. 10d. per barrel.

$$\begin{array}{r l} 6.5 & \textit{gallons.} \\ 5.5 & \textit{factor.} \\ \hline 325 & \\ 325 & \\ \hline 35.75 & \textit{gallons.} \end{array}$$

2. It is required to reduce 7.5 gallons of table beer, at 2s. per barrel, to strong beer, at 10s. per barrel.

$$\begin{array}{r l} 7.5 & \textit{gallons.} \\ .2 & \textit{factor.} \\ \hline 1.50 & \textit{gallons.} \end{array}$$

Note. These reductions may be easily made by the Sliding Rule, in the following manner: As one on A, is to the factor on B; so is the given number of gallons on A, to the required number of gallons on B.

SECTION VII.

The Method of finding the Areas and Contents of Vessels, in Irish Malt Bushels, and Irish Liquid Gallons; also the Process of reducing Irish to English Measure; and vice versa. Likewise Specific Gravity, or the Method of finding the Magnitude of a Body from its Weight, or the Weight of a Body from its Magnitude; also the Method of estimating the Tonnage of Ships, according to the Parliamentary Rules.

IRISH MALT AND LIQUID MEASURES.

PRELIMINARY OBSERVATIONS.

THE Irish malt or corn bushel contains 2178 cubic inches; consequently, it exceeds the English malt bushel by 27.58 cubic inches; and as eight gallons make one bushel, the content of the Irish malt gallon is 272.25 cubic inches. The Irish liquid gallon contains 217.6 cubic inches; consequently, it is 13.4 cubic inches less than the English wine gallon, and 64.4 cubic inches less than the English ale gallon.

All the Rules given in this Work, for finding the areas and contents of vessels, in English measure, may be applied with the same success, in finding the areas and contents of similar vessels in Irish measure; but the Irish multipliers, divisors, and gauge-points, must be used, instead of those given in Part IV.

The factors, divisors, and gauge-points for Irish measures, are found precisely in the same manner as those for English measures; and are given in the following Table.

A TABLE OF IRISH FACTORS, DIVISORS, AND GAUGE-POINTS, FOR SQUARES AND CIRCLES.

	Factors for		Divisors for		Gauge-Points for	
	Squares.	Circles.	Squares.	Circles.	Squares.	Circles.
Irish malt bushel	.000459	.000361	2178.00	2773.10	46.67	52.66
Irish malt gallon	.003673	.002884	272.25	346.64	16.50	18.62
Irish liquid gallon	.004595	.003609	217.60	277.05	14.75	16.64

PROBLEM I.

To find the area and content of any angular vessel, in Irish malt bushels, and liquid gallons.

RULE.

By the Rules given in Part IV., find the area of the base in square inches; then multiply this area by .000459 and .004595; or divide it by 2178 and 217.6, and the respective products or quotients will be the area of the base, in Irish malt bushels, and liquid gallons. Multiply this area by the depth; and the product will be the content.

Note. The preceding Rule will give the content of all angular vessels, whose sides are perpendicular to the base.

EXAMPLES.

1. The length of a vessel in the form of a parallelopipedon, is 84 inches, its breadth 56 inches, and its depth 42 inches; required its area and content in Irish malt bushels, and liquid gallons.

By Prob. II., Part IV., we have $84 \times 56 = 4704$, *the area of the base, in square inches; and* $4704 \div 2178 = 2.159$, *the area in Irish malt bushels; also* $4704 \div 217.6 = 21.617$, *the area in Irish liquid gallons; then* $2.159 \times 42 = 90.678$, *the content of the vessel in Irish malt bushels; and* $21.617 \times 42 = 907.914$, *the content in Irish liquid gallons.*

The length of a rectangular vessel is 96.8, its breadth
and its depth 55.4 inches; required its area and
t in Irish malt bushels, and liquid gallons.

The area and content are 2.826 *and* 156.5604 *Irish*
ushels; and 28.292 *and* 1567.3768 *Irish liquid*

REMARK.

his Problem we have made choice of the Parallel-
on, because no other angular figure occurs so fre-
y in Gauging; however, if the content of any
be found in cubic inches, by the Rules given in
.; and this content divided by 217.6, and 2178;
pective quotients will be the content in Irish liquid
, and malt bushels.

PROBLEM II.

d the area and content of a cylindrical vessel, in Irish malt bushels, and liquid gallons.

RULE.

ltiply the square of the diameter by .000361, and
9; or divide it by 2773.1, and 277.05; and the
tive products or quotients will be the area of the
in Irish malt bushels, and liquid gallons. Multiply
ea by the depth, and the product will be the con-

EXAMPLES.

The diameter of a cylindrical vessel is 56 inches, and
th 48 inches; required its area and content, in Irish
ushels and liquid gallons.

e $56 \times 56 = 3136$, *the square of the diameter; and*
$\div 2773.1 = 1.1308$, *the area in Irish malt bushels;*
$3136 \div 277.05 = 11.3192$, *the area in Irish liquid*
; then $1.1308 \times 48 = 54.2784$, *the content of the*
in Irish malt bushels; and $11.3192 \times 48 = 543.3216$,
tent in Irish liquid gallons.

The diameter of a cylindrical vessel is 82.4 inches,
depth 52.6 inches; required its area and content in
nalt bushels and liquid gallons.

Ans. The area and content are 2.448 and 128.7648 Irish malt bushels; and 24.507 and 1289.0682 Irish liquid gallons.

REMARKS.

1. Wherever the circular divisors 359.05, 294.12, or 2738.00, are used in this Work, to find the areas or contents of vessels, in English ale and wine gallons, and malt bushels; if, instead of these, we substitute 277.05, and 2773.10, the results will give the areas or contents of the same or similar vessels, in Irish liquid gallons, and malt bushels.

2. The Irish circular divisors, for the frustum of a cone, in order to find its content in Irish liquid gallons, and malt bushels, are 831.15, and 8319.3; for a sphere, or a spheroid, 415.575, and 4159.65; for the middle frustum of a spheroid, 831.15, and 8319.3; for a parabolic spindle, 519.46, and 5199.56; for the middle frustum of a parabolic spindle, 4155.75, and 41596.5; and for the frustum of a parabolic conoid, 554.1, and 5546.2.

3. In order to find the content of a circular vessel, in Irish liquid gallons, and malt bushels, by Prob. XIX., Part V., the divisors 1662.3, and 16638.6 must be used; if the content be required by Prob. XX.; then 3324.6, and 33277.2 must be used as divisors; and lastly, if the content be required by Rule I., Prob. XXI., we must then use the square divisors 217.6, and 2178; and the results will be the contents of the vessels in Irish liquid gallons, and malt bushels.

PROBLEM III.

To reduce Irish measure to English measure.

RULE.

Multiply the given number of bushels, Irish measure, by 1.0128; and the product will be the number of bushels, English measure. Also, multiply the given number of gallons, Irish measure, by .7716, and .9410; and the respective products will be the number of ale and wine gallons, English measure.

Note. The multipliers given in the preceding Rule, are found by dividing the Irish by the English square divisors.

EXAMPLES.

In 186 Irish, how many English bushels?
re $1.0128 \times 186 = 188.3808$ *English bushels.*
In 106 Irish gallons, how many English ale and wine
s?
re $.7716 \times 106 = 81.7896$ *English ale gallons; and*
$\times 106 = 99.8414$ *English wine gallons.*
A Maltster in Dublin, ships for Liverpool, 2685
s of malt, according to the *gauge* in Ireland; for
number of bushels must the duty be charged, when
alt arrives in England?

Ans. 2719.368 *English malt bushels.*

A Distiller in Waterford, ships for Bristol, 25 casks
iskey, containing 2864 gallons, according to the
in Ireland; for how many gallons must the duty
rged, when the liquor arrives in England?

Ans. 2697.6016 *English wine gallons.*

A Brewer in Cork, ships for Milford, 56 barrels of
ch containing 32 gallons, according to the gauge in
d; for how many barrels of 36 gallons each, must
ity be charged, when the liquor arrives in England?
Ans. 38 barrels, 14.7 *gallons, English ale measure.*

PROBLEM IV.

To reduce English measure to Irish measure.

RULE.

ltiply the given number of bushels, English measure,
373; and the product will be the number of bushels,
measure. Also, multiply the given number of gal-
ale measure, by 1.2959; and the given number of
s, wine measure, by 1.0615; and the respective
cts will be the number of gallons, Irish measure.

. The multipliers given in the preceding Rule, are found by
g the English by the Irish square divisors.

EXAMPLES.

In 198 English, how many Irish bushels?
re $.9873 \times 198 = 195.4854$ *Irish malt bushels.*

2. In 2685 English ale gallons, how many gallons, Irish measure? *Ans.* 3479.4915 *gallons.*

3. In 3634 English wine gallons, how many gallons, Irish measure? *Ans.* 3857.491 *gallons.*

Remark.

All the directions and Rules, given in Part VI., for gauging, fixing, and inching the Utensils of Victuallers, Common Brewers, Maltsters, and Distillers, may be applied with the same success in gauging, fixing, and inching similar vessels, according to the Irish measures; care however, must always be taken to use the Irish divisors, in order that the areas and contents of the vessels may be obtained in Irish liquid gallons, and malt bushels.

SPECIFIC GRAVITY.

PRELIMINARY OBSERVATIONS.

Specific Gravity is the relative weight or gravity of any body or substance, considered with regard to some other body which is assumed as a standard of comparison; and this standard, by universal consent and practice, is rain-water, in consequence of its being less subject to variation than any other body. By a very fortunate coincidence, it happens that a cubic foot of rain-water weighs exactly 1000 ounces avoirdupois; consequently, assuming this as the specific gravity of rain-water, and comparing all other bodies with it, the same numbers that express the specific gravities of bodies, will also express the weight of a cubic foot of each body, in avoirdupois ounces, which is a great convenience in numerical computations.

A Table of the Specific Gravities of Bodies.

Gold	19640	Chalk	1793
lard Gold	18888	Sand	1520
ɛ Silver	14000	Lignum vitæ	1333
	11325	Ebony	1331
Silver	11091	Coal	1250
lard Silver	10535	Pitch	1150
er...	9000	Mahogany	1063
Metal	8784	Box-wood	1030
Brass	8000	Sea-water	1030
	7850	Common water	1000
	7645	Oak	925
Iron	7425	Gunpowder, shaken ...	922
	7320	Logwood	913
l-stone	4930	Beech	852
iite	3000	Ash	800
ɔle	2700	Yew	797
s	2642	Apple-tree	793
t	2570	Plum-tree	785
;ol-stone	2510	Maple	755
land-stone	2496	Cherry-tree	715
-stone	2484	Pear-tree	661
‹shire-stone	2442	Cedar of Lebanon	613
	2160	Elm	600
ıd-stone	2143	Willow	585
ford-stone	2049	Fir	550
:k	2000	Poplar	383
ht earth	1984	Cork	240
y	1825	Atmospheric air	1.2

ote. The several sorts of wood, mentioned in the preceding Table, upposed to be dry.

A Table of the Specific Gravities of Liquids.

Milk, Beer, &c.		*Various Wines.*	
's and goat's milk	1036	Madeira	1038
wn beer	1034	Canary	1033
ı's milk	1032	Malaga	1022
ɜ beer	1023	Champagne	998
ler	1018	Port, red and white ...	997
of spirits	922	Bourdeaux, or Claret...	994
rit of wine or alcohol	833	Burgundy	992
;hly rectified ditto...	829	Xeres, or Sherry	992

Note. Madeira and Canary wines are imported from two islands of the same name, lying in the Atlantic ocean. Malaga and Sherry are brought from Malaga and Xeres, two towns situated in Granada and Andalusia, in Spain. Champagne, Claret, and Burgundy, are brought from France; and Port is brought from Oporto and Lisbon, in Portugal.

PROBLEM I.

To find the magnitude of a body from its weight.

RULE.

As the tabular specific gravity of the body,
Is to its weight in avoirdupois ounces;
So is one cubic foot, or 1728 cubic inches,
To its magnitude in feet, or inches, respectively.

EXAMPLES.

1. A quantity of Madeira wine weighs $94\frac{1}{2}$ pounds avoirdupois; required its content in wine gallons.

$$\begin{array}{lrcrcr}
 & \textit{oz.} & & \textit{lb. oz.} & & \textit{cu. in.} \\
\textit{As} & 1038 & : & 94\ 8 & :: & 1728 \\
 & & & \underline{16} & & \underline{1512} \\
 & & & \underline{572} & & \underline{3456} \\
 & & & \underline{94} & & 1728 \\
 & & & \underline{1512} & & 8640 \\
 & & & & & \underline{1728}
\end{array}$$

Divisor $1038)2612736(2517$ *cubic inches.*

And $2517 \div 231 = 10.89$, *the content in wine gallons.*

2. A cask of proof spirits weighs 186 pounds 12 ounces avoirdupois; what is its content in wine gallons; deducting 45 pounds 9 ounces for the weight of the cask?

Ans. 18.32 *wine gallons.*

3. A piece of carved mahogany weighs 58lb. 10 oz. avoirdupois; what is its content in cubic inches?

Ans. 1524 *cubic inches.*

4. An empty cask weighs 56 pounds 9 ounces; required the content of the materials, in cubic inches; the cask being made of dry oak.

Ans. 1690 *cubic inches.*

PROBLEM II.

find the weight of a body from its magnitude.

RULE.

s one cubic foot, or 1728 cubic inches,
; to the content of the body;
o is its tabular specific gravity,
o the weight of the body.

EXAMPLES.

Vhat is the weight of 146 gallons of Port wine?

```
      cu. in.  gal.     cu. in.    oz.
  As 1728  :  146  ×  231  ::  997
                      146
                     ----
                     1386
                     924
                    231
                    -----
                    33726
                      997
                   ------
                   236082
                  303534
                 303534
 Divisor 1728)33624822(19458 ounces.
```

d 19458 *oz.* = 10 *cwt.* 3 *qr.* 12*lb.* 2 *oz.* *the weight* *ed.*

Required the weight of 36 gallons of brown beer.
Ans. 3 *cwt.* 1 *qr.* 15 *lb.* 10 *oz.*

Required the weight of a block of marble, whose h is 63 feet, and its breadth and thickness each 12 these being the dimensions of one of the stones in alls of Balbec.
s. 683.4375 *tons, which is nearly equal to the burthen East India ship.*

e. Balbec is a town of Syria, situated about 37 miles north of scus. Its ruins are stupendously grand. It is supposed to have ounded before the Christian æra.

The Method of Estimating

THE TONNAGE OF SHIPS.

PRELIMINARY OBSERVATIONS.

If the number of cubic feet of water which a ship displaces, in sinking from the *light water-mark* to the *load water-mark*, be divided by 35, (the number of cubic feet of sea-water in one ton,) the quotient will be the number of tons which the vessel is capable of carrying, or the vessel's true burthen.

Now the number of cubic feet of water so displaced, is exactly equal to the solid content of so much of the body of the ship, as is contained between the said two *water-lines* or *marks*; but in consequence of the great variety of forms given to this part of the vessel, no exact practical Rule can be given that will apply in all cases.

If indeed the areas of three or five horizontal sections of that part of the hull contained between the said two water-lines, could be obtained by means of equidistant ordinates, then the content might be correctly found by Prob. XIX., or XX., Part V.; but as the taking of the dimensions of these sections would be attended with great difficulty, we shall give the following Parliamentary Rules for ascertaining the tonnage of Merchants' and King's ships.

CASE I.

When the vessel is laid dry.

RULE.

Measure the length on a straight line along the rabbit of the keel of the ship, from the back of the main stern-post, to a perpendicular line let fall from the fore-part of the main stem, under the bowsprit; from this length, subtract $\frac{3}{5}$ of the extreme breadth; and the remainder will be the length of the keel, for tonnage. The breadth must be taken from outside to outside of the plank, in the broadest part of the ship, whether above or below the main wales,

sive of all manner of doubling planks, or sheathing, nay be wrought upon the sides of the vessel; then, ply the length of the keel, in feet, by the breadth, this product by half the breadth; divide the last ict by 94; and the quotient will be the tonnage re- d.

EXAMPLES.

The length from the back of the stern-post to a line ll from the fore-part of the main stem, is 88 feet 6 s; the extreme breadth from outside to outside of the , 26 feet 6 inches; required the tonnage of the ship.

	Feet.	
Gross length	88.5	*of the keel.*
$26.5 \times \frac{3}{5} =$	15.9	*the deduction.*
True length	72.6	*difference.*
	26.5	*breadth of the beam.*
	3630	
	4356	
	1452	
	1923.90	*first product.*
	13.25	*half the breadth.*
	961950	
	384780	
	577170	
	192390	
	25491.6750	*second product.*

en $25491.675 \div 94 = 271.188$ *tons, the burthen or ge required.*

The length from the back of the stern-post to a line l from the fore-part of the main stem, is 108 feet 9 ; the extreme breadth from outside to outside of ank, 29 feet 6 inches; required the tonnage of the

Ans. 421.469 *tons.*

CASE II.

When the vessel is afloat.

RULE.

fall a plumb-line over the stern of the ship, and re the distance between this line and the aft part of

the stern-post, at the load water-mark; then measure from the top of the said plumb-line, in a parallel direction with the water, to a perpendicular point immediately over the load water-mark, at the fore-part of the main stem; from the last measured distance subtract the former; and the remainder will be the ship's extreme length. From this length, deduct 3 inches for every foot of the load-draught of water, for the rake abaft; and also $\frac{3}{5}$ of the ship's extreme breadth for the rake of the stem; and the remainder will be the true length of the keel, for tonnage. The extreme breadth must be measured, and the tonnage found as directed in the first Case.

EXAMPLE.

The true length of an eighty gun ship, after all deductions are made, in taking the dimensions, is 150 feet 9 inches; and the extreme breadth 50 feet 6 inches; required the tonnage of the vessel.

Ans. 2044.9478 *tons.*

Note 1. It is found by experience, that ships of war carry less; and most merchant-ships carry considerably more tonnage than they are rated at, by the preceding Rules.

2. Some Writers on this subject, divide by 100, instead of 94, for King's ships. On the same principles, the divisor for Merchants' ships should be decreased, perhaps to 90 or 92.

PART VII.

TABLES

OF THE

AREAS OF CIRCLES,

IN

Ale and Wine Gallons,

'O ALL DIAMETERS IN INCHES,

AND

INCHES AND TENTHS,

From 1 to 216 Inches.

Also,

A TABLE

educing Ale Gallons to Victuallers Barrels.

Of the Areas of Circles in Ale Gallons.

Diams. in Inches.	0	.1	.2	.3	.4
1	0.0027	0.0033	0.0040	0.0047	0.0054
2	0.0111	0.0122	0.0134	0.0147	0.0160
3	0.0250	0.0267	0.0285	0.0303	0.0321
4	0.0445	0.0468	0.0491	0.0514	0.0539
5	0.0696	0.0724	0.0753	0.0782	0.0812
6	0.1002	0.1036	0.1070	0.1105	0.1140
7	0.1364	0.1405	0.1443	0.1484	0.1525
8	0.1782	0.1827	0.1872	0.1918	0.1965
9	0.2255	0.2306	0.2357	0.2408	0.2460
10	0.2785	0.2841	0.2897	0.2954	0.3012
11	0.3369	0.3431	0.3493	0.3556	0.3619
12	0.4010	0.4077	0.4145	0.4213	0.4282
13	0.4706	0.4779	0.4852	0.4926	0.5000
14	0.5458	0.5537	0.5615	0.5695	0.5775
15	0.6266	0.6350	0.6434	0.6519	0.6605
16	0.7129	0.7219	0.7309	0.7399	0.7490
17	0.8048	0.8143	0.8239	0.8335	0.8432
18	0.9023	0.9124	0.9225	0.9327	0.9429
19	1.0054	1.0160	1.0266	1.0374	1.0482
20	1.1140	1.1252	1.1364	1.1477	1.1590
21	1.2282	1.2399	1.2517	1.2635	1.2754
22	1.3479	1.3602	1.3726	1.3850	1.3974
23	1.4733	1.4861	1.4990	1.5120	1.5250
24	1.6042	1.6176	1.6310	1.6445	1.6581
25	1.7406	1.7546	1.7686	1.7827	1.7968
26	1.8827	1.8972	1.9118	1.9264	1.9411
27	2.0308	2.0454	2.0605	2.0757	2.0909
28	2.1835	2.1991	2.2148	2.2305	2.2463
29	2.3422	2.3584	2.3746	2.3909	2.4073
30	2.5065	2.5233	2.5401	2.5569	2.5738
31	2.6764	2.6937	2.7111	2.7285	2.7459
32	2.8519	2.8698	2.8877	2.9056	2.9236
33	3.0330	3.0513	3.0698	3.0884	3.1069
34	3.2196	3.2385	3.2575	3.2766	3.2957
35	3.4117	3.4312	3.4508	3.4704	3.4901
36	3.6095	3.6295	3.6497	3.6698	3.6901

A TABLE

Of the Areas of Circles in Ale Gallons.

Diams. in Inches.	.5	.6	.7	.8	.9
1	0.0062	0.0071	0.0080	0.0090	0.0100
2	0.0174	0.0188	0.0203	0.0218	0.0234
3	0.0341	0.0360	0.0381	0.0402	0.0423
4	0.0564	0.0589	0.0615	0.0641	0.0668
5	0.0842	0.0873	0.0904	0.0936	0.0969
6	0.1176	0.1213	0.1250	0.1287	0.1325
7	0.1566	0.1608	0.1651	0.1694	0.1738
8	0.2012	0.2059	0.2108	0.2156	0.2206
9	0.2513	0.2566	0.2620	0.2674	0.2729
10	0.3070	0.3129	0.3188	0.3248	0.3308
11	0.3683	0.3747	0.3812	0.3877	0.3943
12	0.4351	0.4421	0.4492	0.4563	0.4634
13	0.5075	0.5151	0.5227	0.5303	0.5381
14	0.5855	0.5936	0.6018	0.6100	0.6183
15	0.6691	0.6777	0.6864	0.6952	0.7041
16	0.7582	0.7674	0.7767	0.7860	0.7954
17	0.8529	0.8627	0.8725	0.8824	0.8923
18	0.9532	0.9635	0.9739	0.9843	0.9948
19	1.0590	1.0699	1.0808	1.0918	1.1029
20	1.1704	1.1818	1.1933	1.2049	1.2165
21	1.2874	1.2994	1.3114	1.3235	1.3357
22	1.4099	1.4225	1.4351	1.4478	1.4605
23	1.5380	1.5511	1.5643	1.5775	1.5908
24	1.6717	1.6854	1.6991	1.7129	1.7267
25	1.8110	1.8252	1.8395	1.8538	1.8682
26	1.9558	1.9706	1.9854	2.0003	2.0153
27	2.1062	2.1215	2.1369	2.1524	2.1679
28	2.2621	2.2781	2.2940	2.3100	2.3261
29	2.4237	2.4401	2.4567	2.4732	2.4899
30	2.5908	2.6078	2.6249	2.6420	2.6592
31	2.7635	2.7810	2.7987	2.8164	2.8341
32	2.9417	2.9598	2.9780	2.9963	3.0146
33	3.1255	3.1442	3.1630	3.1818	3.2006
34	3.3149	3.3342	3.3535	3.3728	3.3922
35	3.5099	3.5297	3.5495	3.5694	3.5894
36	3.7104	3.7308	3.7512	3.7716	3.7922

Of the Areas of Circles in Ale Gallons.

Diams. in Inches.	0	.1	.2	.3	.4
37	3.8128	3.8334	3.8541	3.8748	3.8956
38	4.0216	4.0428	4.0641	4.0854	4.1067
39	4.2361	4.2578	4.2796	4.3015	4.3234
40	4.4561	4.4784	4.5008	4.5232	4.5457
41	4.6817	4.7046	4.7275	4.7505	4.7735
42	4.9129	4.9363	4.9598	4.9833	5.0069
43	5.1496	5.1736	5.1976	5.2217	5.2459
44	5.3919	5.4164	5.4410	5.4657	5.4904
45	5.6398	5.6649	5.6900	5.7152	5.7405
46	5.8932	5.9189	5.9446	5.9703	5.9962
47	6.1522	6.1784	6.2047	6.2310	6.2574
48	6.4168	6.4436	6.4704	6.4973	6.5242
49	6.6870	6.7143	6.7417	6.7691	6.7966
50	6.9627	6.9906	7.0185	7.0465	7.0745
51	7.2440	7.2724	7.3009	7.3295	7.3581
52	7.5309	7.5599	7.5889	7.6180	7.6472
53	7.8233	7.8528	7.8825	7.9121	7.9418
54	8.1213	8.1514	8.1816	8.2118	8.2421
55	8.4249	8.4555	8.4863	8.5170	8.5479
56	8.7340	8.7652	8.7965	8.8279	8.8592
57	9.0487	9.0805	9.1124	9.1442	9.1762
58	9.3690	9.4014	9.4338	9.4662	9.4987
59	9.6949	9.7278	9.7607	9.7937	9.8268
60	10.0263	10.0598	10.0933	10.1268	10.1604
61	10.3683	10.3973	10.4314	10.4655	10.4997
62	10.7059	10.7404	10.7751	10.8097	10.8445
63	11.0540	11.0891	11.1243	11.1595	11.1948
64	11.4077	11.4434	11.4791	11.5149	11.5508
65	11.7670	11.8032	11.8396	11.8759	11.9123
66	12.1318	12.1686	12.2055	12.2424	12.2793
67	12.5023	12.5396	12.5770	12.6145	12.6520
68	12.8783	12.9162	12.9541	12.9921	13.0302
69	13.2598	13.2983	13.3368	13.3754	13.4140
70	13.6469	13.6860	13.7250	13.7642	13.8034
71	14.0396	14.0792	14.1188	14.1585	14.1983
72	14.4379	14.4780	14.5182	14.5585	14.5988

Of the Areas of Circles in Ale Gallons.

Diams. in Inches.	.5	.6	.7	.8	.9
37	3.9165	3.9374	3.9584	3.9794	4.0005
38	4.1282	4.1496	4.1712	4.1928	4.2144
39	4.3454	4.3674	4.3895	4.4117	4.4339
40	4.5682	4.5908	4.6134	4.6361	4.6589
41	4.7966	4.8197	4.8429	4.8662	4.8895
42	5.0305	5.0542	5.0780	5.1018	5.1257
43	5.2701	5.2943	5.3186	5.3430	5.3674
44	5.5151	5.5400	5.5648	5.5898	5.6147
45	5.7658	5.7912	5.8166	5.8421	5.8676
46	6.0220	6.0480	6.0739	6.1000	6.1261
47	6.2838	6.3103	6.3369	6.3635	6.3901
48	6.5512	6.5782	6.6053	6.6325	6.6597
49	6.8241	6.8517	6.8794	6.9071	6.9349
50	7.1027	7.1308	7.1590	7.1873	7.2156
51	7.3867	7.4154	7.4442	7.4730	7.5019
52	7.6764	7.7057	7.7350	7.7644	7.7938
53	7.9716	8.0014	8.0313	8.0613	8.0913
54	8.2724	8.3028	8.3332	8.3638	8.3943
55	8.5788	8.6097	8.6407	8.6717	8.7029
56	8.8907	8.9222	8.9537	8.9854	9.0170
57	9.2082	9.2402	9.2724	9.3046	9.3367
58	9.5313	9.5639	9.5965	9.6293	9.6620
59	9.8599	9.8931	9.9263	9.9596	9.9929
60	10.1941	10.2278	10.2616	10.2955	10.3294
61	10.5339	10.5682	10.6025	10.6369	10.6714
62	10.8792	10.9141	10.9490	10.9839	11.0189
63	11.2302	11.2656	11.3010	11.3365	11.3721
64	11.5867	11.6226	11.6586	11.6947	11.7308
65	11.9487	11.9852	12.0218	12.0584	12.0951
66	12.3164	12.3534	12.3906	12.4277	12.4650
67	12.6896	12.7272	12.7649	12.8026	12.8404
68	13.0683	13.1065	13.1448	13.1831	13.2214
69	13.4527	13.4914	13.5302	13.5691	13.6080
70	13.8426	13.8819	13.9212	13.9607	14.0001
71	14.2381	14.2779	14.3178	14.3578	14.3978
72	14.6391	14.6795	14.7200	14.7605	14.8011

Of the Areas of Circles in Ale Gallons.

Diams. in Inches.	0	.1	.2	.3	.4
1	0.0027	0.0033	0.0040	0.0047	0.0054
2	0.0111	0.0122	0.0134	0.0147	0.0160
3	0.0250	0.0267	0.0285	0.0303	0.0321
4	0.0445	0.0468	0.0491	0.0514	0.0539
5	0.0696	0.0724	0.0753	0.0782	0.0812
6	0.1002	0.1036	0.1070	0.1105	0.1140
7	0.1364	0.1403	0.1443	0.1484	0.1525
8	0.1782	0.1827	0.1872	0.1918	0.1965
9	0.2255	0.2306	0.2357	0.2408	0.2460
10	0.2785	0.2841	0.2897	0.2954	0.3012
11	0.3369	0.3431	0.3493	0.3556	0.3619
12	0.4010	0.4077	0.4145	0.4213	0.4282
13	0.4706	0.4779	0.4852	0.4926	0.5000
14	0.5458	0.5537	0.5615	0.5695	0.5775
15	0.6266	0.6350	0.6434	0.6519	0.6605
16	0.7129	0.7219	0.7309	0.7399	0.7490
17	0.8048	0.8143	0.8239	0.8335	0.8432
18	0.9023	0.9124	0.9225	0.9327	0.9429
19	1.0054	1.0160	1.0266	1.0374	1.0482
20	1.1140	1.1252	1.1364	1.1477	1.1590
21	1.2282	1.2399	1.2517	1.2635	1.2754
22	1.3479	1.3602	1.3726	1.3850	1.3974
23	1.4733	1.4861	1.4990	1.5120	1.5250
24	1.6042	1.6176	1.6310	1.6445	1.6581
25	1.7406	1.7546	1.7686	1.7827	1.7968
26	1.8827	1.8972	1.9118	1.9264	1.9411
27	2.0303	2.0454	2.0605	2.0757	2.0909
28	2.1835	2.1991	2.2148	2.2305	2.2463
29	2.3422	2.3584	2.3746	2.3909	2.4073
30	2.5065	2.5233	2.5401	2.5569	2.5738
31	2.6764	2.6937	2.7111	2.7285	2.7459
32	2.8519	2.8698	2.8877	2.9056	2.9236
33	3.0330	3.0513	3.0698	3.0884	3.1069
34	3.2196	3.2385	3.2575	3.2766	3.2957
35	3.4117	3.4312	3.4508	3.4704	3.4901
36	3.6095	3.6295	3.6497	3.6698	3.6901

Of the Areas of Circles in Ale Gallons.

Diams. in Inches.	.5	.6	.7	.8	.9
1	0.0062	0.0071	0.0080	0.0090	0.0100
2	0.0174	0.0188	0.0203	0.0218	0.0234
3	0.0341	0.0360	0.0381	0.0402	0.0423
4	0.0563	0.0589	0.0615	0.0641	0.0668
5	0.0842	0.0873	0.0904	0.0936	0.0969
6	0.1176	0.1213	0.1250	0.1287	0.1325
7	0.1566	0.1608	0.1651	0.1694	0.1738
8	0.2012	0.2059	0.2108	0.2156	0.2206
9	0.2513	0.2566	0.2620	0.2674	0.2729
10	0.3070	0.3129	0.3188	0.3248	0.3308
11	0.3683	0.3747	0.3812	0.3877	0.3943
12	0.4351	0.4421	0.4492	0.4563	0.4634
13	0.5076	0.5151	0.5227	0.5303	0.5381
14	0.5855	0.5936	0.6018	0.6100	0.6183
15	0.6691	0.6777	0.6864	0.6952	0.7041
16	0.7582	0.7674	0.7767	0.7860	0.7954
17	0.8529	0.8627	0.8725	0.8824	0.8923
18	0.9532	0.9635	0.9739	0.9843	0.9948
19	1.0590	1.0699	1.0808	1.0918	1.1029
20	1.1704	1.1818	1.1933	1.2049	1.2165
21	1.2874	1.2994	1.3114	1.3235	1.3357
22	1.4099	1.4225	1.4351	1.4478	1.4605
23	1.5380	1.5511	1.5643	1.5775	1.5908
24	1.6717	1.6854	1.6991	1.7129	1.7267
25	1.8110	1.8252	1.8395	1.8538	1.8682
26	1.9558	1.9706	1.9854	2.0003	2.0153
27	2.1062	2.1215	2.1369	2.1524	2.1679
28	2.2621	2.2781	2.2940	2.3100	2.3261
29	2.4237	2.4401	2.4567	2.4732	2.4899
30	2.5908	2.6078	2.6249	2.6420	2.6592
31	2.7635	2.7810	2.7987	2.8164	2.8341
32	2.9417	2.9598	2.9780	2.9963	3.0146
33	3.1255	3.1442	3.1630	3.1818	3.2006
34	3.3149	3.3342	3.3535	3.3728	3.3922
35	3.5099	3.5297	3.5495	3.5694	3.5894
36	3.7104	3.7308	3.7512	3.7716	3.7922

Of the Areas of Circles in Ale Gallons.

Diams. in Inches.	.0	.1	.2	.3	.4
37	3.8128	3.8334	3.8541	3.8748	3.8956
38	4.0216	4.0428	4.0641	4.0854	4.1067
39	4.2361	4.2578	4.2796	4.3015	4.3234
40	4.4561	4.4784	4.5008	4.5232	4.5457
41	4.6817	4.7046	4.7275	4.7505	4.7735
42	4.9129	4.9363	4.9598	4.9833	5.0069
43	5.1496	5.1736	5.1976	5.2217	5.2459
44	5.3919	5.4164	5.4410	5.4657	5.4904
45	5.6398	5.6649	5.6900	5.7152	5.7405
46	5.8932	5.9189	5.9446	5.9703	5.9962
47	6.1522	6.1784	6.2047	6.2310	6.2574
48	6.4168	6.4436	6.4704	6.4973	6.5242
49	6.6870	6.7143	6.7417	6.7691	6.7966
50	6.9627	6.9906	7.0185	7.0465	7.0745
51	7.2440	7.2724	7.3009	7.3295	7.3581
52	7.5309	7.5599	7.5889	7.6180	7.6472
53	7.8233	7.8528	7.8825	7.9121	7.9418
54	8.1213	8.1514	8.1816	8.2118	8.2421
55	8.4249	8.4555	8.4863	8.5170	8.5479
56	8.7340	8.7652	8.7965	8.8279	8.8592
57	9.0487	9.0805	9.1124	9.1442	9.1762
58	9.3690	9.4014	9.4338	9.4662	9.4987
59	9.6949	9.7278	9.7607	9.7937	9.8268
60	10.0263	10.0598	10.0933	10.1268	10.1604
61	10.3633	10.3973	10.4314	10.4655	10.4997
62	10.7059	10.7404	10.7751	10.8097	10.8445
63	11.0540	11.0891	11.1243	11.1595	11.1948
64	11.4077	11.4434	11.4791	11.5149	11.5508
65	11.7670	11.8032	11.8396	11.8759	11.9123
66	12.1318	12.1686	12.2055	12.2424	12.2793
67	12.5023	12.5396	12.5770	12.6145	12.6520
68	12.8783	12.9162	12.9541	12.9921	13.0302
69	13.2598	13.2983	13.3368	13.3754	13.4140
70	13.6469	13.6860	13.7250	13.7642	13.8034
71	14.0396	14.0792	14.1188	14.1585	14.1983
72	14.4379	14.4780	14.5182	14.5585	14.5988

Of the Areas of Circles in Ale Gallons.

Diams. in Inches.	.5	.6	.7	.8	.9
37	3.9165	3.9374	3.9584	3.9794	4.0005
38	4.1282	4.1496	4.1712	4.1928	4.2144
39	4.3454	4.3674	4.3895	4.4117	4.4339
40	4.5682	4.5908	4.6134	4.6361	4.6589
41	4.7966	4.8197	4.8429	4.8662	4.8895
42	5.0305	5.0542	5.0780	5.1018	5.1257
43	5.2701	5.2943	5.3186	5.3430	5.3674
44	5.5151	5.5400	5.5648	5.5898	5.6147
45	5.7658	5.7912	5.8166	5.8421	5.8676
46	6.0220	6.0480	6.0739	6.1000	6.1261
47	6.2838	6.3103	6.3369	6.3635	6.3901
48	6.5512	6.5782	6.6053	6.6325	6.6597
49	6.8241	6.8517	6.8794	6.9071	6.9349
50	7.1027	7.1308	7.1590	7.1873	7.2156
51	7.3867	7.4154	7.4442	7.4730	7.5019
52	7.6764	7.7057	7.7350	7.7644	7.7938
53	7.9716	8.0014	8.0313	8.0613	8.0913
54	8.2724	8.3028	8.3332	8.3638	8.3943
55	8.5788	8.6097	8.6407	8.6717	8.7029
56	8.8907	8.9222	8.9537	8.9854	9.0170
57	9.2082	9.2402	9.2724	9.3046	9.3367
58	9.5313	9.5639	9.5965	9.6293	9.6620
59	9.8599	9.8931	9.9263	9.9596	9.9929
60	10.1941	10.2278	10.2616	10.2955	10.3294
61	10.5339	10.5682	10.6025	10.6369	10.6714
62	10.8792	10.9141	10.9490	10.9839	11.0189
63	11.2302	11.2656	11.3010	11.3365	11.3721
64	11.5867	11.6226	11.6586	11.6947	11.7308
65	11.9487	11.9852	12.0218	12.0584	12.0951
66	12.3164	12.3534	12.3906	12.4277	12.4650
67	12.6896	12.7272	12.7649	12.8026	12.8404
68	13.0683	13.1065	13.1448	13.1831	13.2214
69	13.4527	13.4914	13.5302	13.5691	13.6080
70	13.8426	13.8819	13.9212	13.9607	14.0001
71	14.2381	14.2779	14.3178	14.3578	14.3978
72	14.6391	14.6795	14.7200	14.7605	14.8011

Of the Areas of Circles in Ale Gallons.

Diams. in Inches.	.0	.1	.2	.3	.4
73	14.8417	14.8824	14.9232	14.9640	15.0048
74	15.2512	15.2924	15.3337	15.3751	15.4165
75	15.6661	15.7079	15.7498	15.7917	15.8337
76	16.0867	16.1290	16.1715	16.2139	16.2565
77	16.5128	16.5557	16.5987	16.6417	16.6848
78	16.9445	16.9880	17.0315	17.0751	17.1187
79	17.3818	17.4258	17.4699	17.5140	17.5583
80	17.8246	17.8692	17.9138	17.9585	18.0033
81	18.2730	18.3181	18.3633	18.4086	18.4539
82	18.7270	18.7727	18.8184	18.8642	18.9101
83	19.1865	19.2328	19.2791	19.3255	19.3719
84	19.6516	19.6984	19.7453	19.7922	19.8392
85	20.1223	20.1697	20.2171	20.2646	20.3121
86	20.5985	20.6465	20.6945	20.7425	20.7906
87	21.0804	21.1289	21.1774	21.2260	21.2747
88	21.5678	21.6168	21.6659	21.7151	21.7643
89	22.0607	22.1103	22.1600	22.2097	22.2595
90	22.5593	22.6094	22.6596	22.7099	22.7602
91	23.0634	23.1141	23.1649	23.2157	23.2666
92	23.5730	23.6243	23.6750	23.7271	23.7785
93	24.0883	24.1401	24.1920	24.2439	24.2959
94	24.6091	24.6615	24.7139	24.7664	24.8190
95	25.1355	25.1884	25.2414	25.2945	25.3476
96	25.6674	25.7209	25.7745	25.8281	25.8818
97	26.2050	26.2590	26.3131	26.3673	26.4215
98	26.7481	26.8027	26.8573	26.9121	26.9668
99	27.2967	27.3519	27.4071	27.4624	27.5177
100	27.8510	27.9067	27.9625	28.0183	28.0742
101	28.4108	28.4670	28.5234	28.5798	28.6362
102	28.9761	29.0330	29.0899	29.1468	29.2039
103	29.5471	29.6045	29.6619	29.7194	29.7770
104	30.1236	30.1815	30.2396	30.2976	30.3558
105	30.7057	30.7642	30.8228	30.8814	30.9401
106	31.2933	31.3524	31.4115	31.4707	31.5300
107	31.8866	31.9462	32.0059	32.0656	32.1254
108	32.4854	32.5455	32.6058	32.6661	32.7264

A TABLE

Of the Areas of Circles in Ale Gallons.

Diams. in Inches.	.5	.6	.7	.8	.9
73	15.0458	15.0867	15.1277	15.1688	15.2100
74	15.4580	15.4995	15.5411	15.5827	15.6244
75	15.8757	15.9178	15.9599	16.0021	16.0444
76	16.2991	16.3417	16.3844	16.4271	16.4699
77	16.7280	16.7712	16.8144	16.8577	16.9011
78	17.1624	17.2062	17.2500	17.2939	17.3378
79	17.6025	17.6468	17.6912	17.7356	17.7801
80	18.0481	18.0930	18.1379	18.1829	18.2279
81	18.4993	18.5447	18.5902	18.6357	18.6813
82	18.9560	19.0020	19.0481	19.0941	19.1403
83	19.4184	19.4649	19.5115	19.5581	19.6049
84	19.8863	19.9384	19.9805	20.0277	20.0750
85	20.3597	20.4074	20.4551	20.5029	20.5507
86	20.8388	20.8870	20.9352	20.9836	21.0319
87	21.3234	21.3722	21.4210	21.4698	21.5188
88	21.8135	21.8629	21.9123	21.9617	22.0112
89	22.3093	22.3592	22.4091	22.4591	22.5092
90	22.8106	22.8611	22.9115	22.9621	23.0127
91	23.3175	23.3685	23.4195	23.4707	23.5218
92	23.8300	23.8815	23.9331	23.9848	24.0365
93	24.3480	24.4001	24.4523	24.5045	24.5568
94	24.8716	24.9243	24.9770	25.0298	25.0826
95	25.4008	25.4540	25.5073	25.5606	25.6140
96	25.9355	25.9893	26.0431	26.0970	26.1510
97	26.4758	26.5301	26.5845	26.6390	26.6935
98	27.0217	27.0766	27.1315	27.1865	27.2416
99	27.5731	27.6286	27.6841	27.7397	27.7953
100	28.1302	28.1862	28.2422	28.2983	28.3545
101	28.6927	28.7493	28.8059	28.8626	28.9193
102	29.2609	29.3180	29.3752	29.4324	29.4897
103	29.8346	29.8923	29.9501	30.0078	30.0657
104	30.4189	30.4722	30.5305	30.5888	30.6472
105	30.9988	31.0577	31.1165	31.1754	31.2343
106	31.5893	31.6486	31.7080	31.7675	31.8270
107	32.1858	32.2452	32.3051	32.3652	32.4252
108	32.7868	32.8473	32.9078	32.9684	33.0290

Of the Areas of Circles in Ale Gallons.

Diams. in Inches.	.0	.1	.2	.3	.4
109	33.0897	33.1505	33.2113	33.2721	33.3330
110	33.6997	33.7610	33.8223	33.8837	33.9452
111	34.3152	34.3770	34.4389	34.5009	34.5629
112	34.9362	34.9987	35.0611	35.1237	35.1862
113	35.5629	35.6259	35.6889	35.7520	35.8151
114	36.1951	36.2586	36.3222	36.3859	36.4496
115	36.8329	36.8970	36.9611	37.0253	37.0896
116	37.4763	37.5409	37.6056	37.6703	37.7352
117	38.1252	38.1904	38.2556	38.3209	38.3863
118	38.7797	38.8454	38.9113	38.9771	39.0430
119	39.4398	39.5061	39.5724	39.6389	39.7053
120	40.1054	40.1723	40.2392	40.3062	40.3732
121	40.7766	40.8440	40.9115	40.9790	41.0466
122	41.4534	41.5214	41.5894	41.6575	41.7256
123	42.1357	42.2043	42.2729	42.3415	42.4102
124	42.8237	42.8927	42.9619	43.0311	43.1004
125	43.5171	43.5868	43.6565	43.7263	43.7961
126	44.2162	44.2864	44.3567	44.4270	44.4974
127	44.9208	44.9916	45.0624	45.1333	45.2042
128	45.6310	45.7024	45.7737	45.8452	45.9167
129	46.3468	46.4187	46.4906	46.5626	46.6347
130	47.0681	47.1406	47.2131	47.2856	47.3582
131	47.7951	47.8680	47.9411	48.0142	48.0874
132	48.5275	48.6011	46.6747	48.7484	48.8221
133	49.2656	49.3397	49.4139	49.4881	49.5624
134	50.0092	50.0839	50.1586	50.2334	50.3082
135	50.7584	50.8336	50.9089	50.9842	51.0596
136	51.5182	51.5889	51.6648	51.7407	51.8166
137	52.2735	52.3498	52.4262	52.5027	52.5792
138	53.0394	53.1163	53.1932	53.2703	53.3473
139	53.8109	53.8883	53.9658	54.0434	54.1210
140	54.5879	54.6659	54.7440	54.8221	54.9003
141	56.3705	55.4491	55.5277	55.6064	55.6851
142	56.1587	56.2378	56.3170	56.3962	56.4755
143	56.9525	57.0321	57.1119	57.1917	57.2715
144	57.7518	57.8320	57.9123	57.9927	58.0731

Of the Areas of Circles in Ale Gallons.

Diams. in Inches.	.5	.6	.7	.8	.9
109	33.3940	33.4550	33.5161	33.5772	33.6384
110	34.0067	34.0683	34.1299	34.1916	34.2534
111	34.6250	34.6871	34.7493	34.8116	34.8739
112	35.2489	35.3116	35.3743	35.4371	35.5000
113	35.8783	35.9416	36.0049	36.0682	36.1316
114	36.5133	36.5771	36.6410	36.7049	36.7689
115	37.1539	37.2182	37.2827	37.3471	37.4117
116	37.8000	37.8649	37.9299	37.9950	38.0600
117	38.4517	38.5172	38.5827	38.6483	38.7140
118	39.1090	39.1751	39.2411	39.3073	39.3735
119	39.7719	39.8385	39.9051	39.9718	40.0386
120	40.4403	40.5074	40.5747	40.6419	40.7092
121	41.1143	41.1820	41.2498	41.3176	41.3854
122	41.7939	41.8621	41.9304	41.9988	42.0672
123	42.4790	42.5478	42.6167	42.6856	42.7546
124	43.1697	43.2391	43.3085	43.3780	43.4475
125	43.8660	43.9359	44.0059	44.0759	44.1460
126	44.5678	44.6383	44.7089	44.7795	44.8501
127	45.2752	45.3463	45.4174	45.4885	45.5598
128	45.9882	46.0598	46.1315	46.2032	46.2750
129	46.7068	46.7789	46.8512	46.9234	46.9958
130	47.4309	47.5036	47.5764	47.6492	47.7221
131	48.1606	48.2339	48.3072	48.3806	48.4540
132	48.8959	48.9697	49.0436	49.1175	49.1915
133	49.6367	49.7111	49.7855	49.8600	49.9346
134	50.3831	50.4581	50.5331	50.6081	50.6832
135	51.1351	51.2106	51.2861	51.3618	51.4374
136	51.8926	51.9687	52.0448	52.1210	52.1972
137	52.6557	52.7324	52.8090	52.8858	52.9626
138	53.4244	53.5016	53.5788	53.6561	53.7335
139	54.1987	54.2764	54.3542	54.4321	54.5100
140	54.9785	55.0568	55.1352	55.2136	55.2920
141	55.7639	55.8428	55.9217	56.0006	56.0796
142	56.5549	56.6343	56.7137	56.7933	56.8728
143	57.3514	57.4314	57.5114	57.5915	57.6716
144	58.1535	58.2341	58.3146	58.3953	58.4759

Of the Areas of Circles in Ale Gallons.

Diams. in Inches.	.0	.1	.2	.3	.4
145	58.5567	58.6375	58.7183	58.7992	58.8802
146	59.3672	59.4485	59.5299	59.6114	59.6929
147	60.1832	60.2651	60.3471	60.4291	60.5111
148	61.0048	61.0872	61.1698	61.2523	61.3350
149	61.8320	61.9150	61.9981	62.0812	62.1644
150	62.6647	62.7483	62.8319	62.9156	62.9994
151	63.5030	63.5872	63.6713	63.7556	63.8399
152	64.3469	64.4316	64.5163	64.6012	64.6860
153	65.1964	65.2816	65.3669	65.4523	65.5377
154	66.0514	66.1372	66.1281	66.3090	66.3950
155	66.9120	66.9983	67.0848	67.1712	67.2578
156	67.7781	67.8651	67.9520	68.0391	68.1262
157	68.6499	68.7374	68.8249	68.9125	69.0001
158	69.5272	69.6152	69.7033	69.7915	69.8797
159	70.4101	70.4987	70.5873	70.6760	70.7648
160	71.2985	71.3877	71.4769	71.5661	71.6554
161	72.1925	72.2822	72.3720	72.4618	72.5517
162	73.0921	73.1824	73.2727	73.3631	73.4535
163	73.9973	74.0881	74.1790	74.2699	74.3609
164	74.9080	74.9994	75.0908	75.1823	75.2739
165	75.8243	75.9162	76.0082	76.1003	76.1924
166	76.7462	76.8387	76.9312	77.0238	77.1165
167	77.6736	77.7667	77.8598	77.9529	78.0461
168	78.6066	78.7002	78.7939	78.8876	78.9814
169	79.5452	79.6394	79.7336	79.8279	79.9222
170	80.4893	80.5841	80.6788	80.7737	80.8686
171	81.4391	81.5343	81.6297	81.7251	81.8205
172	82.3943	82.4902	82.5861	82.6820	82.7780
173	83.3552	83.4516	83.5480	83.6446	83.7411
174	84.3216	84.4186	84.5156	84.6127	84.7098
175	85.2986	85.3911	85.4887	85.5863	85.6840
176	86.2712	86.3693	86.4674	86.5656	86.6637
177	87.2543	87.3530	87.4516	87.5504	87.6492
178	88.2431	88.3422	88.4415	88.5408	88.6401
179	89.2373	89.3371	89.4369	89.5367	89.6366
180	90.2372	90.3375	90.4378	90.5382	90.6387

A TABLE

Of the Areas of Circles in Ale Gallons.

Diams. in Inches.	.5	.6	.7	.8	.9
145	58.9612	59.0423	59.1234	59.2046	59.2858
146	59.7745	59.8561	59.9378	60.0195	60.1013
147	60.5932	60.6755	60.7577	60.8400	60.9224
148	61.4177	61.5004	61.5832	61.6661	61.7490
149	62.2476	62.3309	62.4143	62.4977	62.5812
150	63.0832	63.1670	63.2509	63.3349	63.4189
151	63.9243	64.0087	64.0931	64.1777	64.2623
152	64.7709	64.8559	64.9409	65.0260	65.1112
153	65.6232	65.7087	65.7943	65.8799	65.9656
154	66.4810	66.5671	66.6532	66.7394	66.8257
155	67.3444	67.4310	67.5177	67.6045	67.6913
156	68.2133	68.3005	68.3878	68.4751	68.5625
157	69.0878	69.1756	69.2634	69.3513	69.4392
158	69.9679	70.0562	70.1446	70.2330	70.3215
159	70.8536	70.9425	71.0314	71.1204	71.2094
160	71.7448	71.8343	71.9237	72.0133	72.1029
161	72.6416	72.7316	72.8217	72.9118	73.0019
162	73.5440	73.6345	73.7251	73.8158	73.9065
163	74.4519	74.5430	74.6342	74.7254	74.8167
164	75.3655	75.4571	75.5488	75.6406	75.7324
165	76.2845	76.3767	76.4690	76.5613	76.6537
166	77.2092	77.3020	77.3948	77.4877	77.5806
167	78.1391	78.2327	78.3261	78.4196	78.5131
168	79.0752	79.1691	79.2630	79.3570	79.4511
169	80.0166	80.1110	80.2055	80.3001	80.3947
170	80.9635	81.0585	81.1536	81.2487	81.3438
171	81.9160	82.0116	82.1072	82.2028	82.2986
172	82.8741	82.9702	83.0664	83.1626	83.2589
173	83.8377	83.9344	84.0311	84.1279	84.2247
174	84.8069	84.9042	85.0015	85.0988	85.1962
175	85.7817	85.8795	85.9774	86.0752	86.1732
176	86.7621	86.8604	86.9588	87.0573	87.1558
177	87.7480	87.8469	87.9459	88.0449	88.1439
178	88.7395	88.8390	88.9385	89.0380	89.1377
179	89.7366	89.8366	89.9366	90.0368	90.1370
180	90.7392	90.8398	90.9404	91.0411	91.1418

Of the Areas of Circles in Wine Gallons.

Diams. in Inches.	.0	.1	.2	.3	.4
37	4.6546	4.6797	4.7050	4.7303	4.7557
38	4.9096	4.9354	4.9614	4.9874	5.0135
39	5.1714	5.1979	5.2245	5.2512	5.2780
40	5.4400	5.4672	5.4945	5.5219	5.5493
41	5.7154	5.7433	5.7712	5.7993	5.8274
42	5.9976	6.0261	6.0548	6.0835	6.1123
43	6.2866	6.3158	6.3452	6.3746	6.4041
44	6.5824	6.6123	6.6423	6.6724	6.7026
45	6.8850	6.9156	6.9463	6.9771	7.0079
46	7.1944	7.2257	7.2570	7.2885	7.3200
47	7.5106	7.5425	7.5746	7.6067	7.6389
48	7.8336	7.8662	7.8990	7.9318	7.9647
49	8.1634	8.1967	8.2301	8.2636	8.2972
50	8.5000	8.5340	8.5681	8.6023	8.6365
51	8.8434	8.8781	8.9128	8.9477	8.9826
52	9.1936	9.2289	9.2644	9.2999	9.3355
53	9.5506	9.5866	9.6228	9.6590	9.6953
54	9.9144	9.9511	9.9879	10.0248	10.0618
55	10.2850	10.3224	10.3599	10.3975	10.4351
56	10.6624	10.7005	10.7386	10.7769	10.8152
57	11.0466	11.0853	11.1242	11.1631	11.2021
58	11.4376	11.4770	11.5166	11.5562	11.5959
59	11.8354	11.8755	11.9157	11.9560	11.9964
60	12.2400	12.2808	12.3217	12.3627	12.4037
61	12.6514	12.6929	12.7344	12.7761	12.8178
62	13.0696	13.1117	13.1540	13.1963	13.2387
63	13.4946	13.5374	13.5804	13.6234	13.6665
64	13.9264	13.9699	14.0135	14.0572	14.1010
65	14.3650	14.4092	14.4535	14.4979	14.5423
66	14.8104	14.8553	14.9002	14.9453	14.9904
67	15.2626	15.3081	15.3538	15.3995	15.4453
68	15.7216	15.7678	15.8142	15.8606	15.9071
69	16.1874	16.2343	16.2813	16.3284	16.3756
70	16.6600	16.7076	16.7553	16.8031	16.8509
71	17.1394	17.1877	17.2360	17.2845	17.3330
72	17.6256	17.6745	17.7236	17.7727	17.8219

Of the Areas of Circles in Ale Gallons.

Diams. in Inches.	.5	.6	.7	.8	.9
181	91.7474	91.8485	91.9497	92.0510	92.1523
182	92.7612	92.8629	92.9646	93.0664	93.1683
183	93.7805	93.8828	93.9851	94.0874	94.1898
184	94.8055	94.9082	95.0111	95.1140	95.2170
185	95.8359	95.9393	96.0427	96.1462	96.2497
186	96.8720	96.9759	97.0799	97.1839	97.2880
187	97.9136	98.0181	98.1226	98.2272	98.3318
188	98.9608	99.0658	99.1709	99.2761	99.3813
189	100.0136	100.1192	100.2248	100.3305	100.4363
190	101.0719	101.1781	101.2843	101.3905	101.4968
191	102.1358	102.2425	102.3493	102.4561	102.5630
192	103.2053	103.3126	103.4199	103.5272	103.6347
193	104.2804	104.3882	104.4960	104.6040	104.7119
194	105.3610	105.4693	105.5778	105.6863	105.7948
195	106.4472	106.5561	106.6651	106.7741	106.8832
196	107.5389	107.6484	107.7579	107.8675	107.9772
197	108.6363	108.7463	108.8564	108.9665	109.0767
198	109.7392	109.8498	109.9604	110.0711	110.1819
199	110.8476	110.9588	111.0700	111.1813	111.2926
200	111.9617	112.0734	112.1851	112.2970	112.4088
201	113.0813	113.1985	113.3059	113.4182	113.5307
202	114.2065	114.3193	114.4322	114.5451	114.6581
203	115.3372	115.4506	115.5640	115.6775	115.7911
204	116.4735	116.5875	116.7015	116.8155	116.9296
205	117.6154	117.7299	117.8445	117.9591	118.0737
206	118.7629	118.8779	118.9930	119.1082	119.2234
207	119.9159	120.0315	120.1472	120.2629	120.3787
208	121.0745	121.1907	121.3069	121.4232	121.5395
209	122.2387	122.3554	122.4722	122.5890	122.7059
210	123.4084	123.5257	123.6430	123.7604	123.8779
211	124.5837	124.7016	124.8195	124.9374	125.0554
212	125.7646	125.8830	126.0015	126.1200	126.2385
213	126.9511	127.0700	127.1890	127.3081	127.4272
214	128.1431	128.2626	128.3822	128.5018	128.6215
215	129.3407	129.4608	129.5809	129.7011	129.8213

Of the Areas of Circles in Wine Gallons.

Diams. in Inches.	.0	.1	.2	.3	.4
1	0.0034	0.0041	0.0049	0.0057	0.0066
2	0.0136	0.0149	0.0164	0.0179	0.0195
3	0.0306	0.0326	0.0348	0.0370	0.0393
4	0.0544	0.0571	0.0599	0.0628	0.0658
5	0.0850	0.0884	0.0919	0.0955	0.0991
6	0.1224	0.1265	0.1306	0.1349	0.1392
7	0.1666	0.1713	0.1762	0.1811	0.1861
8	0.2176	0.2230	0.2286	0.2342	0.2399
9	0.2754	0.2815	0.2877	0.2940	0.3004
10	0.3400	0.3468	0.3537	0.3607	0.3677
11	0.4114	0.4189	0.4264	0.4341	0.4418
12	0.4896	0.4977	0.5060	0.5143	0.5227
13	0.5746	0.5834	0.5924	0.6014	0.6105
14	0.6664	0.6759	0.6855	0.6952	0.7050
15	0.7650	0.7752	0.7855	0.7959	0.8063
16	0.8704	0.8813	0.8922	0.9033	0.9144
17	0.9826	0.9941	1.0058	1.0175	1.0293
18	1.1016	1.1188	1.1262	1.1386	1.1511
19	1.2274	1.2403	1.2533	1.2664	1.2796
20	1.3600	1.3736	1.3873	1.4011	1.4149
21	1.4994	1.5137	1.5280	1.5425	1.5570
22	1.6456	1.6605	1.6756	1.6907	1.7059
23	1.7986	1.8142	1.8300	1.8458	1.8617
24	1.9584	1.9747	1.9911	2.0076	2.0242
25	2.1250	2.1420	2.1591	2.1763	2.1935
26	2.2984	2.3161	2.3338	2.3517	2.3696
27	2.4786	2.4969	2.5154	2.5339	2.5525
28	2.6656	2.6846	2.7038	2.7230	2.7423
29	2.8594	2.8791	2.8989	2.9188	2.9388
30	3.0600	3.0804	3.1009	3.1215	3.1421
31	3.2674	3.2885	3.3096	3.3309	3.3522
32	3.4816	3.5034	3.5252	3.5471	3.5691
33	3.7026	3.7250	3.7476	3.7702	3.7929
34	3.9304	3.9535	3.9767	4.0000	4.0234
35	4.1650	4.1888	4.2127	4.2367	4.2607
36	4.4064	4.4309	4.4554	4.4801	4.5048

Of the Areas of Circles in Wine Gallons.

Diams. in Inches.	.5	.6	.7	.8	.9
1	0.0076	0.0087	0.0098	0.0110	0.0122
2	0.0212	0.0229	0.0247	0.0266	0.0285
3	0.0416	0.0440	0.0465	0.0490	0.0517
4	0.0688	0.0719	0.0751	0.0783	0.0816
5	0.1028	0.1066	0.1104	0.1143	0.1183
6	0.1436	0.1481	0.1526	0.1572	0.1618
7	0.1912	0.1963	0.2015	0.2068	0.2121
8	0.2456	0.2514	0.2573	0.2632	0.2693
9	0.3068	0.3133	0.3199	0.3265	0.3332
10	0.3748	0.3820	0.3892	0.3965	0.4039
11	0.4496	0.4575	0.4654	0.4734	0.4814
12	0.5312	0.5397	0.5483	0.5570	0.5657
13	0.6196	0.6288	0.6381	0.6474	0.6569
14	0.7148	0.7247	0.7347	0.7447	0.7548
15	0.8168	0.8274	0.8380	0.8487	0.8595
16	0.9256	0.9369	0.9482	0.9596	0.9710
17	1.0412	1.0531	1.0651	1.0772	1.0893
18	1.1636	1.1762	1.1889	1.2016	1.2145
19	1.2928	1.3061	1.3195	1.3329	1.3464
20	1.4288	1.4428	1.4568	1.4709	1.4851
21	1.5716	1.5863	1.6010	1.6158	1.6306
22	1.7212	1.7365	1.7519	1.7674	1.7829
23	1.8776	1.8936	1.9097	1.9258	1.9421
24	2.0408	2.0575	2.0743	2.0911	2.1080
25	2.2108	2.2282	2.2456	2.2631	2.2807
26	2.3876	2.4057	2.4238	2.4420	2.4602
27	2.5712	2.5899	2.6087	2.6276	2.6465
28	2.7616	2.7810	2.8005	2.8200	2.8397
29	2.9588	2.9789	2.9991	3.0193	3.0396
30	3.1628	3.1836	3.2044	3.2253	3.2463
31	3.3736	3.3951	3.4166	3.4382	3.4598
32	3.5912	3.6133	3.6355	3.6579	3.6801
33	3.8156	3.8384	3.8613	3.8842	3.9073
34	4.0468	4.0703	4.0939	4.1175	4.1412
35	4.2848	4.3090	4.3332	4.3575	4.3819
36	4.5296	4.5545	4.5794	4.6044	4.6294

Of the Areas of Circles in Wine Gallons.

Diams. in Inches.	.0	.1	.2	.3	.4
73	18.1186	18.1682	18.2180	18.2678	18.3177
74	18.6184	18.6687	18.7191	18.7696	18.8202
75	19.1250	19.1760	19.2271	19.2783	19.3295
76	19.6384	19.6901	19.7418	19.7937	19.8456
77	20.1586	20.2109	20.2634	20.3159	20.3685
78	20.6856	20.7386	20.7918	20.8450	20.8983
79	21.2194	21.2731	21.3269	21.3808	21.4348
80	21.7600	21.8144	21.8689	21.9235	21.9781
81	22.3074	22.3625	22.4176	22.4729	22.5282
82	22.8616	22.9173	22.9732	23.0291	23.0851
83	23.4226	23.4790	23.5356	23.5922	23.6489
84	23.9904	24.0475	24.1047	24.1620	24.2194
85	24.5650	24.6228	24.6807	24.7387	24.7967
86	25.1464	25.2049	25.2634	25.3221	25.3808
87	25.7346	25.7937	25.8530	25.9123	25.9717
88	26.3296	26.3894	26.4494	26.5094	26.5695
89	26.9314	26.9919	27.0525	27.1132	27.1740
90	27.5400	27.6012	27.6625	27.7239	27.7853
91	28.1554	28.2173	28.2792	28.3413	28.4034
92	28.7776	28.8401	28.9028	28.9655	29.0283
93	29.4066	29.4698	29.5332	29.5962	29.6601
94	30.0424	30.1063	30.1703	30.2344	30.2986
95	30.6850	30.7496	30.8143	30.8791	30.9439
96	31.3344	31.3997	31.4650	31.5305	31.5960
97	31.9906	32.0565	32.1226	32.1887	32.2549
98	32.6536	32.7202	32.7870	32.8538	32.9207
99	33.3234	33.3907	33.4581	33.5256	33.5932
100	34.0000	34.0680	34.1361	34.2043	34.2725
101	34.6834	34.7521	34.8208	34.8897	34.9586
102	35.3736	35.4429	35.5124	35.5819	35.6515
103	36.0706	36.1406	36.2108	36.2810	36.3513
104	36.7744	36.8451	36.9159	36.9868	37.0578
105	37.4850	37.5564	37.6279	37.6995	37.7711
106	38.2024	38.2745	38.3466	38.4189	38.4912
107	38.9266	38.9993	39.0722	39.1451	39.2181
108	39.6567	39.7310	39.8046	39.8782	39.9519

A TABLE

Of the Areas of Circles in Wine Gallons.

Diams. in Inches.	.5	.6	.7	.8	.9
73	18.3676	18.4176	18.4677	18.5178	18.5681
74	18.8708	18.9215	18.9728	19.0231	19.0740
75	19.3808	19.4322	19.4836	19.5351	19.5867
76	19.8976	19.9497	20.0018	20.0540	20.1062
77	20.4212	20.4739	20.5267	20.5796	20.6325
78	20.9516	21.0050	21.0585	21.1120	21.1657
79	21.4888	21.5429	21.5971	21.6513	21.7056
80	22.0328	22.0876	22.1424	22.1973	22.2523
81	22.5836	22.6391	22.6946	22.7502	22.8058
82	23.1412	23.1972	23.2535	23.3098	23.3661
83	23.7056	23.7624	23.8193	23.8762	23.9333
84	24.2768	24.3343	24.3919	24.4495	24.5072
85	24.8548	24.9130	24.9712	25.0295	25.0879
86	25.4396	25.4985	25.5574	25.6164	25.6754
87	26.0312	26.0907	26.1503	26.2100	26.2697
88	26.6296	26.6898	26.7501	26.8104	26.8709
89	27.2348	27.2957	27.3567	27.4177	27.4788
90	27.8468	27.9084	27.9700	28.0317	28.0935
91	28.4656	28.5279	28.5902	28.6526	28.7150
92	29.0912	29.1541	29.2171	29.2802	29.3433
93	29.7236	29.7872	29.8509	29.9146	29.9785
94	30.3628	30.4271	30.4915	30.5559	30.6204
95	31.0088	31.0738	31.1388	31.2039	31.2691
96	31.6616	31.7273	31.7930	31.8588	31.9246
97	32.3212	32.3875	32.4539	32.5204	32.5869
98	32.9876	33.0546	33.1217	33.1888	33.2561
99	33.6608	33.7285	33.7963	33.8641	33.9320
100	34.3408	34.4092	34.4776	34.5461	34.6147
101	35.0276	35.0967	35.1658	35.2350	35.3042
102	35.7212	35.7909	35.8607	35.9306	36.0005
103	36.4216	36.4920	36.5625	36.6330	36.7037
104	37.1288	37.1999	37.2711	37.3423	37.4136
105	37.8428	37.9146	37.9864	38.0583	38.1303
106	38.5636	38.6361	38.7086	38.7812	38.8538
107	39.2912	39.3643	39.4375	39.5108	39.5841
108	40.0256	40.0994	40.1733	40.2472	40.3213

Of the Areas of Circles in Wine Gallons.

Diams. in Inches.	.0	.1	.2	.3	.4
145	71.4850	71.5836	71.6823	71.7811	71.8799
146	72.4744	72.5737	72.6730	72.7725	72.8720
147	73.4706	73.5705	73.6706	73.7707	73.8709
148	74.4736	74.5742	74.6750	74.7758	74.8767
149	75.4834	75.5847	75.6861	75.7876	75.8892
150	76.5000	76.6020	76.7041	76.8063	76.9085
151	77.5234	77.6261	77.7288	77.8317	77.9346
152	78.5536	78.6569	78.7604	78.8639	78.9675
153	79.5906	79.6946	79.7988	79.9030	80.0073
154	80.6344	80.7391	80.8439	80.9488	81.0538
155	81.6850	81.7904	81.8959	82.0015	82.1071
156	82.7424	82.8485	82.9546	83.0609	83.1672
157	83.8066	83.9133	84.0202	84.1271	84.2341
158	84.8776	84.9850	85.0926	85.2002	85.3079
159	85.9554	86.0635	86.1717	86.2800	86.3884
160	87.0400	87.1488	87.2577	87.3667	87.4757
161	88.1314	88.2409	88.3504	88.4601	88.5698
162	89.2296	89.3397	89.4500	89.5603	89.6707
163	90.3346	90.4454	90.5564	90.6674	90.7785
164	91.4464	91.5579	91.6695	91.7812	91.8930
165	92.5650	92.6772	92.7895	92.9019	93.0143
166	93.6904	93.8033	93.9162	94.0293	94.1424
167	94.8226	94.9361	95.0498	95.1635	95.2773
168	95.9616	96.0758	96.1902	96.3046	96.4191
169	97.1074	97.2223	97.3373	97.4524	97.5676
170	98.2600	98.3756	98.4913	98.6071	98.7229
171	99.4194	99.5357	99.6520	99.7685	99.8850
172	100.5856	100.7025	100.8196	100.9367	101.0539
173	101.7586	101.8762	101.9940	102.1118	102.2297
174	102.9384	103.0567	103.1751	103.2936	103.4122
175	104.1250	104.2440	104.3631	104.4823	104.6015
176	105.3184	105.4381	105.5578	105.6777	105.7976
177	106.5186	106.6389	106.7594	106.8799	107.0005
178	107.7256	107.8466	107.9678	108.0890	108.2103
179	108.9394	109.0611	109.1829	109.3048	109.4268
180	110.1600	110.2824	110.4049	110.5275	110.6501

A TABLE

Of the Areas of Circles in Wine Gallons.

Diams. in Inches.	.5	.6	.7	.8	.9
145	71.9788	72.0778	72.1768	72.2759	72.3751
146	72.9716	73.0713	73.1710	73.2708	73.3706
147	73.9712	74.0715	74.1719	74.2724	74.3729
148	74.9776	75.0786	75.1797	75.2808	75.3821
149	75.9908	76.0925	76.1943	76.2961	76.3980
150	77.0108	77.1132	77.2156	77.3181	77.4207
151	78.0376	78.1407	78.2438	78.3470	78.4502
152	79.0712	79.1749	79.2787	79.3826	79.4865
153	80.1116	80.2160	80.3205	80.4250	80.5297
154	81.1588	81.2639	81.3691	81.4743	81.5796
155	82.2128	82.3186	82.4244	82.5303	82.6363
156	83.2736	83.3801	83.4866	83.5932	83.6998
157	84.3412	84.4483	84.5555	84.6628	84.7701
158	85.4156	85.5234	85.6313	85.7392	85.8473
159	86.4968	86.6053	86.7139	86.8225	86.9312
160	87.5848	87.6940	87.8032	87.9125	88.0219
161	88.6796	88.7895	88.8994	89.0094	89.1194
162	89.7812	89.8917	90.0023	90.1130	90.2237
163	90.8896	91.0008	91.1121	91.2234	91.3349
164	92.0048	92.1167	92.2287	92.3407	92.4528
165	93.1268	93.2394	93.3520	93.4647	93.5775
166	94.2556	94.3689	94.4822	94.5956	94.7090
167	95.3912	95.5051	95.6191	95.7332	95.8473
168	96.5336	96.6482	96.7629	96.8776	96.9925
169	97.6828	97.7981	97.9135	98.0289	98.1444
170	98.8388	98.9548	99.0708	99.1869	99.3031
171	100.0016	100.1183	100.2350	100.3518	100.4686
172	101.1712	101.2885	101.4059	101.5234	101.6409
173	102.3476	102.4656	102.5837	102.7018	102.8201
174	103.5308	103.6495	103.7683	103.8871	104.0060
175	104.7208	104.8402	104.9596	105.0791	105.1987
176	105.9176	106.0377	106.1578	106.2780	106.3982
177	107.1212	107.2419	107.3627	107.4836	107.6045
178	108.3316	108.4530	108.5745	108.6960	108.8177
179	109.5488	109.6709	109.7931	109.9153	110.0376
180	110.7728	110.8956	111.0184	111.1413	111.2643

Of the Areas of Circles in Wine Gallons.

Diams. in Inches.	.0	.1	.2	.3	.4
181	111.3874	111.5105	111.6336	111.7569	111.8802
182	112.6216	112.7453	112.8692	112.9931	113.1171
183	113.8626	113.9870	114.1116	114.2362	114.3609
184	115.1104	115.2355	115.3607	115.4860	115.6114
185	116.3650	116.4908	116.6167	116.7427	116.8687
186	117.6264	117.7529	117.8794	118.0061	118.1328
187	118.8946	119.0217	119.1490	119.2763	119.4037
188	120.1696	120.2974	120.4254	120.5534	120.6815
189	121.4514	121.5799	121.7085	121.8372	121.9660
190	122.7400	122.8692	122.9985	123.1279	123.2573
191	124.0354	124.1653	124.2952	124.4253	124.5554
192	125.3376	125.4681	125.5988	125.7295	125.8603
193	126.6466	126.7778	126.9092	127.0406	127.1721
194	127.9624	128.0943	128.2263	128.3584	128.4906
195	129.2850	129.4176	129.5503	129.6831	129.8159
196	130.6144	130.7477	130.8810	131.0145	131.1480
197	131.9506	132.0845	132.2186	132.3527	132.4869
198	133.2936	133.4282	133.5630	133.6978	133.8327
199	134.6434	134.7787	134.9141	135.0496	135.1852
200	136.0000	136.1360	136.2721	136.4083	136.5445
201	137.3634	137.5001	137.6368	137.7737	137.9106
202	138.7336	138.8709	139.0084	139.1459	139.2835
203	140.1106	140.2486	140.3868	140.5250	140.6633
204	141.4944	141.6331	141.7719	141.9108	142.0498
205	142.8850	143.0244	143.1639	143.3035	143.4431
206	144.2824	144.4225	144.5626	144.7029	144.8432
207	145.6866	145.8273	145.9682	146.1091	146.2501
208	147.0976	147.2390	147.3806	147.5222	147.6639
209	148.5154	148.6575	148.7997	148.9420	149.0844
210	149.9400	150.0828	150.2257	150.3687	150.5117
211	151.3714	151.5149	151.6584	151.8021	151.9458
212	152.8096	152.9537	153.0980	153.2423	153.3867
213	154.2546	154.3994	154.5444	154.6894	154.8345
214	155.7064	155.8519	155.9975	156.1432	156.2890
215	157.1650	157.3112	157.4575	157.6039	157.7503
216	158.6304				

Of the Areas of Circles in Wine Gallons.

Diams. in Inches.	.5	.6	.7	.8	.9
181	112.0036	112.1271	112.2506	112.3742	112.4978
182	113.2412	113.3653	113.4895	113.6138	113.7381
183	114.4856	114.6104	114.7353	114.8602	114.9853
184	115.7368	115.8623	115.9879	116.1135	116.2392
185	116.9948	117.1210	117.2472	117.3735	117.4999
186	118.2596	118.3865	118.5134	118.6404	118.7674
187	119.5312	119.6587	119.7863	119.9140	120.0417
188	120.8096	120.9378	121.0661	121.1944	121.3229
189	122.0948	122.2237	122.3527	122.4817	122.6108
190	123.3868	123.5164	123.6460	123.7757	123.9055
191	124.6856	124.8159	124.9462	125.0766	125.2070
192	125.9912	126.1221	126.2531	126.3842	126.5153
193	127.3036	127.4352	127.5669	127.6986	127.8305
194	128.6228	128.7551	128.8875	129.0199	129.1524
195	129.9488	130.0818	130.2148	130.3479	130.4811
196	131.2816	131.4153	131.5490	131.6828	131.8166
197	132.6212	132.7555	132.8899	133.0244	133.1589
198	133.9676	134.1026	134.2377	134.3728	134.5081
199	135.3208	135.4565	135.5923	135.7281	135.8640
200	136.6808	136.8172	136.9536	137.0901	137.2267
201	138.0476	138.1847	138.3218	138.4590	138.5962
202	139.4212	139.5589	139.6967	139.8346	139.9725
203	140.8016	140.9400	141.0785	141.2170	141.3557
204	142.1888	142.3279	142.4671	142.6063	142.7456
205	143.5828	143.7226	143.8624	144.0023	144.1423
206	144.9836	145.1241	145.2646	145.4052	145.5458
207	146.3912	146.5323	146.6735	146.8148	146.9561
208	147.8056	147.9474	148.0893	148.2312	148.3733
209	149.2268	149.3693	149.5119	149.6545	149.7972
210	150.6548	150.7980	150.9412	151.0845	151.2279
211	152.0896	152.2335	152.3774	152.5214	152.6654
212	153.5312	153.6757	153.8203	153.9650	154.1097
213	154.9796	155.1248	155.2701	155.4154	155.5609
214	156.4348	156.5807	156.7267	156.8727	157.0188
215	157.8968	158.0434	158.1900	158.3367	158.4835

For Reducing Ale Gallons into Victuallers' Barrels of 34 Gallons.

Barrels	0	1/4	1/2	3/4	Barrels	0	1/4	1/2	3/4
0	0	8.5	17	25.5	36	1224	1232.5	1241	1249.5
1	34	42.5	51	59.5	37	1258	1266.5	1275	1283.5
2	68	76.5	85	93.5	38	1292	1300.5	1309	1317.5
3	102	110.5	119	127.5	39	1326	1334.5	1343	1351.5
4	136	144.5	153	161.5	40	1360	1368.5	1377	1385.5
5	170	178.5	187	195.5	41	1394	1402.5	1411	1419.5
6	204	212.5	221	229.5	42	1428	1436.5	1445	1453.5
7	238	246.5	255	263.5	43	1462	1470.5	1479	1487.5
8	272	280.5	289	297.5	44	1495	1504.5	1513	1521.5
9	306	314.5	323	331.5	45	1530	1538.5	1547	1555.5
10	340	348.5	357	365.5	46	1564	1572.5	1581	1589.5
11	374	382.5	391	399.5	47	1598	1606.5	1615	1623.5
12	408	416.5	425	433.5	48	1632	1640.5	1649	1657.5
13	442	450.5	459	467.5	49	1666	1674.5	1683	1691.5
14	476	484.5	493	501.5	50	1700	1708.5	1717	1725.5
15	510	518.5	527	535.5	51	1734	1742.5	1751	1759.5
16	544	552.5	561	569.5	52	1768	1776.5	1785	1793.5
17	578	586.5	595	603.5	53	1802	1810.5	1819	1827.5
18	612	620.5	629	637.5	54	1836	1844.5	1853	1861.5
19	646	654.5	663	671.5	55	1870	1878.5	1887	1895.5
20	680	688.5	697	705.5	56	1904	1912.5	1921	1929.5
21	714	722.5	731	739.5	57	1938	1946.5	1955	1963.5
22	748	756.5	765	773.5	58	1972	1980.5	1989	1997.5
23	782	790.5	799	807.5	59	2006	2014.5	2023	2031.5
24	816	824.5	833	841.5	60	2040	2048.5	2057	2065.5
25	850	858.5	867	875.5	61	2074	2082.5	2091	2099.5
26	884	892.5	901	909.5	62	2108	2116.5	2125	2133.5
27	918	926.5	935	943.5	63	2142	2150.5	2159	2167.5
28	952	960.5	969	977.5	64	2176	2184.5	2193	2201.5
29	986	994.5	1003	1011.5	65	2210	2218.5	2227	2235.5
30	1020	1028.5	1037	1045.5	66	2244	2252.5	2261	2269.5
31	1054	1062.5	1071	1079.5	67	2278	2286.5	2295	2303.5
32	1088	1096.5	1105	1113.5	68	2312	2320.5	2329	2337.5
33	1122	1130.5	1139	1147.5	69	2346	2354.5	2363	2371.5
34	1156	1164.5	1173	1181.5	70	2380	2388.5	2397	2405.5
35	1190	1198.5	1207	1215.5	71	2414	2422.5	2431	2439.5

The Use of the foregoing Table.

The process of reducing ale gallons into barrels and firkins, by the preceding Table, is extremely easy; for Example, suppose it be required to reduce 828 gallons into barrels and firkins; then opposite to 24 barrels in the first column, we find 824.5 gallons in the third column; this being the nearest to the given number. Hence the given number of gallons is equal to 24 barrels and 1 firkin; and by subtraction we have $828 - 824.5 = 3.5$ odd gallons to be transferred to the next charge.

Again, suppose it were required to reduce 2336.5 gallons to barrels and firkins; we find 2329 opposite to $68\frac{1}{2}$ barrels; hence 68 barrels and 2 firkins must be charged with the duty; and $2336.5 - 2329 = 7.5$ odd gallons, which must be carried to the next charge.

Note. The fractions at the top of the columns of the preceding Table, denote one-quarter, one-half, and three-fourths of a barrel.

FINIS.

ERRATA.

Page.	Line.
5,	14, *for* $4.27 \times 1 = 042.7$, *read* $4.27 \times 10 = 42.7$.
7,	15, *for* $29\frac{3}{4}$, *read* $27\frac{3}{4}$.
27,	12, *for* 1s. 10d. *read* 1l. 10s.
169,	21, *for* $277781.2 \times .000454 = 126.112$, *read* $187690 \times .000454 = 85.211$; also, $187690 \div 2200.16 = 85.307$, the content in cubic feet.
199,	The figure on this page should have appeared on page 212; and *vice versa*.
256,	68 Ques. *for* 70, *read* 90.
259,	3, *for* 513.883, *read* 856.471.
259,	18, *for* 615.401, *read* 675.560.
383,	16, *for* 107.070, *read* 101.070.

Excise Establishment.

Commissioners, Robert Nicholas, esq. Lord Geo. Seymour, *Chairmen.*—Earl of Carhampton, Hon. Aug. Phipps, Alex. Campbell, esq. W. Manley, esq. F. H. Doyle, esq sir John C. Mortlock, knt. and the hon. C. R. Trefusis.
Secretary, Thomas Burton, esq.
Clerks, John Storer, Chas. Brown, F. C. Wingrave, Thomas Eley, Richard Murphy, Thomas Langford, Thomas S. Chapman, Wm. Murray, Charles Slater, Charles Clarkson, and Joseph Wingrave, jun.
Correspondent, Wm. Wardley.
Assistants, Francis Thompson, Jer. Otley, Wm. Trent, Jn. Boyers, Thomas Warr.
Clerks, James Wilson, T. White, John Thos. Goodwin, Chas. Hancock, and Henry Paull.
Solicitor, Thomas W. Carr, esq.
Solicitor for Criminal Prosecutions, &c. Philip W. Mayo, esq.
Registrer, Ed. Jackson, Esq.
1st Clerk, Gabriel Riddle.

General Accomptants.

General Business, John Hodgson.
Beer, &c. David Langton.
Malt, &c. William Lorimer.
Foreign Spirits, Tea, &c. Joshua Savage.
Auctions, &c. Joseph Wingrave.
Debentures, &c. William Leese.
Paper, &c. Charles Lyall.
Licences, George Hardy.
Bills of Exchange, James Ewbank.
Accomptants to London Brewery, Wm. Nicholls and Wm. Dodd.
Receiver Gen. G. J. Cholmondeley, esq.
Clerks, Wm. Smith, John Dennis, Geo. Argles, Robt. Drury, Thos. Baldwin.
Receivers, Messrs. Wm. Manser, Edward Powell, John Mullis, David Panchard, John King.
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1st Clerk, Robert Stother.
Comptroller Gen. Edw. Milward, esq.
Deputy, John Rotton, esq.
Auditor, Geo. August. Bouverie, esq.
Assistant, Oliver Naylor.
Clerk of the Securities, Henry Canning, esq.
Assistant, William Gowland.
Clerk of Incidents, Richard Moore.
Assistant, Richard Gilbert.
Clerk of the Diaries, Chas. Mattock.
Clerk of the Entries, Robert Bath.
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Inspector of Permit Printing-Office, John Frederick Walpole.
Warehouse-keepers of seized Goods at the Chief Office, John Saunders, and James West.
At the Port, John Stanford.
General Surveyors, G. Newton, Jas. E. Titterton, John Townley, Edward Hind, John Phipps, Wm. Dowsing, Luke Allsop, Chas. Serancke, Will. Charles Knoff, John Storey, Geo. Gibbs.
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Assistant, Robert Leete.
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Surveyor General of the Tea Warehouses, John R. Payne, esq.
Surveyors, John Sanders, and John Trulock.
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1st Clerk, A. M. Dufaur.
Inspector General of Imports, Thos. Groves.
Assistants, John Archer, Thomas Tanner, Robert Simpson.
Surveyors of Imports, George Hill, Charles Hillam, James Sturges,

John H. Reynolds, Thomas Bignell, John Field, Christ. Dredge.
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THAT KEYS to Works on Arithmetic, Mensuration, and other books intended for the purposes of educa-ion, are of great utility, both to teachers and private tudents, is now universally admitted. They tend to iminish the labour that is the inseparable attendant on ıe important, but arduous occupation of instructing outh; as by referring to them, the tutor is able to scover the accuracy or inaccuracy of his pupils' utions, without the trouble of investigating every eration.

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Part the Eighth comprehends Precedents of Vouchers and Abstracts, for assisting young Officers of the Excise, in making up their Accounts. These, it is hoped, will be found more concise and perspicuous than any others that have hitherto appeared in Works on Gauging.

The First Five Parts will be found extremely useful to teachers and private students; the two following Parts are adapted for the general use of Officers of the Excise and Customs; and for all Manufacturers of Excisable Commodities; and the Eighth Part is intended to furnish young Officers with such Precedents of Vouchers and Abstracts, as will enable them to make up their Accounts in a clear and perspicuous manner.

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A. NESBIT.
W. LITTLE.

Manchester, July, 1823.

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NESBIT'S AND *LITTLE'S*

PRACTICAL GAUGING.

PART I.

Vulgar and Decimal Fractions, and Square and Cube Roots.

ADDITION OF DECIMALS.

Exam. 2.	Exam. 3.
367.60	53.7
4678.3609	2943.
869.563	1.2
.2003	2.0073
7.5964	1.47
42.67	637.
5965.9906 *Ans.*	3638.3773 *Ans.*

Exam. 4.

124.1
.3492
84.02
6.349
.00879
71.2
286.02699 *Ans.*

SUBTRACTION OF DECIMALS.

Exam. 2.

```
 2.18
  .814
 1.366 Ans.
```

Exam. 3.

```
.794
.0981
.6959 Ans.
```

Exam. 4.

```
.0943
.09281
.00149 Ans.
```

Exam. 5.

```
374.901
 68.14
306.761 Ans.
```

MULTIPLICATION OF DECIMALS.

Exam. 2.

```
   .00741
   .00054
     2964
    3705
.0000040014 Ans.
```

Exam. 3.

```
 .3141
  20.5
 15705
6282
6.43905 Ans.
```

Exam. 4.

```
 .35426
   .025
 177130
 70852
.00885650 Ans.
```

Exam. 5.

```
      9268.456
       846.389
      83416104
     74147648
    27805368
   55610736
  37073824
 74147648
7844719.205384 Ans.
```

DIVISION OF DECIMALS.

EXAM. 2.

```
.325)741.000(2280. Ans.
     650
     .910
      650
      2600
      2600
      ...0
```

EXAM. 3.

```
5.2)839.0(161.3461 Ans.
    52
    319
    312
    ..70
      52
      180
      156
      .240
      .208
       .320
        312
       ..80
          52
          28 rem.
```

EXAM. 4.

```
36).074(.00205 Ans.
    72
    .200
     180
     .20 rem.
```

EXAM. 5.

```
24.05)48324.36(2009.32 Ans.
      4810
      ..22436
        21645
      ..7910
        7215
        .6950
         4810
        .2140 rem.
```

EXAM. 6.

```
256.686)5.65690(.02203 Ans.
        5 13372
         523180
         513372
        ..980800
           770058
           210742 rem.
```

REDUCTION OF DECIMALS.

To reduce a vulgar fraction to a decimal of the same value.

CASE I.

Exam. 2.

```
2)1.0
   .5 Ans.
```

Exam. 3.

```
4)3.00
   .75 Ans.
```

Exam. 4.

```
8)3.000
   .375 Ans.
```

Exam. 5.

```
25)18.00(.72 Ans.
   175
   ..50
     50
     ..
```

Exam. 6.

```
38)24.000000(.631578 Ans.
   228
   .120
    114
    ..60
      38
      220
      190
      .300
       266
       .340
        304
        .36
```

Exam. 7.

```
722)35.000000(.048476 Ans.
    2888
    .6120
     5776
     .3440
      2888
      .5520
       5054
       .4760
        4332
        .428
```

EXAM. 8.

```
144)48.000000(.333333 Ans.
    432
    .480
     432
     .480
      432
      .480
       432
       .480
        432
        .48
```

EXAM. 9.

```
4695)382.000000(.0813631 Ans.
     37560
     ..6400
       4695
       17050
       14085
        29650
        28170
         14800
          14085
          ..7150
            4695
          ..2455
```

EXAM. 10.

```
85367)5869.0000000(.687502 Ans.
      512202
       746980
       682936
        640440
        597569
         428710
         426835
        ..187500
           170734
           .16766
```

EXAM. 11.

Here $\frac{2 \times 3 \times 5}{3 \times 4 \times 9} = \frac{30}{108}$, the equivalent simple fraction; then,

$$\begin{array}{l} 108)30.000(.277 \textit{ Ans.} \\ \quad\ \ 216 \\ \quad\ \ \overline{\ .840} \\ \quad\ \ \ \ 756 \\ \quad\ \ \ \ \overline{.840} \\ \quad\ \ \ \ \ \ 756 \\ \quad\ \ \ \ \ \ \overline{.84} \end{array}$$

EXAM. 12.

Here $\frac{26 \times 22 \times 36}{18 \times 25 \times 48} = \frac{20592}{21600}$, the equivalent simple fraction; then,

$$\begin{array}{l} 216,00)20592.00(.9533 \textit{ Ans.} \\ \qquad\quad 1944 \\ \qquad\quad \overline{\ 1152} \\ \qquad\quad\ 1080 \\ \qquad\quad\ \overline{\ \ \ 720} \\ \qquad\quad\ \ \ \ 648 \\ \qquad\quad\ \ \ \ \overline{\ 720} \\ \qquad\quad\ \ \ \ \ 648 \\ \qquad\quad\ \ \ \ \ \overline{\ \ 72} \end{array}$$

CASE II.

To reduce numbers of different denominations, as money, weights, measures, &c. to their equivalent decimal values.

EXAM. 2.

```
                  s.   d.
                  10   9¾
                  12
                 ---
                 129
£.    far.         4
                 ---------
1  =  960)519.000000(.540625 Ans.
          4800
          ----
           3900
           3840
           ----
            ..6000
              5760
              ----
              .2400
               1920
               ----
                4800
                4800
                ----
                ....
```

EXAM. 3.

```
               s.    d.
               9     3¼
              12
             ---
             111
£.    far.     4
1  =  960)445.0000000(.4635416 Ans.
          384 0
          -----
          . 6100
            5760
            ----
            .3400
             2880
             ----
             . 5200
               4800
               ----
              . 4000
                3840
                ----
                 1600
                  960
                 ----
                 .6400
                  5760
                  ----
                   640   rem.
                  ====
```

EXAM. 4.

```
               d.
               10½
s.     far.     4
1  =   48)42.000(.875 Ans.
          38 4
          ----
          .360
           336
           ---
           . 240
             240
             ---
             . . .
            =====
```

EXAM. 5.

```
              s.   d.
              19   11¾
              12
             ----
             239
£.    far.     4
1  =  960)959.0000000(.9989583 Ans.
          8640
          ----
          .9500
           8640
           ----
            .8600
             7680
             ----
              .9200
               8640
               ----
                .5600
                 4800
                 ----
                  .8000
                   7680
                   ----
                    .3200
                     2880
                     ----
                     .320
                     ====
```

EXAM. 6.

```
gal.       pt.  pt.
 1    =    8 )2.00
              ----
               .25 Ans.
               ===
```

EXAM. 7.

```
tun.       gal.  gal.
 1    =   252)189.00(.75 Ans.
              1764
              ----
              .1260
               1260
               ----
               ....
               ====
```

EXAM. 8.

```
fir.    gal.  gal.
 1  =  9)8.000000(.888888 Ans.
         72
         ---
          .80
           72
           ---
            .80
             72
             ---
              .80
               72
               ---
                .80
                 72
                 ---
                  .80
                   72
                   ---
                    .8
                    ==
```

Or, 8 ÷ 9 = .888888, the answer required.

EXAM. 9.

```
butt.   gal.      gal.
 1  =  108.)56.000000(.518518 Ans.
            540
            ---
             200
             108
             ---
              920
              864
              ---
               560
               540
               ---
                200
                108
                ---
                 .920
                  864
                  ---
                  .56
                  ===
```

EXAM. 10.

```
   pk.
 4)3.00
   ----
    .75 Ans.
   ====
```

```
        EXAM. 11.                     EXAM. 12.
                                   lb.   oz.  oz.
        qrs. bush. pk.              1  = 16)12.00(.75 Ans.
         4    5    2                        11 2
         8                                  ..80
        37                                    80
last. pk. 4                                   ..
1 = 320)150.000000(.46875 Ans.
        128 0
         .2200
          1920
          .2800
           2560
           .2400
            2240
            .1600
             1600
             ....

              EXAM. 13.

               lb.  oz.
               22   9
               16
              141
qr.     oz.   22
1  =   448)361.0000000(.8058035 Ans.
           358 4
            ..2600
               2240
               .3600
                3584
                ..1600
                   1344
                   .2560
                    2240
                    ..320
```

EXAM. 14.

qrs. *lb.* *oz.*
3 14 8
28
98
16
596
cwt. *oz.* 98
1 = 1792)1576.0000(.8794 *Ans.*
14336
.14240
12544
.16960
16128
..8320
7168
1152

EXAM. 15.

cwt. *qrs.* *lb.*
15 2 21
4
62
28
517
ton. *lb.* 124
1 = 2240)1757.000000(.784375 *Ans.*
15680
.18900
17920
..9800
8960
.8400
6720
16800
15680
.11200
11200
.....

CASE III.

To find the value of a decimal fraction in the known parts of an integer.

EXAM. 2.

```
   .8649
      12
 -------
 10.3788
       4
 -------
  1.5152 Ans. 10¼d.
```

EXAM. 3.

```
 .92846
     20
--------
18.56920
     12
--------
 6.83040
      4
--------
 3.32160 Ans. 18s. 6¾d.
```

EXAM. 4.

```
  .8694
     63
 ------
  26082
 52164
-------
54.7722
      4
-------
 3.0888 Ans. 54 gall. 3 qts.
```

EXAM. 5.

```
  .73828
      36
 -------
  442968
 221484
--------
26.57808
       4
--------
 2.31232 Ans. 26 gall. 2 qts.
```

EXAM. 6.

```
 .5694
     8
4.5552
     4
2.2208 Ans. 4 bush. 2 pks.
```

EXAM. 7.

```
 .68828
     10
6.83280
      8
6.66240
      4
2.64960 Ans. 6 qrs. 6 b. 2½ pk.
```

EXAM. 8.

```
  .9326
      4
 3.7304
     28
  58432
 14608
20.4512
     16
  27072
  4512
 7.2192 Ans. 3 qr. 20lb. 7 oz.
```

RULE OF THREE IN DECIMALS.

EXAM. 3.

```
     qt.     s.      qt.     s.     £.  s.  d.
As 3.25 : 9.75 :: 28.5 : 85.5 = 4   5   6 Ans.
          28.5
          4875
         7800
        1950
  3.25)277.875(85.5s.
        2600
         1787
         1625
          1625
          1625
```

EXAM. 4.

Here $\frac{1}{2}$ = .5 cwt.; 6 £. 15s. = 6.75 £.; and 3 cwt. 2 qr. 7 lb. = 3.5625 cwt.; then,

```
    cwt.   £.        cwt.      £.  s.  d.
As  .5  :  6.75  ::  3.5625 :  48  1   10½ Ans.
                     6.75
                   ------
                   178125
                  249375
                 213750
                ---------
              .5)24.046875
                ---------
                 48.09375
                       20
                ---------
                  1.87500
                       12
                ---------
                 10.50000
                        4
                ---------
                  2.00000
                =========
```

EXAM. 5.

Here 3 £. 5s. = 3.25 £.; and 5 cwt. 3 qrs. 14 lb. = 5.875 cwt.; then,

```
    cwt.  £.       cwt.      £.  s.  d.
As  1  :  3.25 ::  5.875  :  19  1   10½ Ans.
                   3.25
                 -------
                  29375
                 11750
                17625
                --------
                19.09375
                      20
                --------
                 1.87500
                      12
                --------
                10.50000
                       4
                --------
                 2.00000
                ========
```

EXAM. 6.

```
     bar.    s.        bar.      £.   s.  d.
As 3  :  27.5  ::  243.25  :  111   9   9½ Ans.
                     27.5
                  -------
                   121625
                  170275
                  48650
                 --------
              3)6689.375
              ----------
           2,0)222,9.791
              ----------
            £.111,9.791
                      12
                   -----
                   9.492
                       4
                   -----
                   1.968
                   =====
```

EXAM. 7.

```
     bar.   s.       bar.     £. s.  d.
As 3  :  5.5  ::  18.75  :  1  14  4½ Ans.
                    5.5
                  -----
                   9375
                  9375
                -------
              3)103.125
                -------
                 34.375
                     12
                 ------
                  4.500
                      4
                  -----
                  2.000
                  =====
```

EXAM. 8.

```
      bar.   s. d.       bar.    £. s. d.
As .75 : 6 10½ :: 16.25 : 7  8 11½ Ans.
          12             82.5
         ----           -----
         82.5            8125
         ====           3250
                       13000
                      ------- (12)
                .75)1340.625(1787.5
                     75      ---------
                    ---   2,0)14,8 11
                    590     ---------
                    525    £.7 8s. 11½d.
                    ---     =========
                     656
                     600
                     ---
                      562
                      525
                      ---
                       375
                       375
                       ===
```

EXAM. 9.

Here ⅞ of a 1000 = 875; ¾ of a 1000 = 750; and 9s. 7½d. = 115.5d.; then,

```
      br.     d.        br.    s.  d.
As 750 : 115.5 :: 875 : 11  2¾ Ans.
            875
          -----
           5775
          8085
          9240    (12)
         ------
  750)101062.5(134.75
      750      --------
      ----     11s. 2¾d.
      2606     ========
      2250
      ----
       3562
       3000
       ----
        5625
        5250
        ----
         3750
         3750
         ----
         ....
         ====
```

EXAM. 10.

Here $\frac{1}{20}$ of a 1000 = 50; and $\frac{39}{40}$ of a 1000 = 975; hen,

$$
\begin{array}{lllllll}
 & br. & d. & br. & & s. & d. \\
\text{As} & 50 : & 3.5 : & 975 & :: & 5 & 8\tfrac{1}{4}\ \textit{Ans.}
\end{array}
$$

```
          975
          3.5
         ----
         4875
        2925   (12)
       ------
    50)3412.5(68.25
       300     -------
       ---      5s. 8¼d.
        412    ========
        400
        ---
         125
         100
         ---
          250
          250
          ---
          ...
          ===
```

EXAM. 11.

Here $\frac{4}{5}$ of a 1000 = 800; and $\frac{1}{40}$ of a 1000 = 25; then,

$$
\begin{array}{lllllll}
 & br. & d. & & br. & s. & d. \\
\text{As} & 25 : & 1.75 & :: & 800 : & 4 & 8\ \textit{Ans.}
\end{array}
$$

```
              800 (12)
         ---------
    25)1400.00(56
       125     -------
       ---      4s. 8d.
        150    ========
        150
        ---
        ...
        ===
```

EXAM. 12.

```
     lb.    d.        lb.      £.  s.  d.
As 5 : 11.25 :: 42364 : 397  3   3 Ans.
                 11.25
               ------
               211820
               84728
              42364
             42364
            ---------
           5)476595.00
           ---------
          12)95319
          ---------
         2,0)794,3 3
         ---------
          £. 397 3 3
```

SQUARE ROOT.

To extract the Square Root of any number.

EXAM. 2.

```
     . . . .
     1048576(1024 Ans.
     1
    -----
02)   485
      404
    -----
044)  8176
      8176
    ======
```

EXAM. 3.

```
      . .
      983(31.352 Ans.
      9
      ---
  61)  83
       61
     ------
  623)2200
      1869
     -------
 6265)33100
      31325
     --------
62702) 177500
       125404
       -------
        52096 rem.
```

Exam. 4.

```
      8104.2634(90.023 Ans.
      81
18002)42634
      36004
180043)663000
       540129
       122871 rem.
```

Exam. 5.

```
   744.3260(27.282 Ans.
   4
47)344
   329
542)1582
    1084
5448)44860
     43584
54562)127600
      109124
       18476 rem.
```

Exam. 6.

```
.0000178929(.00423 Ans.
   16
 82)189
    164
843)2529
    2529
```

EXAM. 7.

(See Note 3.)

```
314)202.0(.643312 = 202/314 nearly.
    1884
    ----
     1360
     1256
     ----
      1040
       942
       ---
        980
        942
        ---
         380
         314
         ---
          660
          628
          ---
           32 rem.
          ===

      .  .  .
     .643312(.802 Ans.
      64
      ----
  1602)3312
       3204
       ----
        108 rem.
        ===
```

EXAM. 8.

```
38416)26896.0(.70012494 = 26896/38416 nearly.
      268912
      ------
         48000
         38416
         -----
          95840
          76832
          ------
          190080
          153664
          ------
           364160
           345744
           ------
            184160
            153664
            ------
             30496 rem.
            ======
```

```
       .70012494(.8367 Ans.
        64
     163)601
         489
    1666)11224
          9996
   16727)122894
         117089
           5805 rem.
```

EXAM. 9.

Here $\frac{18 \times 23 \times 35 \times 42}{20 \times 26 \times 43 \times 54} = \frac{608580}{1207440} = .50402504$;
and $\sqrt{.50402504} = .7099$, the answer required.

SQUARE ROOT,

APPLIED TO THE

Solution of Mathematical Problems.

PROBLEM I.

To find a mean proportional between two given numbers.

EXAM. 2.

```
       243
        48
      1944
       972
     11664(108 Ans.
     1
  208)1664
      1664
```

EXAM. 3.

```
   36
   16
  ---
  216
  36
  ---
  576(24 lb. the weight required.
  4
  ---
44)176
   176
   ===
```

Hence it is evident that the arms of the balance are to each other, as 16 to 24, or as 24 to 36, or as 2 to 3.

PROBLEM II.

To find the side of a square equal in area to any given superficies.

EXAM. 1.

```
 2025(45 feet, Ans.
 16
 ---
85)425
   425
   ===
```

EXAM. 2.

```
 324(18 feet, Ans.
 1
 ---
28)224
   224
   ===
```

EXAM. 3.

```
  144
    4
  ---
  576(24 feet, Ans.
  4
  ---
44)176
   176
   ===
```

PROBLEM III.

To find the diameter of a circle, when the area is given.

EXAM. 1.

```
.7854)8824.70000(11235.93 quotient.
      .7854
      ------
        9707
        7854
        -----
        18530
        15708
        -----
         28220
         23562
         -----
          46580
          39270
          -----
           73100
           70686
           -----
            24140
            23562
            -----
            ..578 rem.
            =====
```

```
        11235.93(105.999 inches, Ans.
        1
       ----------
  205)1235
      1025
      ------
  2109)21093
       18981
       ------
  21189)211200
        190701
        -------
  211989)2049900
         1907901
         -------
         .141999 rem.
          =======
```

EXAM. 2.

.7854)254.5000(324 *quotient.*
235 62
―――
18 880
15 708
―――
31720
31416
―――
304 *rem.*

324(18 *inches, Ans.*
1
―
28)224
224

PROBLEM IV.

To find the hypothenuse of a right angled triangle, when the base and perpendicular are given.

EXAM. 1.

Here $105 \times 105 = 11025$, the square of the base; and $56 \times 56 = 3136$, the square of the perpendicular; then $\sqrt{11025 + 3136} = \sqrt{14161} = 119$ inches, the hypothenuse.

EXAM. 2.

Here $112 \times 112 = 12544$, the square of the length; and $84 \times 84 = 7056$, the square of the breadth; then $\sqrt{12544 + 7056} = \sqrt{19600} = 140$ inches, the diagonal required.

EXAM. 3.

Here $72.5 \times 72.5 \times 2 = 5256.25 \times 2 = 10512.5$, twice the square of the side; and $\sqrt{10512.5} = 102.53$ inches, the diagonal required.

D

PROBLEM V.

Given the hypothenuse of a right angled triangle, and either of the legs, to find the other leg.

EXAM. 1.

Here $153^2 - 135^2 = 23409 - 18225 = 5184$; and $\sqrt{5184} = 72$ inches, the perpendicular required.

EXAM. 2.

Here $60^2 - 36^2 = 3600 - 1296 = 2304$; and $\sqrt{2304} = 48$ inches, the perpendicular depth required.

EXAM. 3.

Here $125^2 - 100^2 = 15625 - 10000 = 5625$, and $\sqrt{5625} = 75$, the depth.

Also, $100^2 - 78^2 = 10000 - 6084 = 3916$; and $\sqrt{3916} = 62.577$ the breadth.

PROBLEM VI.

Given the head and bung diameters and length of a cask, to find the diagonal or distance between the centre of the bung hole, and that point where the middle of the opposite staff and head of the cask intersect each other.

EXAM. 1.

Here $\frac{24 + 18}{2} = \frac{42}{2} = 21$, half the sum of the diameters; and $\frac{30}{2} = 15$, half the length of the cask; then $21^2 + 15^2 = 441 + 225 = 666$; and $\sqrt{666} = 25.8$ inches, the diagonal required.

EXAM. 2.

Here $\frac{32 + 24}{2} = \frac{56}{2} = 28$, half the sum of the diameters; and $\frac{40}{2} = 20$, half the length of the cask; then $28^2 + 20^2 = 784 + 400 = 1184$; and $\sqrt{1184} = 34.4$ inches, the diagonal required.

PROBLEM VII.

Given the diagonal and diameters of a cask, to find its length.

EXAM. 1.

Here $\frac{24 + 18}{2} = \frac{42}{2} = 21$, half the sum of the diameters; then $25.8^2 - 21^2 = 665.64 - 441 = 224.64$; and $\sqrt{224.64} \times 2 = 14.98 \times 2 = 29.96$ inches, the length of the cask.

EXAM. 2.

Here $\frac{35 + 29}{2} = \frac{64}{2} = 32$, half the sum of the diameters; then $40^2 - 32^2 = 1600 - 1024 = 576$; and $\sqrt{576} \times 2 = 24 \times 2 = 48$ inches, the length of the cask.

CUBE ROOT.

To extract the Cube Root of any number.

EXAM. 2.

```
63044792(398 Ans.
27
36044 resolvend.
    9 triple of 3.
  27  triple square of 3.
  279 divisor.
  729 cube of 9.
 729   square of 9 × by the triple of 3.
243    triple of 9 × by the square of 3.
32319 subtrahend.
 3725792 second resolvend.
     117 triple of 39.
   4563  triple square of 39.
   45747 second divisor.
     512 cube of 8.
    7488  square of 8 × by the triple of 39.
  36504   triple of 8 × by the square of 39.
 3725792 second subtrahend.
```

PROOF.

Here 398 × 398 × 398 = 158404 × 398 = 63044792, the same as the given number.

EXAM. 3.

958585256(986 *Ans.*

729

229585 resolvend.

27 triple of 9.

243 triple square of 9.

2457 divisor.

512 cube of 8.

1728 square of 8 × by triple of 9.

1944 triple of 8 × by square of 9.

212192 subtrahend.

17393256 second resolvend.

294 triple of 98.

28812 triple square of 98.

288414 second divisor.

216 cube of 6.

10584 square of 6 × by the triple 98.

172872 triple of 6 × by the square of 98.

17393256 second subtrahend.

PROOF.

Here 986 × 986 × 986 = 972196 × 986 = 58585256, the same as the given number.

EXAM. 4.

633.839779(8.59 *Ans.*	
512	
121839	resolvend.
24	triple of 8.
192	triple square of 8.
1944	divisor.
125	cube of 5.
600	square of 5 × by triple of 8.
960	triple of 5 × by square of 8.
102125	subtrahend.
19714779	second resolvend.
255	triple of 85.
21675	triple square of 85.
217005	second divisor.
729	cube of 9.
20655	square of 9 × by triple of 85.
195075	triple of 9 × by square of 85.
19714779	second subtrahend.

PROOF.

Here $8.59 \times 8.59 \times 8.59 = 73.7881 \times 8.59 =$ 633.839779, the same as the given number.

EXAM. 5.

```
1006.012008(10.02 Ans.
1
006  resolvend.
   3 triple of 1.
 3  triple square of 1.
 33    divisor.
  6012  second resolvend.
    30 triple of 10.
  300  triple square of 10.
  3030     second divisor.
  6012008 third resolvend.
      300 triple of 100.
    30000  triple square of 100.
   300300 third divisor.
        8 cube of 2.
     1200  square of 2 × by triple of 100.
   60000   triple of 2 × by square of 100.
  6012008  subtrahend.
```

PROOF.

Here 10.02 × 10.02 × 10.02 = 100.4004 × 10.02 = 1006.012008, the same as the given number.

EXAM. 6.

```
.898632125(.965 Ans.
729
169632 resolvend.
    27 triple of 9.
   243 triple square of 9.
  2457 divisor.
   216 cube of 6.
   972 square of 6 × by triple of 9.
 1458  triple of 6 × by square of 9.
155736      subtrahend.
 13896125 second resolvend.
      288 triple of 96.
    27648 triple square of 96.
   276768 second divisor.
      125 cube of 5.
     7200 square of 5 × by triple 96.
  138240  triple of 5 × by square of 96.
 13896125 second subtrahend.
```

PROOF.

Here .965 × .965 × .965 = .931225 × .965 = .898632125, the same as the given number.

Exam. 7.

Here $\frac{19}{21}$ = .904761904.

.904761904).967 *Ans.*	
729	
175761	resolvend.
27	triple of 9.
243	triple square of 9.
2457	divisor.
216	cube of 6.
972	square of 6 × by triple of 9.
1458	triple of 6 × by square of 9.
155736	subtrahend.
20025904	second resolvend.
288	triple of 96.
27648	triple square of 96.
276768	second divisor.
343	cube of 7.
14112	square of 7 × by triple of 96.
193536	triple of 7 × by square of 96.
19495063	second subtrahend.
530841	remainder.

Proof.

ere .967 × .967 × .967 = .935089 × .967 = *231063,* the cube of the root; to which add the inder, and we obtain .904761904, the proof re- :d.

CUBE ROOT,

APPLIED TO THE

Solution of several useful Mathematical Problems.

PROBLEM I.

To find the side of a cubical vessel that shall be equal in content to any given vessel, whose form is that of a parallelopipedon, a cylinder, a prism, a cone, &c. &c.

EXAM. 1.

Here $\sqrt[3]{216000} = 60$ inches, the answer required.

EXAM. 2.

Here $231 \times 43 = 9933$, the number of cubic inches in 43 wine gallons; and $\sqrt[3]{9933} = 21.496$ inches, the side required.

EXAM. 3.

Here $2150.42 \times 48 = 103220.16$, the number of cubic inches contained in 48 malt bushels; and $\sqrt[3]{103220.16} = 46.908$ inches, the answer required.

PROBLEM II.

Given the dimensions of any vessel, to find the dimensions of another similar vessel, that shall be any number of times greater or less than the given vessel.

EXAM. 1.

Here $216^3 = 10077696$, the cube of the given length; and 10077696×8 (the ratio) $= 80621568$; hence $\sqrt[3]{80621568} = 432$ inches, the length of the required vessel.

Also, $125^3 = 1953125$, the cube of the given breadth; and $1953125 \times 8 = 15625000$; hence $\sqrt[3]{15625000} = 250$ inches, the breadth of the required vessel.

Lastly, $64^3 = 262144$, the cube of the given depth; and $262144 \times 8 = 2097152$; hence $\sqrt[3]{2097152} = 128$ inches, the depth of the required vessel.

EXAM. 2.

Here $64^3 = 262144$, the cube of the given depth; and 262144×27 (the ratio) $= 7077888$; hence $\sqrt[3]{7077888} = 192$ inches, the depth of the required vessel.

Also, $27^3 = 19683$, the cube of the given diameter; and $19683 \times 27 = 531441$; hence $\sqrt[3]{531441} = 81$ inches, the diameter of the required vessel.

EXAM. 3.

Here $343^3 = 40353607$, the cube of the given depth; and $40353607 \div \frac{1}{8}$ (the ratio) $= 5044200.875$; hence $\sqrt[3]{5044200.875} = 171.5$ inches, the depth of the required vessel.

Also, $125^3 = 1953125$, the cube of the given diameter; and $1953125 \div \frac{1}{8} = 244140.625$; hence

$\sqrt[3]{244140.625} = 62.5$ inches, the diameter of the required vessel.

EXAM. 4.

Here $30^3 = 27000$, the cube of the given length; and 27000×3 (the ratio) $= 81000$; hence $\sqrt[3]{81000} = 43.267$ inches, the length of the required cask.

Also, $24^3 = 13824$, the cube of the given bung diameter; and $13824 \times 3 = 41472$; hence $\sqrt[3]{41472} = 34.614$ inches, the bung diameter of the required cask.

Lastly, $18^3 = 5832$, the cube of the given head diameter; and $5832 \times 3 = 17496$; hence $\sqrt[3]{17496} = 25.96$ inches, the head diameter of the required cask.

PROBLEM III.

Given the dimensions and content of any vessel, to find the dimensions of a similar vessel of a given content.

EXAM. 1.

Here, as $2625 : 25^3 :: 21000 : 125000$, the cube of the length; and $\sqrt[3]{125000} = 50$ inches, the length of the required vessel.

Also, as $2625 : 15^3 :: 21000 : 27000$, the cube of the breadth; and $\sqrt[3]{27000} = 30$ inches, the breadth of the required vessel.

Lastly, as $2625 : 7^3 :: 21000 : 2744$, the cube of the depth; and, $\sqrt[3]{2744} = 14$ inches, the depth of the required vessel.

EXAM. 2.

Here $125.33 \times 282 = 35343.06$ cubic inches, the content of the given vessel.

Also, $68.5 \times 282 = 19317$ cubic inches, the content of the vessel, of which the dimensions are required.

Then, as $35343.06 : 30^3 :: 19317 : 14757.041410$, the cube of the diameter of the said vessel; hence $\sqrt[3]{14757.041410} = 24.52$ inches, its diameter.

Lastly, as $35343.06 : 50^3 :: 19317 : 68319.636160$, the cube of the depth of the said vessel; hence $\sqrt[3]{68319.636160} = 40.88$ inches, the depth required.

EXAM. 3.

Here $599.45 \times 231 = 138472.95$ cubic inches, the content of the given vessel.

Also, $946 \times 231 = 218526$ cubic inches, the content of the vessel, of which the dimensions are required.

Then, as $138472.95 : 46^3 :: 218526 : 153607.233297$, the cube of the bottom diameter of the said vessel; hence $\sqrt[3]{153607.233297} = 53.55$ inches, its bottom diameter.

And, as $138472.95 : 62^3 :: 218526 : 376108.500731$, the cube of the top diameter of the said vessel; hence $\sqrt[3]{376108.500731} = 72.18$ inches, its top diameter.

Lastly, as $138472.95 : 60^3 :: 218526 : 340872.466427$ the cube of the depth of the said vessel; hence $\sqrt[3]{340872.466427} = 69.85$ inches, the depth required.

EXAM. 4.

Here $38.75 \times 282 = 10927.5$ cubic inches, the content of the given vessel.

Also, $55.5 \times 282 = 15651$ cubic inches, the content of the vessel, whose dimensions are required.

Then, as $10927.5 : 30^3 :: 15651 : 38670.967741$ the cube of the length, of the said vessel; hence $\sqrt[3]{38670.967741} = 33.82$ inches, its length.

Again, as $10927.5 : 24^3 :: 15651 : 19799.535483$

the cube of the bung diameter, of the said vessel; hence $\sqrt[3]{19799.535483} = 27.05$ inches, its bung diameter.

Lastly, as 10927.5 : 18^3 :: 15651 : 8352.929123 the cube of the head diameter, of the said vessel; and $\sqrt[3]{8352.929123} = 2028$ inches, the head diameter required.

PART II.

THE USE OF THE SLIDING RULE.

PROBLEM I.

Multiplication by the lines A and B.

EXAM. 2.

As 1 on A : 8 on B :: 15 on A : 120 on B.

EXAM. 3.

As 1 on B : 24 on A :: 64 on B : 1536 on A.

EXAM. 4.

As 1 on A : 128 on B :: 265 on A : 33920 on B.

EXAM. 5.

As 1 on B : 5.3 on A :: 8.5 on B : 45.05 on A.

EXAM. 6.

As 1 on A : 3.5 on B :: 4.7 on A : 16.45 on B.

EXAM. 7.

As 1 on B : 1.7 on A :: 3.5 on B : 5.95 on A.

EXAM. 8.

As 1 on A : 2.4 on B :: 6.2 on A : 14.88 on B.

EXAM. 9.

As 1 on B : .9 on A :: 1.8 on B : 1.62 on A.

EXAM. 10.

As 1 on A : 32.8 on B :: 64.7 on A : 2122.16 on B.

EXAM. 11.

As 1 on B : .32 on A :: 86.3 on B : 27.616 on A.

EXAM. 12.

As 1 on A : .238 on B :: .562 on A : .133756 on B.

PROBLEM II.

Division by the lines A and B.

EXAM. 2.

As 7 on A : 1 on B :: 42 on A : 6 on B.

EXAM. 3.

As 8 on B : 1 on A :: 96 on B : 12 on A.

EXAM. 4.

As 24 on A : 1 on B :: 1536 on A : 64 on B.

EXAM. 5.

As 128 on B : 1 on A :: 33920 on B : 265 on A.

EXAM. 6.

As 5.3 on A : 1 on B :: 45.05 on A : 8.5 on B.

EXAM. 7.

As 4.7 on B : 1 on A :: 16.45 on B : 3.5 on A.

EXAM. 8.

As 2.4 on A : 1 on B :: 14.88 on A : 6.2 on B.

EXAM. 9.

As .9 on B : 1 on A :: 1.62 on B : 1.8 on A.

EXAM. 10.

As 32.8 on A : 1 on B :: 2122.16 on A : 64.7 on B.

EXAM. 11.

As 125 on B : 1 on A :: 35 on B : .28 on A.

EXAM. 12.

As 632 on A : 1 on B :: 15 on A : .0237 on B.

PROBLEM III.

To find a fourth proportional to three numbers; or to perform the Rule of Three by the lines A and B.

EXAM. 2.

gal. *s.* *gal.* *s.*
As 44 on B : 55 on A :: 8 on B : to 10 on A.

EXAM. 3.

cand. *lb.* *cand.* *lb.*
As 24 on A : 1 on B :: 1200 on A : 150 on B.

EXAM. 4.

As 12 on B : 24 on A :: 36 on B : 72 on A.

EXAM. 5.

bu. *s.* *bu.* *s.*
As 3.25 on A : 28.5 on B :: 8.75 on A : 76.73 on B.

EXAM. 6.

gal. *s.* *gal.* *s.*
As 1 on B : 19.75 on A :: 45.5 on B : 898.6 on A.

PROBLEM IV.

Inverse Proportion by the lines A and B.

EXAM. 2.

	s.		*gal.*		*s.*		*s.*
As	16 on B	:	8 on A	::	20 on B	:	10 on A.

EXAM. 3.

	men.		*da.*		*men.*		*da.*
As	48 on A	:	18 on B	::	8 on A	:	3 on B.

EXAM. 4.

	offi.		*da.*		*offi.*		*da.*
As	6 on B	:	12 on A	::	2 on B	:	4 on A.

PROBLEM V.

To reduce a vulgar fraction to a decimal, by the lines A and B.

EXAM. 2.

	Den.		*Unity.*		*Num.*		*Dec.*
As	2 on B	:	1 on A	::	1 on B	:	.5 on A.

EXAM. 3.

	Den.		*Unity.*		*Num.*		*Dec.*
As	4 on A	:	1 on B	::	3 on A	:	.75 on B.

EXAM. 4.

	Den.		*Unity.*		*Num.*		*Dec.*
As	96 on B	:	1 on A	::	24 on B	:	.25 on A.

EXAM. 4.

Set 96 on C to 96 on D, then against 486 on C, is 216 on D.

PROBLEM XI.

To cube any number by the lines C and D.

EXAM. 2.

Set 1 on D to 12 on C, then against 12 on D is 1728 on C.

EXAM. 3.

Set 1 on D to 18 on C, then against 18 on D is 5832 on C.

EXAM 4.

Set 1 on D to 35 on C, then against 35 on D is 42875 on C.

PROBLEM XII.

To extract the Cube Root of any number by the lines C and D.

EXAM. 2.

Find 64 on the line C, then move the slide until 1 on D, 64 on C, and 10 on D, cut the same number on the opposite lines, which you will find to be 4, the root required.

EXAM. 3.

Having found 120 on the line C, move the slide until 1 on D, 120 on C, and 10 on D, stand against the same number on the opposite lines, which in this case is 4.93 the root required.

EXAM. 4.

Find 200 on the line C, then move the slide until 1 on D, 200 on C, and 10 on D, cut the same number on the opposite lines, which you will find to be 5.84, the root required.

EXAM. 5.

Having found 1728 on the line C, move the slide until 1 on D, 1728 on C, and 10 on D, stand against the same number on the opposite lines, which in this case is 12, the root required.

EXAM. 6.

Find 1000 on the line C, then move the slide until 1 on D, 1000 on C, and 10 on D, cut the same number on the opposite lines, which you will find to be 10, the root required.

PART III.

Problems in Practical Geometry, for the exercise of the Learner.

PROBLEM I.

Solution.

Draw an indefinite right line with your pencil; and from any scale of equal parts, take 15 in your compasses, and transfer it to the right line; and you will have the given line AB, which you may then draw with ink. Bisect this line by Problem I. of the Guaging; and each part will measure $7\frac{1}{2}$ if the work be right.

PROBLEM II.

Solution.

Draw two lines to form any angle ABC, at pleasure; then bisect the angle by Problem II. of the Gauging; and the work will be done.

PROBLEM III.

Solution.

Draw the right line AB at pleasure, then take eight tenths of an inch in your compasses, and by proceeding as directed in Case 2, Problem III., you will be able to draw the line CD, parallel to the line AB, as required.

PROBLEM IV.

Solution.

Draw a right line at pleasure, then take in your compasses 18 inches, and lay it off upon this line; and you will obtain the given line. Take 8 inches in your compasses, and lay it off from one end of the given line, and you will obtain the point at which the perpendicular must be erected. At this point, erect the perpendicular as directed in Problem IV., and the work will be completed.

PROBLEM V.

Solution.

Draw a right line at pleasure, then take 24 inches in your compasses, and lay it off upon this line, and you will obtain the base of the given triangle.

With 20 in your compasses, as a radius, and one end of the base as a centre, describe an arc; and with 16 in your compasses, and the other end of the base as a centre, describe another arc, cutting the former arc; then proceed as directed in Problem VII.; and the work will be finished.

PROBLEM VI.

Solution.

Draw a right line at pleasure; then take 24 inches in your compasses, lay it off upon this line, and you will have the base of the triangle.

Take 14 in your compasses, and lay it off from one end of the base, and you will have the point, at which the perpendicular must be raised.

From this point, erect a perpendicular at pleasure by Problem IV.; and upon this line lay off 12 inches; proceed as directed in Problem VIII., and the work will be finished.

PROBLEM VII.

Solution.

Draw a right line at pleasure; then take 16 inches in your compasses, and lay it off upon this line, and you will obtain one side of the square; proceed as directed in Problem IX, and you will obtain the square required.

PROBLEM VIII.

Solution.

Having drawn a right line at pleasure, take 24 inches in your compasses, lay it off upon this line, and you will have the length of the rectangle.

At one extremity of the line already laid down, erect a perpendicular at pleasure; then take 12 inches in your compasses, lay it off upon this line, and you will have the breadth of the rectangle.

Proceed as directed in Problem X., and you will obtain the rectangle required.

PROBLEM IX.

Solution.

Draw a line at pleasure; take 24 inches in your compasses, and lay it off upon this line, and you will have the base of the regular rhombus.

Proceed as directed in Problem XI., and you will be able to complete the required figure.

PROBLEM X.

Solution.

Having drawn a right line at pleasure, take 16 in your compasses, and lay it off upon this line, and you will have the base of the irregular rhombus.

At one end of the base, erect a perpendicnlar, which make equal to 12 inches; and then by proceeding as directed in Problem XII., you will be able to construct the required figure.

PROBLEM XI.

Solution.

Draw a right line at pleasure, then take 36 in your compasses, and lay it off upon this line, and you will have the base of the regular rhomboid.

Take 18 in your compasses, and lay it off from one end of the base; and by proceeding as directed in Problem XIII., you will be able to complete the required figure.

PROBLEM XII.

Solution.

Having drawn a right line at pleasure, take in your compasses 28 inches, lay it off upon this line, and you will have the transverse diameter of the ellipse.

Bisect this diameter perpendicularly, and you will obtain the centre of the figure. From this centre, set off 8 inches both ways upon the bissecting line; and you will have the conjugate diameter. Then by proceeding as directed in Problem XV., you will be able to construct the ellipse required.

PROBLEM XIII.

Solution.

Take 14 inches in your compasses, and with this radius, describe a circle; then apply your compasses six times to the circle, and you will have the angular points of the hexagon. Join these points by right lines, and the figure will be completed. (See Problem XVII.)

PROBLEM XIV.

Solution.

Draw an unlimited line at pleasure; then take 60° in in your compasses, and with one end of this line as a centre, describe an arc. Take 42° 30′ in the compasses, and set it off upon this arc; and by proceeding as directed in Problem XXIII., you will be able to complete the work.

PROBLEM XV.

Solution.

Draw the indefinite line AB; and with the chord of 60° in your compasses, describe the arc DE; set off 90° from D to C, and from C to G, set off the excess above 90°, which is 46° 45′. Draw the line AG, and GAB will be the required angle. (See Problem XXIV.)

PART IV.

MENSURATION OF SUPERFICIES

APPLIED TO

GAUGING.

PROBLEM I.

The side of a square being given, to find the area in ale gallons, wine gallons, mash-tun gallons, and malt bushels.

EXAM. 3.

By Multiplication.

```
         Inches.
          153.4 side.
          153.4 side.
          -----
          6136
         4602
        7670
       1534
       ---------
       23531.56 area in square inches.
        .003546 multiplier.
      -----------
      14118936
      9412624
    11765780
    7059468
    -----------
    83.44291176 area in ale gallons.
```

Square Inches.

```
  23531.56 area.
   .004329 multiplier.
  --------
  21178404
  4706312
 7059468
9412624
-----------
101.86812324 wine gallons.
```

Square Inches.

```
  23531.56 area.
   .000465 multiplier.
  --------
  11765780
 14118936
 9412624
-----------
10.94217540 malt bushels.
```

By Division.

Square Inches.

```
Divisor 282)23531.56(83.445 ale gallons.
            2256
            ----
            ..971
              846
              ----
              1255
              1128
              ----
              .1276
               1128
               ----
               .1480
                1410
                ----
                ..70
```

Square Inches.

```
Divisor 231)23531.56(101.868 wine gallons.
            231
            ..431
              231
              2005
              1848
              .1576
               1386
               .1900
                1848
                ..52
```

Square Inches.

```
Divisor 2150.42)23531.56(10.942 malt bushels.
                215042
                .2027360
                 1935378
                 ..919820
                   860168
                   .596520
                    430084
                    166436
```

EXAM. 4.

Here $82.2 \times 82.2 = 6756.84$, the area in square inches; and $6756.84 \div 282 = 23.96$, the area in ale gallons; then $23.96 \times 5.4 = 129.384$, the content in ale gallons.

EXAM. 5.

Here $43.2 \times 43.2 = 1866.24$, the area in square inches; and $1866.24 \div 231 = 8.078$, the area in wine gallons; then $8.078 \times 52.7 = 425.710$, the content in wine gallons.

EXAM. 6.

Here $75 \times 75 = 5625$, the area in square inches; and $5625 \div 2150.42 = 2.615$, the area in malt bushels; then $2.615 \times 16.4 = 42.886$, the content in malt bushels.

EXAM. 7.

1. Here $84 \times 84 = 7056$, the area in square inches; and $7056 \div 282 = 25.021$, the area in ale gallons; then $25.021 \times 14 = 350.294$, the content in ale gallons.
2. Here $7056 \div 231 = 30.545$, the area in wine gallons; and $30.545 \times 14 = 427.63$, the content in wine gallons.
3. Here $7056 \div 227 = 31.083$, the area in mash-tun gallons; and $31.083 \times 14 = 435.162$, the content in mash-tun gallons.
4. Here $7056 \div 2150.42 = 3.281$, the area in malt bushels; and $3.281 \times 14 = 45.934$, the content in malt bushels.

PROBLEM II.

To find the area of a rectangle.

EXAM. 3.

Inches.

```
          162.7 length.
           86.3 breadth.
          -----
           4881
          9762
        13016
        --------
Divisor 282)14041.01(49.79 area in ale gallons.
        1128
        ----
         .2761
          2538
          ----
          .2230
           1974
           ----
           .2561
            2538
            ----
            ..23
```

Gallons.

$$\begin{array}{r} 49.79 \text{ area.} \\ 6.8 \text{ depth.} \\ \hline 39832 \\ 29874 \\ \hline 338.572 \text{ the content.} \end{array}$$

EXAM. 4.

Here $74.6 \times 65.7 = 4901.22$, the area in square inches; and $4901.22 \div 231 = 21.217$, the area in wine gallons; then $21.217 \times 24.7 = 524.0599$ gallons, the content.

EXAM. 5.

Here $64 \times 52 = 3328$, the area in square inches; and $3328 \div 227 = 14.660$, the area in mash-tun gallons, which, by the question, is also the area in malt gallons; then $14.660 \times 32 = 469.12$, the content in malt gallons; hence $469.12 \div 8 = 58.64$, the number of bushels required.

EXAM. 6.

Here $124.5 \times 98.2 = 12225.90$, the area in square inches; and $12225.90 \div 2150.42 = 5.685$, the area in malt bushels; then $5.685 \times 14.8 = 84.138$ malt bushels, the content.

EXAM. 7.

Here $245 \times 152 = 37240$, the area in square inches; and $37240 \div 2150.42 = 17.317$, the area in malt bushels; then $17.317 \times 5.9 = 102.1703$ malt bushels, the content.

PROBLEM III.

To find the area of a rhombus or rhomboides.

EXAM. 3.

Here $73.6 \times 56.2 = 4136.32$, the area in square inches; and $4136.32 \div 282 = 14.667$, the area in ale

gallons; then $14.667 \times 2.7 = 39.6009$, the content in ale gallons.

EXAM. 4.

Here $108.4 \times 70.6 = 7653.04$, the area in square inches; and $7653.04 \div 282 = 27.138$, the area in ale gallons; then $27.138 \times .8 = 21.7104$, the content in ale gallons.

PROBLEM IV.

To find the area of a triangle, when the base and perpendicular are given.

EXAM. 3.

Here $\frac{124 \times 94.7}{2} = \frac{11742.8}{2} = 5871.4$, the area in square inches; and $5871.4 \div 282 = 20.82$, the area in ale gallons; then $20.82 \times 4.1 = 85.362$, the content in ale gallons.

EXAM. 4.

Here $\frac{136.7 \times 86.8}{2} = \frac{11865.56}{2} = 5932.78$, the area in square inches; and $5932.78 \div 2150.42 = 2.758$, the area in malt bushels; then $2.758 \times 42.3 = 116.6634$, the content in malt bushels.

PROBLEM V.

To find the area of a triangle, when the three sides are given.

EXAM. 3.

Here $\frac{114 + 112 + 98}{2} = 162$, half the sum of the sides; then $162 - 114 = 48$, the first remainder;

$162 - 112 = 50$, the second remainder; and $162 - 98 = 64$, the third remainder; whence $\sqrt{162 \times 48 \times 50 \times 64} = \sqrt{24883200} = 4988.306$, the area in square inches; then $4988.306 \div 282 = 17.689$, the are in ale gallons; $4988.306 \div 231 = 21.594$, the area in wine gallons; and $4988.306 \div 2150.42 = 2.319$, the area in malt bushels.

EXAM. 4.

Here $\frac{74 + 74 + 74}{2} = \frac{222}{2} = 111$, half the sum of the sides; then $111 - 74 = 37$, the first remainder; $111 - 74 = 37$, the second remainder; and $111 - 74 = 37$, the third remainder; whence $\sqrt{111 \times 37 \times 37 \times 37} = \sqrt{5622483} = 2371.177$, the area in square inches; then $2371.177 \div 282 = 8.408$, the area in ale gallons; and $8.408 \times 28.6 = 240.4688$, the content in ale gallons.

PROBLEM VI.

To find the area of a trapezium.

EXAM. 3.

Ale gallons.

$$\begin{array}{r} 14.967 \text{ area.} \\ 3.7 \text{ depth.} \\ \hline 104769 \\ 44901 \\ \hline 55.3779 \text{ Ans.} \end{array}$$

EXAM. 4.

Here $\overline{28.2 + 47} \times 94.4 = 75.2 \times 94.4 = 7098.88$; and $7098.88 \div 2 = 3549.44$, the area in square

inches; then $3549.44 \div 231 = 15.365$, the area in wine gallons.

EXAM. 5.

Here $\overline{58.4 + 46.8} \times 126.6 = 105.2 \times 126.6 = 13318.32$; and $13318.32 \div 2 = 6659.16$, the area in square inches; then $6659.16 \div 2150.42 = 3.096$, the area in malt bushels; whence $3.096 \times 39.3 = 121.6728$ bushels, the whole content; and $121.6728 - \frac{121.6728}{5} = 121.6728 - 24.3345 = 97.3383$ bushels, the answer required.

PROBLEM VII.

To find the area of a trapezoid.

EXAM. 3.

Here $\overline{214 + 178} \times 146 = 392 \times 146 = 57232$; and $57232 \div 2 = 28616$, the area in square inches; then $28616 \div 282 = 101.475$, the area in ale gallons.

EXAM. 4.

Here $101.475 \times 1.7 = 172.5075$ ale gallons, the content required.

PROBLEM VIII.

To find the area of an irregular polygon of any number of sides.

EXAM. 2.

Ale gallons.

$$\begin{array}{r l}
24.414 & \text{area.} \\
6.7 & \text{depth.} \\
\hline
170898 & \\
146484 & \\
\hline
163.5738 & \textit{Ans.}
\end{array}$$

EXAM. 3.

Here $\frac{160 + 102 + 94}{2} = \frac{356}{2} = 178$, half the sum of the sides; then $178 - 160 = 18$, the first remainder; $178 - 102 = 76$, the second remainder; and $178 - 94 = 84$, the third remainder; whence $\sqrt{178 \times 18 \times 76 \times 84} = \sqrt{20454336} = 4522.647$ inches, the area of the first triangle.

Again, $\frac{160 + 130 + 64}{2} = \frac{354}{2} = 177$, half the sum of the sides; then $177 - 160 = 17$, the first remainder; $177 - 130 = 47$, the second remainder; and $177 - 64 = 113$, the third remainder; whence $\sqrt{177 \times 17 \times 47 \times 113} = \sqrt{15980799} = 3997.599$ inches, the area of the second triangle.

Also, $\frac{140 + 130 + 100}{2} = \frac{370}{2} = 185$, half the sum of the sides; then $185 - 140 = 45$, the first remainder; $185 - 130 = 55$, the second remainder; and $185 - 100 = 85$, the third remainder; whence $\sqrt{185 \times 45 \times 55 \times 85} = \sqrt{38919375} = 6238.539$ inches, the area of the third triangle; hence we have $4522.647 + 3997.599 + 6238.539 = 14758.785$ square inches, the area of the whole polygon.

Then, $\frac{14758.785}{282} = 52.336$ ale gallons; $\frac{14758.785}{231} = 63.890$ wine gallons; and $\frac{14758.785}{2150.42} = 6.863$ malt bushels.

EXAM. 4.

Trapezoids on the right.

232 perp.
100 perp.
332 sum.
335 base.
1660
996
996
111220 prod.

125 perp.
52 perp.
177 sum.
195 base.
885
1593
177
34515 prod.

100 perp.
152 perp.
252 sum.
220 base.
5040
504
55440 prod.

52 perp.
126 perp.
178 sum.
334 base.
712
534
534
59452 prod.

152 perp.
125 perp.
277 sum.
230 base.
8310
554
63710 prod.

Double areas collected.
111220
55440
63710
34515
59452
324337 sum.

Trapezoids on the left.

```
 360 perp.             312 perp.
 336 perp.             234 perp.
 696 sum.              546 sum.
   260 base.           305 base.
 41760                2730
1392                 16380
180960 prod.         166530 prod.

 336 perp.             234 perp.
 215 perp.             309 base.
 551 sum.             2106
    200 base.         7020
 110200 prod.         72306 prod.

 215 perp.           Double areas collected.
 312 perp.             180960
 527  sum.             110200
   240 base.           126480
 21080                 166530
1054                    72306
126480 prod.           656476 sum.
```

Whole double areas.

Square inches.

```
  324337
  656476
2)980813 sum.
  490406 area.
```

Then $490406 \div 282 = 1739.028$, the area in ale gallons; $490406 \div 231 = 2122.969$, the area in wine gallons; and $490406 \div 2150.42 = 228.051$, the area in malt bushels.

PROBLEM IX.

To find the area of a regular polygon.

EXAM. 2.

Ale gallons.

```
 15.603 area.
   38.9 depth.
 ------
 140427
124824
46809
-----------
606.9567 Ans.
```

EXAM. 3.

Here $64.8 \times 5 \times 44.6 = 14450.4$, and $\frac{14450.4}{2} =$ 7225.2, the area in square inches; then $7225.2 \div 282 = 25.621$, the area in ale gallons.

EXAM. 4.

Here $46.3 \times 7 \times 48.1 = 15589.21$; and $\frac{15589.21}{2} = 7794.605$, the area in square inches; then $\frac{7794.605}{231} = 33.742$, the area in wine gallons; and $33.742 \times 48.6 = 1639.8612$, the content in wine gallons.

PROBLEM X.

To find the area of a regular polygon, when the side only is given.

EXAM. 2.

Here $21.7 \times 21.7 \times .009213 = 470.89 \times .009213 = 4.33830957$, the area in ale gallons; and $4.3383 \times 41.6 = 180.47328$, the content in ale gallons.

Exam. 3.

Here $24.3 \times 24.3 \times .015731 = 590.49 \times .015731 = 9.28899819$, the area in wine gallons.

Exam. 4.

Here $31.2 \times 31.2 \times .002245 = 973.44 \times .002245 = 2.1853728$, the area in malt bushels.

PROBLEM XI.

Given the diameter of a circle to find the circumference; or the circumference to find the diameter.

Exam. 2.

Here $265.4652 \div 3.1416 = 84.5$ inches, the diameter required.

Exam. 3.

Here $36.9 \times 3.1416 = 115.92504$ inches, the circumference required.

Exam. 4.

Here $86.7 \div 3.1416 = 27.5974$ inches, the diameter required.

PROBLEM XII.

To find the length of the arc of a circle.

Exam. 2.

Here $\dfrac{60 \times 8 - 96}{3} = \dfrac{480 - 96}{3} = \dfrac{384}{3} = 128$ inches, the length of the arc required.

Exam. 3.

(See the figure in the Gauging.)

Here we have given AD = 30, and CD = 16, which are the base and perpendicular of the right-

angled triangle ADC; then by Prob. IV., Part I., $\sqrt{30^2+16^2}=\sqrt{900+256}=\sqrt{1156}=34=$ AC, the chord of half the arc; and by the Rule, $\frac{34\times8-60}{3}=\frac{272-60}{3}=\frac{212}{3}=70.666$ inches, the length of the arc ACB.

Exam. 4.

Here AC is the hypothenuse, and CD the perpendicular of a right angled triangle; then by Prob. V., Part I., $\sqrt{42.5^2-20^2}=\sqrt{1806.25-400}=\sqrt{1406.25}$ $=37.5=$ AD, the base; and $37.5\times2=75=$ AB, the chord of the whole arc; hence by the Rule, we have $\frac{42.5\times8-75}{3}=\frac{340-75}{3}=\frac{265}{3}=88.333$ inches, the length of the arc ACB.

Exam. 5.

Here $\frac{30\times8-48}{3}=\frac{240-48}{3}=\frac{192}{3}=64$, the ength of the remaining arc ACB.

Also, $3.1416\times50=157.08$, the circumference of he circle ACBE; then $157.08-64=93.08$ inches, he length of the required arc AEB. *(See Note 2.)*

PROBLEM XIII.

To find the area of a circle.

Exam. 4.

By Multiplication.

Here $38.5\times38.5\times.002785=1482.25\times.002785$ $=4.12806625$, the area in ale gallons; $1482.25\times$ $.03399=5.03816775$, the area in wine gallons; 1482.25 $.0346=5.1285850$, the area in mash-tun gallons; ıd $1482.25\times.000365=.54102125$, the area in malt ıshels.

EXAM. 5.

By Division.

Here $78.6 \times 78.6 \div 294.12 = 6177.96 \div 294.12 = 21.0046$, the area in wine gallons.

By the Table of Wine Areas.

Having found 78 in the first column, and .6 at the top of the page; then in the common angle of meeting, we have 21.0050, the area in wine gallons.

EXAM. 6.

By Multiplication.

Here $53.2 \times 53.2 \times .002785 = 2830.24 \times .002785 = 7.8822184$, the area in ale gallons; and $7.8822 \times 35.7 = 281.39454$, the content in ale gallons.

PROBLEM XIV.

To find the area of the sector of a circle.

EXAM. 2.

Here $\dfrac{42.6 \times 31.5}{2} = \dfrac{1341.9}{2} = 670.95$, the area in square inches; and $670.95 \div 231 = 2.9045$, the area in wine gallons.

EXAM. 3.

(See the figure in the Gauging.)

Here we have given AD $= 60$, the hypothenuse, and DE $= 36$, one of the legs of the right-angled triangle AED, to find AE, the other leg. Then by Prob. V., Part I., $\sqrt{60^2 - 36^2} = \sqrt{3600 - 1296} = \sqrt{2304} = 48 =$ AE; and AC $-$ AE $= 60 - 48 = 12$, the versed sine EC.

Again, in the right-angled DEC, by Prob. IV., Part I., we have $\sqrt{36^2 + 12^2} = \sqrt{1296 + 144} = \sqrt{1440} = 37.947 =$ DC, the chord of half the arc; then

by Prob. XII., Part IV., we have $\frac{37.947 \times 8 - 72}{3} = \frac{303.576 - 72}{3} = \frac{231.576}{3} = 77.192$, the length of the arc DCB.

Lastly, $\frac{77.192 \times 60}{2} = \frac{4631.52}{2} = 2315.76$, the area of the sector ABCD, in square inches; and $2315.76 \div 2150.42 = 1.0768$, the area in malt bushels.

Exam. 4.

By Prob. XII., we have $\frac{69.3 \times 8 - 124.8}{3} = \frac{55.4 - 124.8}{3} = \frac{429.6}{3} = 143.2$, the length of the arc; then $\frac{143.2 \times 80}{2} = \frac{11456}{2} = 5728$, the area of the sector, in square inches; and $5728 \div 282 = 20.312$, the area in ale gallons; hence $20.312 \times 8.7 = 176.7144$, the content in ale gallons.

PROBLEM XV.

To find the area of the segment of a circle.

RULE I.

Exam. 2.

Here $128 \times 32 \times \frac{2}{3} = 4096 \times \frac{2}{3} = \frac{8192}{3} = 2730.666$, two-thirds of the product of the chord and height of the segment; and $\frac{32^3}{128 \times 2} = \frac{32768}{256} = 128$, the cube of the height divided by twice the chord; then $2730.666 + 128 = 2858.666$, the area in square inches; whence, $2858.666 \div 282 = 10.137$, the area in ale gallons; and $2858.666 \div 231 = 12.375$, the area in wine gallons.

EXAM. 3.

By Note 2, we have $\frac{30^2}{40}+40=\frac{900}{40}+40=22.5$ $+40=62.5$, the diameter CD; then $62.5^2 \times .7854=3906.25\times .7854=3067.96875$, square inches, the area of the whole circle.

Again, $62.5-40=22.5$, the versed sine CE; and $22.5\times 60\times \frac{2}{3}=1350\times \frac{2}{3}=\frac{2700}{3}=900$, two-thirds of the product of the chord and height of the remaining segment ABC; also $\frac{22.5^3}{60\times 2}=\frac{11390.625}{120}=94.921$, the cube of the height divided by twice the chord; then $900+94.921=994.921$ square inches, the area of the segment ABC.

Now, $3067.96875-994.921=2073.04775$, the area of the required segment ADB, in square inches; then $2073.047\div 282=7.351$, the area in ale gallons; $2073.047\div 231=8.974$, the area in wine gallons; and $2073.04775\div 2150.42=.964$, the area in malt bushels.

RULE II.

EXAM. 2.

Here $\frac{32.0}{160}=.2$, the quotient or tabular height; and the corresponding *Area Seg.* is .111823; hence $.111823\times 160^2=.111823\times 25600=2862.6688$, the area of the segment, in square inches; then 2862.668 $\div 282=10.151$, the area in ale gallons; and 2862.668 $\div 231=12.392$, the area in wine gallons.

EXAM. 3.

Here, $\frac{40.0}{62.5}=.64$, the tabular height; then $1-.64$ $=.36$; and the *Area Seg.* answering to .36, is .254550, which being taken from .785398, leaves .530848, the *Area Seg.* corresponding to .64; hence $.530848\times 62.5^2$

$= .530848 \times 3906.25 = 2073.625$, the area of the segment in square inches; then $2073.625 \div 282 = 7.353$, the area in ale gallons; $2073.625 \div 231 = 8.976$, the area in wine gallons; and $2073.625 \div 2150.42 = .9642$, the area in malt bushels. *(See Note 2.)*

PROBLEM XVI.

To find the area of an ellipse.

EXAM. 2.

Ale gallons.

$$
\begin{array}{r}
21.467 \text{ area.} \\
10.8 \text{ depth.} \\
\hline
171736 \\
21467 \\
\hline
231.8436 \text{ Ans.}
\end{array}
$$

EXAM. 3.

Here $85.9 \times 63.8 \div 294.12 = 5480.42 \div 294.12 = 18.633$, the area in wine gallons.

EXAM. 4.

Here $96.8 \times 73.2 \div 2738 = 7085.76 \div 2738 = 2.587$, the area in malt bushels; and $2.587 \times 34.7 = 89.7689$, the content in malt bushels.

EXAM. 5.

Here $96.4 \times 82.3 \div 359.05 = 7933.72 \div 359.05 = 22.096$, the area in ale gallons; and $22.096 \times 53.8 = 1188.7648$, the content in ale gallons.

PROBLEM XVII.

To find the area of an elliptical segment, the base of which is parallel to either of the diameters of the ellipse.

EXAM. 2.

By Prob. XVI., we have $120.8 \times 75.4 \div 294.12 = 9108.32 \div 294.12 = 30.9680$ wine gallons, the area of the whole ellipse.

By the last example, the area of the segment ACB is 1813.3298 square inches; hence $1813.3298 \div 231 = 78499$; the area in wine gallons; then $30.9680 - 7.8499 = 23.1181$ wine gallons, the area of the segment ADB.

EXAM. 3.

(See the figure in the Gauging.)

Here $\frac{180}{2} + 27 = 90 + 27 = 117 =$ DG, the height of the segment; then $117 \div 180 = .650$, the tabular height; and by Prob. XV., Rule 2, Note 2, we have $1 - .650 = .350$; and the *Area Seg.* answering to .350, is 244980, which being taken from .785398, leaves .540418, the *Area Seg.* corresponding to .650; hence $180 \times 120 \times .540418 = 21600 \times .540418 = 11673.0288$ square inches, the area of the segment ADB; and $11673.0288 \div 2150.42 = 5.4282$, the area in malt bushels.

PROBLEM XVIII.

To find the area of a parabola, its base and height being given.

EXAM. 2.

Here $108.6 \times 36.2 \times \frac{2}{3} = \frac{108.6 \times 36.2 \times 2}{3} = \frac{3931.32 \times 2}{3} = \frac{7862.64}{3} = 2620.88$, the area in square

inches; and $2620.88 \div 231 = 11.3458$, the area in wine gallons.

PROBLEM XIX.

To find the area of compound figures.

EXAM. 2.

By Prob. V., we have $\frac{75 + 100 + 125}{2} = \frac{300}{2} = 150$, half the sum of the sides; then, $150 - 75 = 75$, the first remainder; $150 - 100 = 50$, the second remainder; $150 - 125 = 25$, the third remainder; whence $\sqrt{150 \times 75 \times 50 \times 25} = \sqrt{14062500} = 3750$, the area of the triangle in square inches.

By Prob. XV., we have $125 \times 26 \times \frac{2}{3} = 3250 \times \frac{2}{3} = \frac{6500}{3} = 2166.666$, two thirds of the product of the chord and versed sine; and $\frac{26^3}{125 \times 2} = \frac{17576}{250} = 70.304$, the cube of the height divided by twice the chord; then $2166.666 + 70.304 = 2236.97$, the area of the segment in square inches.

Now, $3750 + 2236.97 = 5986.97$, the area of the whole compound figure; hence $5986.97 \div 282 = 21.23$, the area in ale gallons; and, $21.23 \times 86 = 182.578$, the content in ale gallons.

PROBLEM XX.

To find the area of any curvilineal figure, by means of equidistant ordinates.

EXAM. 6.

	Inches.
First ordinate	35.2
Last ditto	35.3
Sum	70.5

Exam. 7.

	Inches.
First ordinate	31.2
Last ditto	32.0
Sum	63.2

Even ordinates.

	Inches.
Second	50.8
Fourth	68.2
Sixth	68.8
Eighth	52.0
Sum	239.8
Multiply by	4
Four times the sum	959.2

Odd ordinates.

	Inches.
Third	62.0
Fifth	71.0
Seventh	63.0
Sum	196.0
Multiply by	2
Twice the sum	392.0

	Inches.
Sum of the first and last ordinates	63.2
Four times the sum of the even ordinates	959.2
Twice the sum of the odd ordinates	392.0
Sum total	1414.4
Multiply by 14 (the common distance)	14
	56576
	14144
Divide the product by	3)19801.6
The area in square inches	6600.53

The two Segments.

	Inches.
Sum of the two extreme ordinates	63.2
Sum of the heights	10
Divide the product by	3)632.0
Area of both segments	210.66
Area brought forward	6600.53
Area of the whole figure in square inches	6811.19

Square Inches.

Divisor 282)6811.19(24.153 *ale gallons.*

```
564
----
1171
1128
----
  431
  282
  ----
  1499
  1410
  ----
    890
    746
    ---
     44
```

Square Inches.

Divisor 231)6811.19(29.485 *wine gallons.*

```
462
----
2191
2079
----
 1121
  924
 ----
  1979
  1848
  ----
   1310
   1155
   ----
    155
```

Cubic Inches.

```
   202262.003 content.
       .004 329 multiplier.
  ___________
  1820358 027
  4045240 06
 60678600 9
809048012
___________
875.592210987 wine gallons.
```

Cubic Inches.

```
 202262.003 content.
     :000 465 multiplier.
 ___________
 1011310 015
12135720 18
80904801 2
___________
94.051831 395 malt bushels.
```

By Division.

Cubic Inches.

```
Divisor 282)202262.003(717.241 ale gallons.
            1974
            ----
             486
             282
             ----
             2042
             1974
             ----
               68.0
               56 4
               ----
               11 60
               11 28
               -----
                  323
                  282
                  ---
                   41
```

Cubic Inches.

```
Divisor 231)202262.003(875.593 wine gallons.
            1848
            ----
             1746
             1617
             ----
              1292
              1155
              ----
               1370
               1155
               ----
                2150
                2079
                ----
                 713
                 693
                 ---
                  20
```

Cubic Inches.

```
Divisor 150.42)202262.003(94.056 malt bushels.
               193537 8
               --------
                 8724 20
                 8601 68
                 -------
                  1225230
                  1075210
                  -------
                   1500200
                   1290252
                   -------
                    209948
```

EXAM. 3.

Here $86.5 \times 86.5 \times 86.5 = 748.25 \times 86.5 = 647214.625$, the content in cubic inches; and $647214.625 \div 231 = 2801.794$, the content of the vat in wine gallons.

EXAM. 4.

(See the Note.)

Gallons.

862 *content.*
282 *cubic inches.*

1724
6896
1724

243084 *content in cubic inches.*

Cubic Inches.

243084(62.4 inches, the side required.
216

27084 resolvend.

18 triple of 6.
108 triple square of 6.

1098 divisor.

8 cube of 2.
72 square of 2 × by the triple of 6.
216 triple of 2 × by the square of 6.

22328 subtrahend.

4756000 second resolvend.

186 triple of 62.
11532 triple square of 62.

115506 second divisor.

64 cube of 4.
2976 square of 4 × by the triple of 62.
46128 triple of 4 × by the square of 62.

4642624 second subtrahend.

113376 remainder.

PROBLEM II.

To find the content of a vessel in the form of a parallelopipedon.

EXAM. 2.

```
              Inches.
               92.8  length.
               64.5  breadth.
              -----
              464 0
             3712
            5568
            -------
            5985.60  product.
               46.2  depth.
            -------
          11971 20
         3591360
        2394240
Divisor 282)276534.720(980.619 ale gallons.
        2538
        ----
         2273
         2256
         ----
           1747
           1692
           ----
             552
             282
             ----
             2700
             2538
             ----
              162
              ===
```

Also, $276534.720 \div 231 = 1197.12$, the content in wine gallons; and $276534.720 \div 2150.42 = 128.595$, the content in malt bushels.

EXAM. 3.

Here $104 \times 92.4 \times 38.6 = 9609.6 \times 38.6 = 370930.56$, the content in cubic inches; and $370930.56 \div 231 = 1605.76$, the content in wine gallons.

EXAM. 4.

Here 1[illegible] x [illegible] x 33.5 = 13482 x 33.5 = 4[illegible], the content in cubic inches; and 4[illegible] ÷ 2150.42 = [illegible], the content in malt bushels.

EXAM. 5.

Here [illegible] x [illegible] x 6.3 = 38730 x 6.3 = [illegible], the content in cubic inches; and [illegible] ÷ 2150.42 = [illegible] the content in malt bushels.

EXAM. 6.

(See the Note.)

By the question, the content of the vessel is 456.[illegible] ale gallons; and 456.5 x 282 = 12[illegible], the content in cubic inches.

Also, 83.3 x 45.3 = 3,795.14, the area of the base in inches; hence 12[illegible] ÷ 3,795.14 = 32.646 inches, the depth required.

PROBLEM III.

To find the content of a vessel in the form of a prism.

EXAM. 2.

(See Prob. X., Part IV.)

Inches.	
24.5	side.
24.5	side.
1225	
980	
490	
600.25	square of the side.
.0061 01	multiplier.
60025	
60025	
360150	
3.66212525	area in ale gallons.
57.4	depth.
14 64850100	
256 3487675	
1831 062625	
210.2 05989350	content in ale gallons.

Also, $600.25 \times .007448 = 4.470662$, the area of the base, in wine gallons; and $4.470662 \times 57.4 = 256.615998$, the content in wine gallons.

Likewise, $600.25 \times .000800 = .4802$, the area of the base, in malt bushels; and $.4802 \times 57.4 = 27.56348$, the content in malt bushels.

EXAM. 3.

Here $32.6 \times 32.6 = 1062.76$, the square of the side; and $1062.76 \times .011247 = 11.95286172$, the area of the base, in wine gallons; then $11.95286172 \times 49.7 = 594.057227484$, the content in wine gallons.

EXAM. 4.

By Prob. V., Part IV., we have $\frac{96 + 125 + 159}{2} = \frac{380}{2} = 190$, half the sum of the sides, then $190 - 96 = 94$, the first remainder; $190 - 125 = 65$, the second remainder; and $190 - 159 = 31$, the third remainder; whence $\sqrt{190 \times 94 \times 65 \times 31} = \sqrt{35987900} = 5998.991$, the area of the base, in inches; and $5998.991 \div 282 = 21.273$, the area in ale gallons; hence $21.273 \times 33.4 = 710.5182$, the content in ale gallons.

EXAM. 5.

Here $112 + 93 = 205$, the sum of the perpendiculars; and $\frac{205 \times 274}{2} = \frac{56170}{2} = 28085$, the area in inches; then $28085 \div 2150.42 = 13.060$, the area in malt bushels; hence $13.060 \times 39.4 = 514.564$, the content in malt bushels.

EXAM. 6.

By the question, the content of the vessel is 579.3 wine gallons; and $579.3 \times 231 = 133818.3$, the content in cubic inches.

By Prob. X., Part IV., we have 24.6 × 24.6 × 4.828427 = 605.16 × 4.828427 = 2921.97088332, the area of the base, in inches; then by Note 3, of this Prob. we have 133818.3 ÷ 2921.97088332 = 45.79 inches, the depth required.

PROBLEM IV.

To find the content of a vessel in the form of a cylinder.

EXAM. 2.

```
                    Inches.
                     68.6 diameter.
                     68.6 ditto.
                    -----
                    411 6
                   5488
                  4116
                  -------
                  4705.96 square of ditto.
                       74 depth.
                 --------
                 18823 84
                329417 2
                ---------
Divisor 359.05)348241.04(969.895 ale gallons.
               323145
               ------
                250960
                215430
                ------
                 355304
                 323145
                 ------
                  321590
                  287240
                  ------
                   343500
                   323145
                   ------
                    203550
                    179525
                    ------
                     24025
```

Exam. 3.

Here, $43.1 \times 43.1 \times 57.4 = 1857.61 \times 57.4 = 106626.814$; and $106626.814 \div 294.12 = 362.527$, the content in wine gallons.

Exam. 4.

Here $96.8 \times 96.8 \times 84.6 = 6670.04 \times 84.6 = 564285.384$; and $564285.384 \div 2738 = 206.094$, the content in malt bushels.

Exam. 5.

Here $252 \times 252 \times 324 = 63504 \times 324 = 20575296$; and $20575296 \div 294.12 = 69955.446$ wine gallons, the answer required.

PROBLEM V.

To find the content of a vessel in the form of a pyramid.

Exam. 2.

(See Prob. X., Part IV.)

(By Note 4.)

```
     Inches.
       52.5 side.
       52.5 side.
     ------
      262 5
     1050
    2625
    -------
    2756.25 square of the side.
    .0043 29 multiplier.
    --------
    24806 25
    551250
   826875
 1102500
 -----------
 11.931806 25 area of the top in wine gallons.
           42 one third of the per. depth.
 ------------
 23 863612 50
477 2722500
 ------------
501.135862 50 content in wine gallons.
 ============
```

Exam. 3.

(See Prob. V., Part IV.)

By the Rule.

Here $\frac{46+48+54}{2}=\frac{148}{2}=74$, half the sum of the three sides; then $74-46=28$, the first remainder; $74-48=26$, the second remainder; and $74-54=20$, the third remainder; whence $\sqrt{74\times28\times26\times20}=\sqrt{1077440}=1037.949$, area of the base in inches; and $1037.949\times25=25948.725$, the content in cubic inches; then $25948.725\div2150.42=12.066$, the content in malt bushels.

PROBLEM VI.

To find the content of a vessel in the form of the frustum of a pyramid.

Exam. 2.

By Rule II.

Here $92\times92=8464$, the square of a side of the greater end; $54\times54=2916$, the square of a side of the less end; and $92\times54=4968$, the product of the sides; then $(8464+2916+4968)\times42=16348\times42=686616$, the content in cubic inches. Hence $686616\times.011247=7722.370152$, the content in wine gallons.

Exam. 3.

By Rule II.

Here $35\times35=1225$, the square of a side of the greater end; $28\times28=784$, the square of a side of the less end; and $35\times28=980$, the product of the sides; then $(1225+784+980)\times94\div3=2989\times94\div3=280966\div3=93655.33$, the con-

tent in cubic inches. Then $93655.33 \times .017122 = 1603.56656026$, the content in ale gallons; $93655.33 \times .020902 = 1957.58370766$, the content in wine gallons; and $93655.33 \times .002245 = 210.25621585$ the content in malt bushels.

PROBLEM VII.

To find the content of a vessel in the form of a cone.

EXAM. 2.

By Rule I.

By Prob. V., Part I., we have $\sqrt{112^2 - 42^2} = \sqrt{12544 - 1764} = \sqrt{10780} = 103.826$, the perpendicular depth of the vessel, one third of which is 34.608 inches. Then $84^2 \times 34.608 = 7056 \times 34.608 = 2441940.480$; and $2441940.480 \div 359.05 = 680.1115$, the content in ale gallons.

EXAM. 3.

By Rule II.

By Note 2, Prob. XIII., Part IV., we have $216 \times 216 \times .07958 = 46656 \times .07958 = 3712.88448$, the area of the top, in square inches; and $3712.88448 \div 2150.42 = 1.726$, the area of the top in malt bushels; then $1.726 \times 32 = 55.232$, the content in malt bushels.

PROBLEM VIII.

To find the content of a vessel, in the form of the frustum of a cone.

EXAM. 2.

By Rule I.

Inches.

60 top diameter.
40 bottom diameter.
20 difference.
20 ditto.
400 square of the difference.

Inches.

```
          60 top diameter.
          40 bottom diameter.
        ----
        2400 their product.
           3
        ----
        7200 three times their product.
         400 square of their difference.
        ----
        7600 sum.
          74 perpendicular depth.
       -----
       30400
      53200
      ------
Divisor 1077.15)562400(522.118 ale gallons.
      538575
      ------
       238250
       215430
       ------
        228200
        215430
        ------
         127700
         107715
         ------
          199850
          107715
          ------
           921850
           861720
           ------
            59630
            =====
```

EXAM. 3.

By Rule II.

Here $36 \times 40 = 1440$, the product of the diameters; and $\overline{40+36}|^2 = 76^2 = 76 \times 76 = 5776$, the square of the sum of the diameters; then $(5776 - 1440) \times 52 = 4336 \times 52 = 225472$; and $225472 \div 8214 = 27.449$, the content in malt bushels.

PROBLEM IX.

To find the content of a vessel, in the form of a prismoid.

EXAM. 2.

Here $42.5 \times 38.4 = 1632$, the area of the bottom; and $54.6 \times 42.2 = 2304.12$, the area of the top.

Also $\frac{42.5 + 54.6}{2} = \frac{97.1}{2} = 48.55$, the length of the middle section; and $\frac{38.4 + 42.2}{2} = \frac{80.6}{2} = 40.3$, the breadth of the middle section; then $48.55 \times 40.3 \times 4 = 1956.565 \times 4 = 7826.260$, four times the area of the middle section; whence $(1632 + 2304.12 + 7826.260) \times \frac{54}{6} = 11762.380 \times 9 = 105861.420$, the content in cubic inches; and $105861.420 \div 282 = 375.395$, the content in ale gallons.

EXAM. 3.

Here $56 \times 70 = 3920$, the area of the top; and $53.8 \times 66 = 3550.8$ the area of the bottom.

Also $\frac{70 + 66}{2} = \frac{136}{2} = 68$, the length of the middle section; and $\frac{56 + 53.8}{2} = \frac{109.8}{2} = 54.9$, the breadth of the middle section; then $54.9 \times 68 \times 4 = 3733.2 \times 4 = 14932.8$, four times the area of the middle section; whence $(3920 + 3550.8 + 14932.8) \times \frac{48}{6} = 22403.6 \times 8 = 179228.8$, the content in cubic inches; and $179228.8 \div 2150.42 = 83.345$, the content in malt bushels.

EXAM. 4.

Here $\frac{49.6 + 67.2}{2} = \frac{116.8}{2} = 58.4$, the transverse diameter of the middle section; and $\frac{37.8 + 50.4}{2} = \frac{88.2}{2} = 44.1$, the conjugate diameter of the middle section.

Now, $49.6 \times 37.8 \times .7854 = 1874.88 \times .7854 = 1472.530752$, the area of the bottom; $58.4 \times 44.1 \times .7854 \times 4 = 2575.44 \times .7854 \times 4 = 2022.750576 \times 4 = 8091.002304$, four times the area of the middle section; and $67.2 \times 50.4 \times .7854 = 3386.88 \times .7854 = 2660.055552$, the area of the top; then $(1472.530752 + 8091.002304 + 2660.055552) \times \frac{46.8}{6} = 12223.588608 \times 7.8 = 95343.9911424$, the content in cubic inches; hence $95343.991 \div 282 = 338.099$, the content in ale gallons; and $95343.991 \div 231 = 412.744$, the content in wine gallons.

EXAM. 5.

Here $\frac{61.6 + 42.6}{2} = \frac{104.2}{2} = 52.1$, the transverse diameter of the middle section; and $\frac{46.2 + 42.6}{2} = \frac{88.8}{2} = 44.4$, the conjugate diameter of the middle section.

Now, $61.6 \times 46.2 \times .7854 = 2845.92 \times .7854 = 2235.185568$, the area of the bottom, $52.1 \times 44.4 \times .7854 \times 4 = 2313.24 \times .7854 \times 4 = 1816.818696 \times 4 = 7267.274784$, four times the area of the middle section; and $42.6 \times 42.6 \times .7854 = 1814.76 \times .7854 = 1425.312504$, the area of the top; then $(2235.185568 + 7267.274784 + 1425.312504) \times$

52.6 = 10927.772856 × 52.6 = 574800.8522256; which being divided by 6, gives 95800.1420376, the content in cubic inches; hence, 95800.142 ÷ 282 = 339.716, the content in ale gallons; and 95800.142 ÷ 231 = 414.719, the content in wine gallons.

PROBLEM X.

To find the content of a vessel, in the form of a sphere or globe.

EXAM. 2.

By Rule I.

Here $116^3 = 116 \times 116 \times 116 = 1560896$, the cube of the diameter; and $1560896 \times .5236 = 817285.1456$, the content in cubic inches; then $\frac{817285.1456}{282} = 2898.174$, the content in ale gallons.

EXAM. 3.

By Rule II.

By Multiplication.

Here $16.1^3 = 16.1 \times 16.1 \times 16.1 = 4173.281$, the cube of the diameter; and $4173.281 \times .000243 = 1.014107283$, the content in malt bushels.

By Division.

Here 4173.281 ÷ 4107 = 1.016, the content in malt bushels.

PROBLEM XI.

To find the content of a vessel, in the form of the segment of a sphere.

EXAM. 2.

By Rule I.

Here $24^2 \times 3 = 576 \times 3 = 1728$, three times the square of the radius of the top; also $21^2 = 441$, the square of the depth; then $(1728 + 441) \times 21 \times .5236 = 2169 \times 21 \times .5236 = 45549 \times .5236 = 23849.4564$, the content in cubic inches; hence $23849.4564 \div 231 = 103.244$, the content in wine gallons.

EXAM. 3.

By Rule II.

Here $38^2 = 1444$, the square of the diameter of the base; and $15^2 = 225$, the square of the height; also $\frac{225}{3} = 75$, one third of the square of the height; then $(1444 + 225 + 75) \times 7.5 = 1744 \times 7.5 = 13080$; and $13080 \div 2738 = 4.777$, the content in malt bushels.

PROBLEM XII.

To find the content of a vessel in the form of the frustum or zone of a sphere.

EXAM. 2.

Here $\frac{52^2 + 30^2}{2} = \frac{2704 + 900}{2} = \frac{3604}{2} = 1802$, half the sum of the squares of the two diameters; and

$18^2 \times \frac{2}{3} = 324 \times \frac{2}{3} = \frac{648}{3} = 216$, two thirds of the square of the depth; then $1802 + 216 \times 18 = 2018 \times 18 = 36324$; and $36324 \div 294.12 = 123.500$, the content in wine gallons.

EXAM. 3.

Here $\frac{25^2 + 24^2}{2} = \frac{625 + 576}{2} = \frac{1201}{2} = 600.5$, half the sum of the squares of the two diameters; and $20^2 \times \frac{2}{3} = 400 = \times \frac{2}{3} = \frac{800}{3} = 266.66$, two thirds of the square of the altitude; then $600.5 + 266.66 \times 20 = 867.16 \times 20 = 17343.20$; and $17343.20 \div 2738 = 6.334$, the content in malt bushels.

PROBLEM XIII.

To find the content of a vessel, in the form of a prolate spheroid.

EXAM. 2.

Here $56^2 \times 72.5 = 3136 \times 72.5 = 227360$; and $\frac{227360}{441.18} = 515.345$, the content in wine gallons.

EXAM. 3.

Here $42^2 \times 48 = 1764 \times 48 = 84672$; and $\frac{84672}{4107} = 20.616$, the content in malt bushels.

PROBLEM XIV.

To find the content of a vessel, in the form of the middle frustum of a prolate spheroid.

Exam. 2.

Middle Diameter.

$$\begin{array}{rl}
24 & \\
24 & \\
\hline
96 & \\
48 & \\
\hline
576 & \text{square.} \\
2 & \\
\hline
1152 & \text{twice the square of the middle diameter.} \\
400 & = 20 \times 20\text{, the square of the end diameter.} \\
\hline
1552 & \text{sum.} \\
25 & \text{length.} \\
\hline
7760 & \\
3104 & \\
\hline
\end{array}$$

$$\begin{array}{rl}
1077.15)\overline{38800.00}(36.020 & \text{content in ale gallons.} \\
323145 & \\
\hline
648550 & \\
646290 & \\
\hline
226000 & \\
215430 & \\
\hline
105700 & \\
\hline\hline
\end{array}$$

Exam. 3.

Here $35 \times 35 \times 2 = 1225 \times 2 = 2450$, twice the square of the middle diameter; and $30 \times 30 = 900$, the square of the end diameter; then $2450 + 900 \times 38 = 3350 \times 38 = 127300$; hence $127300 \div 882.36 = 144.272$, the content in wine gallons.

EXAM. 4.

Here $52 \times 52 \times 2 = 2704 \times 2 = 5408$, twice the square of the middle diameter; and $40 \times 40 = 1600$, the square of the end diameter; then $5408 + 1600 \times 58 = 7008 \times 58 = 406464$; hence $406464 \div 8214 = 49.484$, the content in malt bushels.

PROBLEM XV.

To find the content of a vessel, in the form of a parabolic spindle.

EXAM. 2.

Here $24 \times 24 = 576$, the square of the middle iameter; and $576 \times 65 = 37440$, the square of the iameter multiplied by the length; then $37440 \div 51.48 = 67.890$, the content in wine gallons.

EXAM. 3.

Here $56 \times 56 = 3136$, the square of the middle iameter; and $3136 \times 150 = 470400$, the square of te diameter multiplied by the length; then 470400 · $5133.75 = 91.628$, the content in malt bushels.

PROBLEM XVI.

find the content of a vessel, in the form of the middle frustum of a parabolic spindle.

EXAM. 2.

Here $27 \times 27 \times 8 = 729 \times 8 = 5832$, eight times square of the middle diameter; $22 \times 22 \times 3 =$

$484 \times 3 = 1452$, three times the square of the head diameter; and $27 \times 22 \times 4 = 594 \times 4 = 2376$, four times the product of the two diameters; then $(5832 + 1452 + 2376) \times 30 = 9660 \times 30 = 289800$; and $289800 \div 5385.79 = 53.808$, the content in ale gallons; and $289800 \div 4411.76 = 65.688$, the content in wine gallons.

EXAM. 3.

Here $25 \times 25 \times 8 = 625 \times 8 = 5000$, eight times the square of the middle diameter; $20 \times 20 \times 3 = 400 \times 3 = 1200$, three times the square of the top or bottom diameters; and $25 \times 20 \times 4 = 500 \times 4 = 2000$, four times the product of the two diameters; then $(5000 + 1200 + 2000) \times 27 = 8200 \times 27 = 221400$; and $221400 \div 41069.92 = 5.390$, the content in malt bushels.

PROBLEM XVII.

To find the content of a vessel, in the form of a parabolic conoid.

EXAM. 2.

Here $24 \times 24 = 576$, the square of the diameter of the base; and $576 \times 21 = 12096$, the square of the diameter multiplied by half the altitude; then $12096 \div 359.05 = 33.688$, the content in ale gallons; and $12096 \div 294.12 = 41.126$, the content in wine gallons.

EXAM. 3.

Here $24 \times 24 = 576$, the square of the diameter of the base; and $576 \times 9 = 5184$, the square of the diameter of the base, multiplied by half the depth;

then $5184 \div 2738 = 1.893$, the content in malt bushels.

PROBLEM XVIII.

To find the content of a vessel, in the form of the frustum of a parabolic conoid.

EXAM. 2.

Here $100 \times 100 = 10000$, the square of the diameter of the greater end; $50 \times 50 = 2500$, the square of the diameter of the less end; and $(10000 + 2500) \times 56 = 12500 \times 56 = 700000$, the sum of the squares of the diameters multiplied by the depth; then $700000 \div 718.1 = 974.794$, the content in ale gallons; and $700000 \div 588.24 = 1189.990$, the content in wine gallons.

EXAM. 3.

Here $28 \times 28 = 784$, the square of the bung diameter; $21 \times 21 = 441$, the square of the head diameter; and $(784 + 441) \times 36 = 1225 \times 36 = 44100$, the sum of the squares of the two diameters multiplied by the length; then $44100 \div 718.1 = 61.412$, the content in ale gallons; and $44100 \div 588.24 = 74.969$, the content in wine gallons.

PROBLEM XIX.

To find the content of a circular vessel when its sides are a little curved.

EXAM. 2.

Here $84 \times 84 = 7056$, the square of the bottom diameter; $81.5 \times 81.5 = 6642.25$, the square of the top diameter; and $94 \times 94 \times 4 = 8836 \times 4 = 35344$,

four times the square of the middle diameter; then $(7056 + 6642{,}25 + 35344) \times 52 = 49042.25 \times 52 = 2550197$; and $2550197 \div 2154.3 = 1183.770$, the content in ale gallons; and $2550197 \div 1764.72 = 1445.100$, the content in malt bushels.

EXAM. 3.

Here $30 \times 30 \times 2 = 900 \times 2 = 1800$, twice the square of the head diameter; and $35 \times 35 \times 4 = 1225 \times 4 = 4900$, four times the square of the bung diameter; then $(1800 + 4900) \times 38 = 6700 \times 38 = 254600$; and $254600 \div 2154.3 = 118.181$, the content in ale gallons; and $254600 \div 1764.72 = 144.272$, the content in wine gallons.

PROBLEM XX.

To find the content of a circular vessel, when its sides are much curved, by taking five diameters at equal distances from each other.

EXAM. 2.

Here $52^2 + 46^2 = 2704 + 2116 = 4820$, the sum of the squares of the top and bottom diameters.

Also, $63^2 \times 2 = 3969 \times 2 = 7938$, twice the square of the middle diameter.

And, $58^2 + 60^2 \times 4 = 3364 + 3600 \times 4 = 6964 \times 4 = 27856$, four times the sum of the squares of the diameters, taken at one-fourth, and at three-fourths of the depth.

Then, $(4820 + 7938 + 27856) \times 60 = 40614 \times 60 = 2436840$; and $2436840 \div 4308.6 = 565.575$, the content in ale gallons.

EXAM. 3.

Here $76^2 + 86^2 = 5776 + 7396 = 13172$, the sum of the squares of the top and bottom diameters.

Also, $104^2 \times 2 = 10816 \times 2 = 21632$, twice the square of the middle diameter.

And, $\overline{95^2 + 100^2} \times 4 = \overline{9025 + 10000} \times 4 = 19025 \times 4 = 76100$, four times the sum of the squares of the diameters taken at one-fourth, and at three-fourths of the depth.

Then, $(13172 + 21632 + 76100) \times 82 = 110904 \times 82 = 9094128$; and $9094128 \div 32856 = 276.787$, the content of the vessel in malt bushels.

PROBLEM XXI.

To find the content of a circular vessel, when its sides are very much curved, by taking a competent number of equidistant, parallel sections.

EXAM. 2.

By Rule II.

Diameters.	Inches.	Areas. Ale gallons.
First	93.0	24.0883
Second	109.5	33.3940
Third	120.8	40.6419
Fourth	123.2	42.2729
Fifth	124.5	43.1697
Sixth	117.8	38.6488
Seventh	108.4	32.7264

Then, 24.0883 + 32.7264 = 56.8147, the sum of the two extreme areas; (33.3940 + 42.2729 + 38.6483) × 4 = 114.3152 × 4 = 457.2608, four times the sum of all the even areas; and (41.5421 + 42.2695) × 2 = 83.8116 × 2 = 167.6232, twice the sum of all the odd areas; also 133.7 ÷ 7 = 19.1, the common distance of the sections; hence (56.8147 + 457.2608 + 167.6232) $\times \frac{19.1}{3} = \frac{681.6987 \times 19.1}{3} = \frac{13020.44517}{3} =$ 4340.14839, the content in ale gallons.

Diameters.		Areas.
	Inches.	*Wine gallons.*
First	93.0	29.4066
Second	109.5	40.7668
Third	120.8	49.6149
Fourth	123.2	51.6060
Fifth	124.5	52.7008
Sixth	117.8	47.1812
Seventh	108.4	39.9519

Then, 29.4066 + 39.9519 = 69.3585, the sum of the two extreme areas; (40.7668 + 51.6060 + 47.1812) × 4 = 139.5540 × 4 = 558.2160, four times the sum of all the even areas; and (49.6149 + 52.7008) × 2 = 102.3157 × 2 = 204.6314, twice the sum of all the odd areas; hence (69.3585 + 558.2160 + 204.6314) $\times \frac{19.1}{3} = \frac{832.2059 \times 19.1}{3}$ $= \frac{15895.13269}{3} =$ 5298.37756, the content in wine gallons.

EXAM. 3.

By Rule I.

Here $108.4^2 \times .7854 = 11750.56 \times .7854 =$ 9228.889824, the area of the first section; $123.6^2 \times .7854 = 15276.96 \times .7854 =$ 11998.524384, the area of the second section; $130.8^2 \times .7854 =$ $17108.64 \times .7854 =$ 13437.125856, the area of the

third section; $136.6^2 \times .7854 = 18659.56 \times .7854 = 14655.218424$, the area of the fourth section; $136.2^2 \times .7854 = 18550.44 \times .7854 = 14569.515576$, the area of the fifth section; $131.8^2 \times .7854 = 17371.24 \times .7854 = 13643.371896$, the area of the sixth section; $124.2^2 \times .7854 = 15425.64 \times .7854 = 12115.297656$, the area of the seventh section; $114.8^2 \times .7854 = 13179.04 \times .7854 = 10350.818016$, the area of the eighth section; and $96.8^2 \times .7854 = 9370.24 \times .7854 = 7359.386496$, the area of the ninth or last section.

Then $9228.889824 + 7359.386496 = 16588.276320$, the sum of the areas of the two end sections; $(11998.524384 + 14655.218424 + 13643.371896 + 10350.818016) \times 4 = 50647.932720 \times 4 = 202591.730880$, four times the sum of all the even sections; and $(13437.125856 + 14569.515576 + 12115.297656) \times 2 = 40121.939088 \times 2 = 80243.878176$, twice the sum of all the odd sections; hence $(16588.276320 + 202591.730880 + 80243.878176) \times \frac{18.3}{3} = 299423.885376 \times 6.1 = 1826485.7007936$, the content in cubic inches; consequently, $1826485.7007936 \div 282 = 6476.8996482$, the content in ale gallons; $1826485.7007936 \div 231 = 7906.8645056$, the content in wine gallons; and $1826485.7007936 \div 2150.42 = 849.36231$, the content in malt bushels.

PROBLEM XXII.

To find the content of the hoof of a cylinder, or the quantity of liquor contained in a cylindrical vessel, placed in an inclining position.

Exam. 2.

Here $74.4^2 = 5535.36$, the square of the diameter of the base; and $\frac{58.6 + 18.2}{2} = \frac{76.8}{2} = 38.4$, half

the sum of the greatest and least depths of the liquor; then $5535.36 \times 38.4 = 212557.824$; and $212557.824 \div 294.12 = 722.690$, the content in wine gallons.

EXAM. 3.

Here $63.8^2 = 4070.44$, the square of the diameter of the base; and $\frac{28.8 + 25.2}{2} = \frac{54}{2} = 27$, half the sum of the greatest and least depths of the malt; then $4070.44 \times 27 = 109901.88$; and $109901.88 \div 2738 = 40.138$, the content in malt bushels.

EXAM. 4.

Here $65^2 = 4225$, the square of the diameter of the base; and $\frac{25.8}{2} = 12.9$, half the greatest depth of the liquor; then $4225 \times 12.9 = 54502.5$; and $54502.5 \div 359.05 = 151.796$, the content in ale gallons.

PROBLEM XXIII.

To find the content of a cylindrical ungula or hoof, when its base is less than a semicircle.

EXAM. 2.

By Rule 1, Problem XV., Part IV., we have $96 \times 36 \times \frac{2}{3} = 3456 \times \frac{2}{3} = \frac{6900}{3} = 2300$, two thirds of the product of the chord and versed sine; and $\frac{36^3}{96 \times 2} = \frac{46656}{192} = 243$, the cube of the height divided by twice the chord; then $2300 + 243 = 2543$ square inches, the area of the base of the hoof.

Also, $\frac{100}{2} - 36 = 50 - 36 = 14$, the difference between half the diameter of the vessel and the versed

sine of the hoof's base; and $2543 \times 14 = 35602$, the area of the base multiplied by the said difference.

Again $48^3 \times \frac{2}{3} = 110592 \times \frac{2}{3} = \frac{221184}{3} = 73728$, two thirds of the cube of half the chord; and $73728 - 35602 = 38126$; then $38126 \times \frac{108}{36} = 38126 \times 3 = 114378$, the content in cubic inches; hence $114378 \div 231 = 495.142$, the content in wine gallons.

PROBLEM XXIV.

To find the content of a cylindrical ungula, when its base is a semicircle.

EXAM. 2.

Here $124 \times 124 \times \frac{126}{6} = 15376 \times 21 = 322896$, the content in cubic inches; and $\frac{322896}{231} = 1397.818$, the content in wine gallons.

PROBLEM XXV.

To find the content of a cylindrical ungula, when its base is greater than a semicircle.

EXAM. 2.

By Rule 2, Prob. XV., Part IV., we have $64 \div 80 = .8$, the tabular height; then by Note 2, of the same Problem, $1 - .8 = .2$; and the Area Seg. answering to .2 is .111823, which being taken from .785398,

leaves .673575, the Area Seg. corresponding to .8; hence $.673575 \times 80^2 = .673575 \times 6400 = 4310.88$, the area of the ungula's base, in square inches.

Also, $64 - 40 = 24$, the difference between the versed sine of the ungula's base and half the diameter of the vessel; and $4310.88 \times 24 = 103461.12$, the area of the base multiplied by the said difference.

Again, $32^3 \times \frac{2}{3} = 32768 \times \frac{2}{3} = \frac{65536}{3} =$ 21845.33, two thirds of the cube of half the chord; and $103461.12 + 21845.333 = 125306.453$; then $125306.453 \times \frac{92}{64} = \frac{11528193.676}{64} = 180128.026$, the content of the ungula, in cubic inches; hence $180128.026 \div 231 = 779.775$, the content in wine gallons.

PROBLEM XXVI.

To find the content of a pyramidical ungula, or the quantity of liquor contained in a vessel in the form of the frustum of a square or rectangular pyramid, when it is placed in such a position that the liquor just covers the bottom, and rises up three of the sides in an oblique direction, forming a figure that is called a cuneus or wedge.

EXAM. 2.

Here $30 \times 2 + 33 = 60 + 33 = 93$, twice the length of the base added to the length of the upper edge of the liquor; then $\frac{93 \times 27 \times 22}{6} = \frac{2511 \times 22}{6}$ $= \frac{55242}{6} = 9207$, the content of the ungula in

cubic inches; and $\frac{9207}{231} = 39.857$, the content in wine gallons.

EXAM. 3.

Here $58 \times 2 + 38 = 116 + 38 = 154$, twice the length of the base added to the length of the upper edge of the liquor; then $\frac{154 \times 45 \times 42}{6} =$ $6930 \times \frac{42}{6} = 6930 \times 7 = 48510$, the content of the ungula in cubic inches; and $\frac{48510}{282} = 172.021$, the content in ale gallons.

PROBLEM XXVII.

To find the content of a conical ungula, or the quantity of liquor contained in a vessel in the form of the frustum of a cone, when it stands upon its greater base, and in such a position that the liquor just covers the whole of its bottom.

EXAM. 2.

Here $\sqrt{82.6 \times 58.7} = \sqrt{4848.62} = 69.62$, the the mean proportional between the diameters; and $4848.62 \times 69.62 = 337560.9244$, the product of the two diameters multiplied by the mean proportional between them.

Also, $82.6^3 = 563559.976$, the cube of the bottom diameter; and $\frac{563559.976 - 337560.9244}{82.6 - 58.7} =$ $\frac{225999.0516}{23.9} = 9456.027$; then $9456.027 \times 47.3 \times .001133 = 447270.0771 \times .001133 = 506.75699735$ wine gallons, the answer required.

PROBLEM XXVIII.

To find the content of a conical ungula, or the quantity of liquor contained in a vessel in the form of the frustum of a cone, when it stands upon its less base, and in such a position that the liquor just covers the whole of its bottom.

EXAM. 2.

Here $\sqrt{67.8 \times 42.3} = \sqrt{2867.94} = 53.553$, the mean proportional between the diameters; and $2867.94 \times 53.553 = 153586.79082$, the product of the two diameters multiplied by the mean proportional between them.

Also $42.3^3 = 75686.967$, the cube of the bottom diameter; and $\frac{153586.79082 - 75686.967}{67.8 - 42.3} =$ $\frac{77899.82382}{25.5} = 3054.895$; then $3054.895 \times 54.6 \times .0009283 = 166797.267 \times .0009283 = 154.8379029561$ ale gallons, the content required.

PROBLEM XXIX.

To find the quantity of liquor contained in a conical vessel, placed in an inclining position so that the liquor intersecting the opposite sides of the vessel, in an oblique direction, disposes itself into a compound figure, consisting of the frustum of a cone, and a conical ungula.

EXAM. 2.

To find the content of the frustum.

By Rule 2, Prob. VIII., we have $40 \times 30 = 1200$, the product of the diameters; and $\overline{40 + 30}|^2 =$

$70^2 = 70 \times 70 = 4900$, the square of the sum of the diameters; then $(4900 - 1200) \times 20 = 3700 \times 20 = 74000$; and $74000 \div 882.36 = 83.865$, the content in wine gallons.

To find the content of the ungula.

By Prob. XXVIII., we have $\sqrt{50 \times 40} = \sqrt{2000} = 44.721$, the mean proportional between the diameters; and $44.721 \times 2000 = 89442$, the product of the two diameters multiplied by the mean proportional between them.

Also, $40^3 = 64000$, the cube of the bottom diameter; and $\frac{89442 - 64000}{50 - 40} = \frac{25442}{10} = 2544.2$; then $2544.2 \times 20 \times .001133 = 50884 \times .001133 = 57.651572$, the content in wine gallons.

Lastly, $83.865 + 57.651572 = 141.516572$ wine gallons, the content of the whole compound figure.

PROBLEM XXX.

To find the quantity of liquor contained in a vessel in the form of the frustum of a cone, when it is placed on its greater end, and such a position that the liquor, covering only part of its bottom, forms an elliptic ungula.

EXAM. 2.

Here $60 - 50 = 10$, the difference of the diameters; and $\frac{40 - 10}{50} = \frac{30}{50} = .6$, the quotient obtained by subtracting the difference of the diameters from the versed sine of the base, and dividing the remainder by the less diameter. The Area Seg. answering to this quotent is .492029.

Now, $50^3 = 125000$, the cube of the less diameter; also, $\frac{40}{40 - 10} = \frac{40}{30} = 1.333$, the quotient arising

from dividing the versed sine of the base by the difference between the said versed sine and the difference of the diameters; and $\sqrt{1.333} = 1.154$, the square root of the said quotient; then $.492029 \times 125000 \times 1.333 \times 1.154 = 61503.625 \times 1.333 \times 1.154 = 81984.332125 \times 1.154 = 94609.91927225$, the reserved product.

Again, $60^3 = 216000$, the cube of the bottom diameter; and $\frac{40}{60} = .666$, the quotient of the versed sine of the base divided by the bottom diameter. The area segment answering to this quotent is .555597; then $.555597 \times 216000 = 120008.952$, the product arising from multiplying the last Area Seg. by the cube of the bottom diameter.

Now, $120008.952 - 94609.919 = 25399.033$, the difference between the last product and the reserved product; also $\frac{54}{3} = 18$, one-third of the perpendicular height of the ungula; then $25399.033 \times \frac{18}{10} = \frac{457182.594}{10} = 45718.2594$, the content of the ungula, in cubic inches; and $\frac{45718.259}{231} = 197.914$, the content in wine gallons.

PROBLEM XXXI.

To find the quantity of liquor, contained in a vessel in the form of the frustum of a cone, when it is placed on its less end, and in such a position that the liquor, covering only part of its bottom, forms an elliptic ungula.

EXAM. 2.

Here $\frac{110 - 80 + 36}{110} = \frac{30 + 36}{110} = \frac{66}{110}$

= .6. The Area Seg. answering to the quotient is .492029; and .492029 × 110³ = .492029 × 1331000 = 654890.599, the first number.

Again, $\frac{36}{36 + 30} = \frac{36}{66} = .545$; and $\sqrt{.545^3}$ = $\sqrt{.161878625}$ = .40234, the second number.

Again, $\frac{36}{80} = .45$. The Area Seg. answering to this quotient is .342782; and .342782 × 80³ = .342782 × 512000 = 175504.384, the third number.

Again, $\frac{42}{30 \times 3} = \frac{42}{90} = .4666$, the fourth number.

Now, 654890.599 × .40234 = 263488.68360166, the product of the first and second numbers; then (263488.68360166 − 175504.384) × .4666 = 87984.29960166 × .4666 = 41053.4719963334556, the content of the ungula, in cubic inches; and $\frac{41053.472}{282}$ = 145.579, the content in ale gallons.

PROBLEM XXXII.

To find the quantity of liquor, contained in a vessel in the form of the frustum of a cone, when it is placed on its greater end, and in such a position that the liquor, covering only part of its bottom, forms a parabolic ungula.

Exam. 2.

Here 100 − 74 = 26, the versed sine of the base of the ungula; and $\frac{26}{100} = .26$. The Area Seg.

answering to this quotient is .162263; then .162263 $\times$ 100^2 = .162263 $\times$ 10000 = 1622.63, the area of the base of the ungula; and $\frac{1622.63 \times 100}{26} = \frac{162263}{26}$ = 6240.884 the reserved quotient.

Again, $\sqrt{26 \times 74} \times 74 \times \frac{4}{3} = \sqrt{1924} \times \frac{296}{3} = 43.863 \times \frac{296}{3} = \frac{12983.448}{3} = 4327.816.$

Then $(6240.884 - 4327.816) \times \frac{34.5}{3} = 1913.068 \times 11.5 = 22000.282$, the content of the ungula, in cubic inches; and $\frac{22000.282}{231} = 95.239$, the content in wine gallons.

PROBLEM XXXIII.

To find the quantity of liquor contained in a vessel in the form of the frustum of a cone, when it is placed upon its greater end, and in such a position that the liquor, covering only part of its bottom, forms a hyperbolic ungula.

EXAM. 2.

CALCULATION.

To find the transverse and conjugate diameters of the hyperbolic section.

Here 64 $\times$ 60 = 3840, the reserved product; and $96 - (60 + 30) = 96 - 90 = 6$; then $\frac{3840}{6} =$ 640, the transverse diameter.

Again, $\sqrt{30 \div 6} = \sqrt{5} = 2.236$; and 2.236 $\times$ 60 = 134.16, the conjugate diameter.

To find the area of the hyperbolic section.

Here $21\sqrt{640 \times 64 + 64^2 \times \frac{5}{7}} = 21\sqrt{40960 + 4096 \times \frac{5}{7}} = 21\sqrt{40960 + 20480 \div 7} = 21\sqrt{40960 + 292.5714} = 21\sqrt{41252.5714} = 203.11 = 21 = 4265.31$.

Again, $(4\sqrt{640 \times 64} + 4265.31) \div 75 = (4\sqrt{40960} + 4265.31) \div 75 = (4 \times 202.385 + 4265.31) \div 75 = (809.54 + 4265.31) \div 75 = 5074.85 \div 75 = 67.664$.

Lastly, $\frac{134.16 \times 64 \times 4}{640} \times 67.664 = \frac{8586.24 \times 4}{640} \times 67.664 = \frac{34344.96}{640} \times 67.664 = 53.664 \times 67.664 = 3631.120896$, the area of the section.

To find the area of the circular segment of the base.

Here $\frac{30}{96} = .3125$, the tabular height. The *Area seg.* answering to this quotient, is .20969; then $.20969 \times 96^2 = .20969 \times 9216 = 1932.50304$, the area of the base.

To find the content of the hyperbolic ungula.

Here $\frac{63 \div 3}{96 - 60} = \frac{21}{36} = .5833$, the first number.

Again, $1932.50304 \times 96 = 185520.29184$, the second number.

Again, $\frac{60 \times 30}{64} \times 3631.120896 = \frac{1800}{64} \times 3631.120896 = 28.125 \times 3631.120896 = 102125.2752$, the third number.

Then, $(185520.29184 - 102125.2752) \times .5833 = 83395.01664 \times .5833 = 48644.31320612$, the content in cubic inches; and $48644.3132 \div 282 = 172.4975$, the content in ale gallons.

MISCELLANEOUS EXAMPLES

IN THE

MENSURATION

OF

SUPERFICIES AND SOLIDS,

APPLIED TO

GAUGING.

Exam. 1.

By Problem I., Part I., Square Root, we have $\sqrt{81 \times 64} = \sqrt{5184} = 72$ inches, the depth required.

Exam. 2.

By Prob. II., Part I., we have $\sqrt{59536} = 244$ inches, the side of the square required.

Exam. 3.

By Prob. III., Part I., we have $6432 \div .7854 = 8189.457601$; and $\sqrt{8189.457601} = 90.495$ inches, the diameter required.

EXAM. 4.

By Prob. IV., Part I., we have $\sqrt{78^2 + 112^2} = \sqrt{6084 + 12544} = \sqrt{18628} = 136.484$ inches. the diagonal required.

EXAM. 5.

Here a perpendicular drawn from the middle of the given diagonal of the opposite angle, will be equal to half the said diagonal; consequently, we have the base and perpendicular of an isosceles right-angled triangle given, to find the hypothenuse, as in the last Example; hence $\sqrt{57.5^2 + 57.5^2} = \sqrt{3306.25 + 3306.25} = \sqrt{6612.50} = 81.317$ inches, the side required.

EXAM. 6.

By Prob. V., Part I., we have $\sqrt{145^2 - 116^2} = \sqrt{21025 - 13456} = \sqrt{7569} = 87$ inches, the breadth required.

EXAM. 7.

By Prob. VI., Part I., we have $\frac{36 + 27}{2} = \frac{63}{2} = 31.5$, half the sum of the diameters; then $\sqrt{31.5^2 + 22.5^2} = \sqrt{992.25 + 506.25} = \sqrt{1498.50} = 38.71$ inches, the diagonal required.

EXAM. 8.

By Prob. VII., Part I., we have $\frac{38.7 + 29.6}{2} = \frac{68.3}{2} = 34.15$, half the sum of the diameters; then $2\sqrt{40.3^2 - 34.15^2} = 2\sqrt{1624.09 - 1166.2225} = 2\sqrt{457.8675} = 21.39 \times 2 = 42.78$ inches, the length required.

EXAM. 9.

By Problem I., Part I., Cube Root, we have $\sqrt[3]{254 \times 282} = \sqrt[3]{71628} = 41.52$ inches, the side required.

EXAM. 10.

By Prob. II., Part I., we have $\sqrt[3]{125^3 \times 5}$ $\sqrt[3]{1953125 \times 5} = \sqrt[3]{9765625} = 213.74$ inches, the length of the required vessel.

Again, $\sqrt[3]{86^3 \times 5} = \sqrt[3]{636056 \times 5} = \sqrt[3]{3180280} = 147.05$ inches, the breadth of the required vessel.

Lastly, $\sqrt[3]{62^3 \times 5} = \sqrt[3]{238328 \times 5} = \sqrt[3]{1191640} = 106.01$ inches, the depth of the required vessel.

EXAM. 11.

By Problem II., Part I., we have $\sqrt[3]{254^3 \div 3} = \sqrt[3]{16387064 \div 3} = \sqrt[3]{5462354.666666} = 176.11$ inches, the depth of the required vessel.

Also, $\sqrt[3]{112^3 \div 3} = \sqrt[3]{1404928 \div 3} = \sqrt[3]{468309.333333} = 77.64$ inches, the diameter of the required vessel.

EXAM. 12.

By Prob. III., Part I., as $121500 : 75^3 :: 562500 : 1953125$, the cube of the length; and $\sqrt[3]{1953125} = 125$ inches, the length of the required vessel.

Again, $121500 : 45^3 :: 562500 : 421875$, the cube of the breadth; and $\sqrt[3]{421875} = 75$ inches, the breadth of the required vessel.

Lastly, as $121500 : 36^3 :: 562500 : 216000$, the cube of the depth; and $\sqrt[3]{216000} = 60$ inches, the depth of the required vessel.

EXAM. 13.

By Prob. I., Part IV., we have $62.8 \times 62.8 = 3943.84$, the area in square inches; and $\frac{3943.84}{282} = 13.981$, the area in ale gallons.

EXAM. 14.

By Prob. II., Part IV., we have $115.3 \times 86.4 = 9961.92$, the area in square inches; $\frac{9961.92}{231} = 43.125$, the area in wine gallons.

EXAM. 15.

By Prob. III., Part IV., we have $84.6 \times 63.4 = 5363.64$, the area in square inches; and $\frac{5363.64}{2150.42} = 2.494$, the area in malt bushels.

EXAM. 16.

By Prob. IV., Part IV., we have $123.6 \times 42.85 = 5296.260$, the area in square inches; and $\frac{5296.260}{282} = 18.781$, the area in ale gallons.

EXAM. 17.

By Prob. V., Part IV., we have $\frac{56 + 68 + 79}{2} = \frac{203}{2} = 101.5$, half the sum of the sides; then $101.5 - 56 = 45.5$ the first remainder; $101.5 - 68 =$

33.5 the second remainder; and 101.5 − 79 = 22.5 the third remainder; whence $\sqrt{101.5 \times 45.5 \times 33.5 \times 22.5} = \sqrt{3481005.9375} = 1865.745$, the area in square inches; and $\frac{1865.745}{231} = 8.076$, the area in wine gallons.

Exam. 18.

By Prob. VI., Part IV., we have $\overline{63.8 + 56.4} \times 138.6 = 120.2 \times 138.6 = 16659.72$; and $\frac{16659.72}{2} = 8329.86$, the area in square inches; then $\frac{8329.86}{2150.42} = 3.873$, the area in malt bushels.

Exam. 19.

By Prob. VII., Part IV., we have $\overline{98 + 124} \times 136 = 222 \times 136 = 30192$; and $30192 \div 2 = 15096$, the area in square inches; then $15096 \div 282 = 53.531$, the area in ale gallons.

Exam. 20.

(See Problems V. and VIII., Part IV.)

Here $\frac{122 + 104 + 168}{2} = \frac{394}{2} = 197$, half the sum of the sides; then 197 − 122 = 75, the first remainder; 197 − 104 = 93, the second remainder; and 197 − 168 = 29, the third remainder; whence $\sqrt{197 \times 75 \times 93 \times 29} = \sqrt{39848175} = 6312.541$ inches, the area of the first triangle.

Again, $\frac{168 + 114 + 112}{2} = \frac{394}{2} = 197$, half the sum of the sides; then 197 − 168 = 29, the first remainder; 197 − 114 = 83, the second

remainder; and $197 - 112 = 85$, the third remainder; whence $\sqrt{197 \times 29 \times 83 \times 85} = \sqrt{40305215} = 6348.638$ inches, the area of the second triangle.

Also, $\frac{114 + 87 + 92}{2} = \frac{293}{2} = 146.5$, half the sum of the sides; then $146.5 - 114 = 32.5$, the first remainder; $146.5 - 87 = 59.5$, the second remainder; $146.5 - 92 = 54.5$, the third remainder; whence $\sqrt{146.5 \times 32.5 \times 59.5 \times 54.5} = \sqrt{1543954343.75}$ $= 3929.318$ inches, the area of the third triangle; hence we have $6312.541 + 6348.638 + 3929.318 = 16590.497$ square inches, the area of the whole polygon.

Then, $\frac{16590.497}{2150.42} = 7.715$, the area in malt bushels; and $7.715 \times 6.4 = 49.376$ the content in malt bushels.

EXAM. 21.

By Prob. IX., Part IV., we have $52.6 \times 8 \times 63.7 = 26804.96$, and $\frac{26804.96}{2} = 13402.48$, the area in square inches; and $13402.48 \div 231 = 58.019$, the area in wine gallons; then, $58.019 \times 84.3 = 4891.0017$, the content in wine gallons.

EXAM. 22.

By Prob. X., Part IV., we have $87.6 \times 87.6 \times .021921 = 7673.76 \times .021921 = 168.21649296$, the area in ale gallons.

EXAM. 23.

By Prob. XIII., Part IV., we have $226.4 \times 226.4 \div 359.05 = 51256.96 \div 359.05 = 142.757$, the area of the base in ale gallons; and $142.757 \times 258.8 = 36945.5116$, the content in ale gallons.

EXAM. 24.

(See the Figure in Problem XIV., Part IV., of the Gauging.)

By the question, AD $= 52$, and DE $= 43$; then by Prob. V., Part I., we have $\sqrt{52^2 - 43^2} = \sqrt{2704 - 1849} = \sqrt{855} = 29.24 =$ AE; and AC — AE $= 52 - 29.24 = 22.76$, the versed sine EC.

Again, by Prob. IV., Part I., we have $\sqrt{43^2 + 22.76^2} = \sqrt{1849 + 518.0176} = \sqrt{2367.0176} = 48.653$ = DC, the chord of half the arc.

Again, by Prob. XII., Part IV., we have $\frac{48.653 \times 8 - 86}{3} = \frac{389.224 - 86}{3} = \frac{303.224}{3}$ $= 101.0746$, the length of the arc DCB.

Lastly, by Problem XIV., Part IV., we have $\frac{101.0746 \times 52}{2} = \frac{5255.8792}{2} = 2627.9396$, the area of the sector ABCD, in square inches; and $2627.9396 \div 231 = 11.3763$, the area in wine gallons.

EXAM. 25.

By Prob. XV., Part IV., we have $92 \times 38 \times \frac{2}{3} = 3496 \times \frac{2}{3} = \frac{6992}{3} = 2330.6$, two-thirds of the product of the chord and versed sine, or height of the segment; and $\frac{38^3}{92 \times 2} = \frac{54872}{184} = 298.217$, the cube of the height divided by twice the chord, then $2330.6 + 298.217 = 2628.817$, the area in square inches; and $2628.817 \div 282 = 9.322$, the area in ale gal-

lons; whence $9.322 \times 6.4 = 59.6608$, the content in ale gallons.

EXAM. 26.

By Prob. XVI., Part IV., we have $125.4 \times 82.8 \div 2738 = 10383.12 \div 2738 = 3.792$, the area in malt bushels.

EXAM. 27.

By Prob. XVII., Part IV., we have $\frac{21.8}{52.4} = .416\frac{1}{131}$, the tabular height; and the corresponding *Area Seg.* is .309125; then $.309125 \times 65.6 \times 52.4 = 1062.59864$, the area of the segment in square inches; and $\frac{1062.59864}{282} = 3.76808$, the area in ale gallons.

EXAM. 28.

By Prob. XVIII., Part IV., we have $98.6 \times 75.4 \times \frac{2}{3} = \frac{98.6 \times 75.4 \times 2}{3} = \frac{7434.44 \times 2}{3} = \frac{14868.88}{3} = 4956.293$, the area in square inches; and $4956.293 \div 231 = 21.455$, the area in wine gallons.

EXAM. 29.

By Prob. IV., Part I., we have $\sqrt{80^2 + 60^2} = \sqrt{6400 + 3600} = \sqrt{10000} = 100$ inches, the hypothenuse or chord of the segment.

Then, by Prob. V., Part IV., we have $\frac{80 \times 60}{2} = \frac{4800}{2} = 2400$, the area of the triangle in square inches.

Again, by Prob. XV., Rule I., Part IV., we have

$100 \times 25 \times \frac{2}{3} = 2500 \times \frac{2}{3} = \frac{5000}{3} = 1666.666$, two-thirds of the product of the chord and versed sine, or height of the segment; and $\frac{25^3}{100 \times 2} = \frac{15625}{200}$ $= 78.125$, the cube of the height divided by twice the chord; then $1666.666 + 78.125 = 1744.791$, the area of the segment in square inches.

Now, $2240 + 1744.791 = 3984.791$, the area of the cooler in square inches, and $3984.791 \div 282 = 14.13$, the area in ale gallons; whence 14.13×9.4 $= 132.822$, the content in ale gallons. *(See Prob. XIX., Part IV.)*

Exam. 30.

By Prob. XX., Part IV., we have $49.5 + 49.6 = 99.1$, the sum of the first and last ordinates; $(51.4 + 52.6 + 52.7 + 51.5) \times 4 = 208.2 \times 4 = 832.8$, four times the sum of the even ordinates; $(52.2 + 53.2 + 52.3) \times 2 = 157.7 \times 2 = 315.4$, twice the sum of the odd ordinates; and $97.6 \div 8 = 12.2$, the common distance of the ordinates; then, $(99.1 + 832.8 + 315.4) \times \frac{12.2}{3} = 1247.3 \times \frac{12.2}{3} = \frac{15217.06}{3}$ $= 5072.353$, the area in square inches; hence $5072.353 \div 231 = 21.958$, the area in wine gallons.

Exam. 31.

By Prob. I., Part V., we have $64.3 \times 64.3 \times 64.3$ $= 4134.49 \times 64.3 = 265847.707$, the content in cubic inches; hence $265847.707 \div 282 = 942.722$, the content in ale gallons; and $265847.707 \div 231 = 1150.855$, the content in wine gallons.

Exam. 32.

By Prob. II., Part V., we have $145 \times 96 \times 84 = 13920 \times 84 = 1169280$, the content in cubic inches;

and 1169280 ÷ 282 = 4146.382, the content in ale gallons.

EXAM. 33.

By Prob. X., Part IV., we have 86.4 × 86.4 × .001689 = 7464.96 × .001689 = 12.60831744, the area of the base in malt bushels; and by Prob. III., Part V., 12.60831744 × 73.6 = 927.972163584, the content in malt bushels.

EXAM. 34.

By Prob. V., Part I., we have $95^2 - 76^2 = 9025 - 5776 = 3249$, the square of the diameter of the base; then by Prob. IV., Part V., 3249 × 76 ÷ 359.05 = 246924 ÷ 359.05 = 687.714, the content in ale gallons.

EXAM. 35.

By Prob. X., Part IV., we have 53.8 × 53.8 × .001875 = 2894.44 × .001875 = 5.42707500, the area of the top in wine gallons; and by Note 4, Prob. ., Part V., 5.42707500 × 20.8 = 112.883, the content wine gallons.

EXAM. 36.

By Rule II., Prob. VI., Part V., we have 45.7 × 45.7 2088.49, the square of a side of the greater end; .3 × 34.3 = 1176.49, the square of a side of the s end; and 45.7 × 34.3 = 1567.51, the product the sides; then (2088.49 + 1176.49 + 1567.51) 22.8 = 4832.49 × 22.8 = 110180.772, the content in cubic inches. Hence 110180.772 × .0008 = .1446176, the content in malt bushels.

EXAM. 37.

By Prob. V., Part I., we have $2\sqrt{85^2 - 75^2} = \sqrt{7225 - 5625} = 2\sqrt{1600} = 40 \times 2 = 80$, diameter of the top; then by Rule I., Prob. VII.,

Part V., $(80 \times 80 \times 25) \div 359.05 = 160000 \div 359.05 = 445.62$, the content in ale gallons.

Exam. 38.

By Rule I., Prob. VIII., Part V., we have $126.3 \times 158.6 \times 3 = 20031.18 \times 3 = 60093.54$, three times the product of the diameter.

Also $158.6 - 126.3 = 32.3$, the difference of the diameters; and $32.3 \times 32.3 = 1043.29$, the square of the difference; then $(60093.54 + 1043.29) \times 132.7 = 61136.83 \times 132.7 = 8112857.341$; and $8112857.341 \div 882.36 = 9194.898$, the content in wine gallons.

Exam. 39.

By Prob. IX., Part V., we have $82.4 \times 42.6 = 3510.24$, the area of the bottom; and $104.2 \times 54.8 = 5710.16$, the area of the top.

Also $\frac{82.4 + 104.2}{2} = \frac{186.6}{2} = 93.3$, the length of the middle section; and $\frac{42.6 + 54.8}{2} = \frac{97.4}{2} = 48.7$, the breadth of the middle section; then $93.3 \times 48.7 \times 4 = 4543.71 \times 4 = 18174.84$, four times the area of the middle section; whence $(3510.24 + 5710.16 + 18174.84) \times \frac{112.2}{6} = 27395.24 \times 18.7 = 512290.988$, the content in cubic inches; and $512290.988 \div 2150.42 = 238.228$, the content in malt bushels.

Exam. 40.

By Prob. IX., Part V., we have $\frac{99.2 + 134.4}{2} = \frac{233.6}{2} = 116.8$, the transverse diameter of the middle

section; and $\frac{75.6 + 100.8}{2} = \frac{176.4}{2} = 88.2$, the conjugate diameter of the middle section.

Now $99.2 \times 75.6 \times .7854 = 7499.52 \times .7854 = 5890.123008$, the area of the bottom; $116.8 \times 88.2 \times .7854 \times 4 = 10301.76 \times .7854 \times 4 = 8091.002304 \times 4 = 32364.009216$, four times the area of the middle section; and $134.4 \times 100.8 \times .7854 = 13547.52 \times .7854 = 10640.222208$, the area of the top; then $(5890.123008 + 32364.009216 + 10640.222208) \times \frac{93.6}{6} = 48894.354432 \times 15.6 = 762751.9291392$, the content in cubic inches; hence $762751.929 \div 282 = 2704.794$, the content in ale gallons.

EXAM. 41.

By Rule II., Prob. X., Part V., we have $52.4 \times 52.4 \times 52.4 \times .00227 = 2745.76 \times 52.4 \times .00227 = 143877.824 \times .00227 = 326.60266048$, the content in wine gallons.

EXAM. 42.

By Rule I., Prob. XI., Part V., we have $17.8 \times 17.8 \times 3 = 316.84 \times 3 = 950.52$, three times the square of half the top diameter; also $15.8 \times 15.8 = 249.64$, the square of the depth; then $(950.52 + 249.64) \times 15.8 \times .5236 = 1200.16 \times 15.8 \times .5236 = 18962.528 \times .5236 = 9928.7796608$, the content in cubic inches; hence $9928.779 \div 282 = 35.208$, the content in ale gallons.

EXAM. 43.

By Rule II., Prob. XI., Part V., we have $53.8 \times 53.8 = 2894.44$, the square of the diameter of the base; and $32.7 \times 32.7 = 1069.29$, the square of the depth; also $\frac{1069.29}{3} = 356.43$, one-third of the

square of the depth; then $(2894.44 + 1069.29 + 356.43) \times 16.35 = 4320.16 \times 16.35 = 70634.616$; and $70634.616 \div 294.12 = 240.155$, the content in wine gallons.

EXAM. 44.

By Prob. XII., Part V., we have $\frac{72^2 + 84^2}{2} = \frac{5184 + 7056}{2} = \frac{12240}{2} = 6120$, half the sum of the squares of the two diameters; and $60^2 \times \frac{2}{3} = 3600 \times \frac{2}{3} = \frac{7200}{3} = 2400$, two-thirds of the square of the depth; then $(6120 + 2400) \times 60 = 8520 \times 60 = 511200$; and $511200 \div 2738 = 186.705$, the content in malt bushels.

EXAM. 45.

By Prob XIII., Part V., we have $82 \times 82 \times 96 = 6724 \times 96 = 645504$; and $645504 \div 538.58 = 1198.529$, the content in ale gallons.

EXAM. 46.

By the Remark at the end of Prob. XIII., Part V., we have $(40 \times 40 \times 28) \div 441.18 = (1600 \times 28) \div 441.18 = 44800 \div 441.18 = 101.545$, the content in wine gallons.

EXAM. 47.

By Prob. XIV., Part V., we have $48 \times 48 \times 2 = 2304 \times 2 = 4608$, twice the square of the middle diameter, $40 \times 40 = 1600$, the square of the end diameter; and $4608 + 1600 \times 50 = 6208 \times 50 = 310400$; hence $310400 \div 1077.15 = 288.167$, the content in ale gallons.

EXAM. 48.

By the Remark at the end of Prob. XIV., Part V., we have $72 \times 72 \times 2 = 5184 \times 2 = 10368$, twice the square of the greater diameter; and $54 \times 54 = 2916$, the square of the less diameter; then $10368 + 2916 \times 80 = 13284 \times 80 = 1062720$; hence $1062720 \div 882.36 = 1204.406$, the content in wine gallons.

EXAM. 49.

By Prob. XV., Part V., we have $50 \times 50 \times 80 = 2500 \times 80 = 200000$, the square of the middle diameter, multiplied by the length; then $200000 \div 673.22 = 297.079$, the content in ale gallons.

EXAM. 50.

By Prob. XVI., Part V., we have $50 \times 50 \times 8 = 2500 \times 8 = 20000$, eight times the square of the middle diameter; $40 \times 40 \times 3 = 1600 \times 3 = 4800$, three times the square of the end diameter; and $50 \times 40 \times 4 = 2000 \times 4 = 8000$, four times the product of the diameters; then $(20000 + 4800 + 8000) \times 54 = 32800 \times 54 = 1771200$; and $1771200 \div 4411.8 = 401.468$, the content in wine gallons.

EXAM. 51.

By Prob. XVII., Part V., we have $54 \times 54 \times 32 = 2916 \times 32 = 93312$, the square of the diameter of the top, multiplied by half the depth; then $93312 \div 359.05 = 259.885$, the content in ale gallons.

EXAM. 52.

By the Remark at the end of Prob. XVII., Part V., we have $30^2 + 42^2 = 900 + 1764 = 2664$, the square of the radius of the top, added to the square of the middle diameter; then $2664 \times 42 \times .5236 =$

$111888 \times .5236 = 58584.5568$, the content in cubic inches; and $58584.5568 \div 231 = 253.6128$, the content in wine gallons.

EXAM. 53.

By Prob XVIII, Part V., we have $72^2 + 48^2 \times 30 = (5184 + 2304) \times 30 = 7488 \times 30 = 224640$, the sum of the squares of the diameters, multiplied by the depth; and $224640 \div 718.1 = 312.825$, the content in ale gallons.

EXAM. 54.

By the Remark at the end of Prob. XVIII., Part V., we have $24^2 + 40^2 + 68^2 = 576 + 1600 + 4624 = 6800$, the sum of the squares of the semi-diameters of the top and bottom, added to the square of the middle diameter; then $6800 \times 96 \times .5236 = 652800 \times .5236 = 341806.08$, the content in cubic inches; and $341806.08 \div 231 = 1479.68$, the content in wine gallons.

EXAM. 55.

By Prob. XIX., Part V., we have $42^2 + 45^2 = 1764 + 2025 = 3789$, the sum of the squares of the top and bottom diameters; and $47^2 \times 4 = 2209 \times 4 = 8836$, four times the square of the middle diameter; then $(3789 + 8836) \times 48 = 12625 \times 48 = 606000$; and $606000 \div 2154.3 = 281.297$, the content in ale gallons.

EXAM. 56.

By Prob. XX., Part V., we have $69^2 + 78^2 = 4761 + 6084 = 10845$, the sum of the squares of the top and bottom diameters; $95^2 \times 2 = 9025 \times 2 = 18050$, twice the square of the middle diameter; and $87^2 + 90^2 \times 4 = (7569 + 8100) \times 4 = 15669 \times 4 = 62676$, four times the sum of the squares of the diameters taken at one-fourth, and at three-fourths

of the depth; then $(10845 + 18050 + 62676) \times 80 = 91571 \times 80 = 7325680$; and $7325680 \div 4308.6 = 1700.246$, the content in ale or porter gallons.

EXAM. 57.

By Rule II., Prob. XXI., Part V., we have the following Solution.

	Diameters. Inches.	Areas. Wine gallons.
First	82.6	23.1972
Second	97.5	32.3212
Third	107.4	39.2181
Fourth	109.3	40.6180
Fifth	110.7	41.6652
Sixth	104.6	37.1999
Seventh	96.2	31.4650

Then, $23.1972 + 31.4650 = 54.6622$, the sum of the two extreme areas; $(32.3212 + 40.6180 + 37.1999) \times 4 = 110.1391 \times 4 = 440.5564$, four times the sum of all the even areas; and $89.2181 + 41.6652 \times 2 = 808833 \times 2 = 161.7666$, twice the sum of all the odd areas; also $90 \div 6 = 15$, the common distance of the sections; hence, $(54.6622 + 440.5564 + 161.7666) \times \frac{15}{3} = 656.9852 \times 5 = 3284.926$, the content in wine gallons.

EXAM. 58.

By Prob. XXII., Part V., we have $50 \times 50 = 2500$, the square of the diameter of the base; and $\frac{32 + 23}{2} = \frac{55}{2} = 27.5$, half the sum of the greater and least depth of the liquor; then $2500 \times 27.5 = 68750$; and $68750 \div 359.05 = 191.477$, the content in ale gallons.

Exam. 59.

By the question, the diameter of the vessel is 85, and the versed sine of the ungula's base, or less segment 36; consequently, $85 - 36 = 49$, the versed sine of the greater segment.

Now, by Theo. 12, Part III., the product of the two parts of the diameters, is equal of the square of half the chord: hence, we have $2\sqrt{49 \times 36} = 2\sqrt{1764} = 42 \times 2 = 84$ inches, the chord of the ungula's base.

By Rule I., Prob. XV., Part IV., we have $84 \times 36 \times \frac{2}{3} = 3024 \times \frac{2}{3} = \frac{6048}{3} = 2016$, two-thirds of the product of the chord and versed sine of the ungula's base; and $\frac{36^3}{84 \times 2} = \frac{46656}{168} = 277.714$, the cube of the versed sine divided by twice the chord: then $2016 + 277.714 = 2293.714$ square inches, the area of the ungula's base.

By Prob. XXIII., Part V., we have $\frac{85}{2} - 36 = 42.5 - 36 = 6.5$, the difference between half the diameter of the vessel, and the versed sine of the ungula's base; then $2293.714 \times 6.5 = 14909.141$, the area of the base multiplied by the said difference.

Again, $42^3 \times \frac{2}{3} = 74088 \times \frac{2}{3} = \frac{148176}{3} = 49392$, two-thirds of the cube of half the chord; and $49392 - 14909.141 = 34482.859$; then $34482.859 \times \frac{60}{36} = \frac{2068971.54}{36} = 57470.143$, the content in cubic inches; hence $57470.143 \div 231 = 248.788$, the content of the ungula, in wine gallons.

EXAM. 60.

By Prob. XXIV., Part V., we have $\frac{60 \times 60 \times 50}{6}$ $= \frac{3600 \times 50}{6} = \frac{180000}{6} = 30000$, the content in cubic inches; and $\frac{30000}{282} = 106.382$, the content of the ungula, in ale gallons.

EXAM. 61.

By Note II., Prob. XV., Part IV., we have $\frac{30 \times 30}{36}$ $+ 36 = \frac{900}{36} + 36 = 25 + 36 = 61$ inches, the diameter of the vessel; hence it appears that the base of the ungula is greater than a semi-circle. By Rule II., Prob. XV., Part IV., we have $\frac{36}{61} = .590\,\frac{10}{61}$ the quotient of the versed sine, divided by the diameter: and by Note 2, we have $1 - .590\,\frac{10}{61} = .409\,\frac{51}{61}$. The *Area Seg.* answering to this remainder is .303025, then $.785398 - .303025 = .482373$, the area segment, corresponding to $.590\,\frac{10}{61}$; hence $.482373 \times 61^2 = .482373 \times 3721 = 1794.909933$ square inches, the area of the ungula's base.

By Prob. XXV., Part V., we have $36 - \frac{61}{2} = 36 - 30.5 = 5.5$, the difference between the versed sine of the ungula's base, and half the diameter of the vessel; then $1794.909933 \times 5.5 = 9872.0046315$, the area of the ungula's base, multiplied by the said difference.

Again, $30^3 \times \frac{2}{3} = 27000 \times \frac{2}{3} = \frac{54000}{3} = 18000$, two-thirds of the cube of half the chord; and 9872 +

$18000 = 27872$; then $27872 \times \frac{50}{36} = \frac{1393600}{36}$ $= 38711.111$, the content in cubic inches; and $\frac{38711.111}{231}$ $= 167.580$, the content of the ungula in wine gallons.

Exam. 62.

By Rule II., Prob. XV., Part IV., we have $40 \div 50 = .8$, the versed sine or depth of the liquor, divided by the diameter of the vessel. The *Area Seg.* answering to this quotient, is $.673575$; then $.673575 \times 50^2 = .673575 \times 2500 = 1683.9375$ square inches, the area of the circular segment of the end; and by the Remark at the end of Prob. XXV., Part V., we have $1683.9375 \times 60 = 101036.25$, the content of the ungula in cubic inches; consequently, $101036.25 \div 282 = 358.284$, the content in ale gallons.

Exam. 63.

By the Solution to the last Examples, we have the area of the greater end $= 1683.9375$ square inches; and as the less end is a semi-circle, we have $(.7854 \times 2500) \div 2 = 1963.5 \div 2 = 981.75$, its area in square inches.

Now $\frac{40 + 25}{2} = \frac{65}{2} = 32.5$ inches, the depth of the liquor, at the middle of the vessel; then $32.5 \div 50 = .65$, the versed sine or depth of the liquor divided by the diameter of the vessel. The *Area Seg.* corresponding to this quotient, is $.540418$; then $.540418 \times 2500 = 1351.045$ square inches, the area of the middle section.

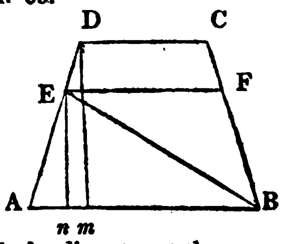
By Prob. IX., Part V., we have $1683.9375 + 981.75 = 2665.6875$, the sum of the areas of the two ends: and $1351.045 \times 4 = 5404.18$, four times the area of the middle section; then $(2665.6875 + 5404.18) \times \frac{60}{6}$

$= 8069.8675 \times 10 = 80698.675$, the content of the ungula in cubic inches; and $80698.675 \div 231 = 349.344$, the content in wine gallons. (See the Remark at the end of Prob. XXV., Part V.)

EXAM. 64.

By Prob. XXVI., Part V., we have $87 \times 2 + 57 = 174 + 57 = 231$, twice the length of the base, added to the length of the upper edge of the liquor; then, $\frac{231 \times 68 \times 63}{6} = \frac{989604}{6} = 164934$, the content of the ungula, in cubic inches; and $164934 \div 282 = 584.872$, the content in ale gallons.

EXAM. 65.

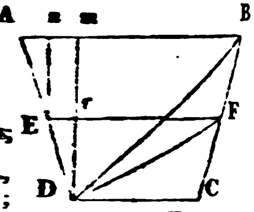


By the question $AB = 60$, $CD = 50$, $Dm = 40$, $En = 30$, and $Am = \frac{60 - 50}{2} = \frac{10}{2} = 5$ inches; then by Theo. 11, Part III., as $40 : 5 :: 30 : 3.75 = An$; and $60 - 3.75 \times 2 = 60 - 7.5 = 52.5 = EF$, the diameter at the upper extremity of the liquor. Also, by Prob I., Part I., $\sqrt{52.5 \times 60} = \sqrt{3150} = 56.12$, the mean proportional between the diameters AB and EF.

By Prob. XXVII., Part V., we have $3150 \times 56.12 = 176778$, the product of the two diameters multiplied by the mean proportional between them.

Also, $60^3 = 216000$, the cube of the bottom diameter; then $\frac{216000 - 176778}{60 - 52.5} = \frac{39222}{7.5} = 5229.6$; and $5229.6 \times 30 \times .001133 = 177.754104$, the content of the ungula A B E, in wine gallons.

Exam. 66.



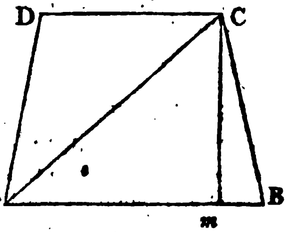
By the question AB = 60, CD = 50, Dm = 40, Dr = En = 20, and An $= \frac{60 - 50}{2} = \frac{10}{2} = 5$ inches; then by Theo. 11, Part III., as 40 : 5 :: 20 : 2.5 = An; and $60 - 2.5 \times 2 = 60 - 5 = 55$ = EF, the diameter at the upper extremity of the liquor, when the vessel is in its first position. Also by Prob. I., Part I., $\sqrt{50 \times 55} = \sqrt{2750} = 52.44$, the mean proportional between the diameters CD and EF.

By Prob. XXVIII., Part V., we have $2750 \times 52.44 = 144210$, the product of the two diameters multiplied by the mean proportional between them.

Also $50^3 = 125000$, the cube of the bottom diameter: then $\frac{144210 - 125000}{55 - 50} = \frac{19210}{5} = 3842$; and $3842 \times 20 \times .0009283 = 71.330572$, the content of the ungula DCF, in ale gallons.

Again, by the same Prob. we have $\sqrt{60 \times 50} = \sqrt{3000} = 54.77$, the mean proportional between the diameters AB and CD; and $54.77 \times 3000 = 164310$, the mean proportional multiplied by the product of the two diameters; then $\frac{164310 - 125000}{60 - 50} = \frac{39310}{10} = 3931$; and $3931 \times 40 \times .0009283 = 145.965892$, the content of the ungula DCB, in ale gallons; then $145.965892 - 71.330572 = 74.63532$ ale gallons, the answer required.

EXAM. 67.



By the question AB = 70, CD = 60, and Cm = 50 inches. Now, as :he ungula ABC is occu-)ied by the liquor, when he vessel is in its first)osition, it is evident hat the content of the ry hoof ACD, will be the answer required.

By Prob. XXVIII., Part V., we have $\sqrt{70 \times 60} =$ $\sqrt{4200} = 64.807$, the mean proportional between the iameters; and $64.807 \times 4200 = 272189.4$, the mean roportional multiplied by the product of the two dia-ieters.

Also, $60^3 = 216000$, the cube of the diameter CD; ien $\dfrac{272189.4 - 216000}{70 - 60} = \dfrac{56189.4}{10}$ 5618.94; and ;18.94 $\times 50 \times .0009283 = 260.8031001$ ale gallons, e content of the dry hoof ACD.

EXAM. 68.

By Prob. XXX., Part V., we have $90 - 75 = 15$, e difference of the diameters; and $\dfrac{60 - 15}{75} = \dfrac{45}{75} =$ the quotient obtained by subtracting the difference the diameters from the versed sine of the base, and viding the remainder by the less diameter. The ea *Seg*. answering to this quotient is .492029.

Now, $75^3 = 421875$, the cube of the less diameter; o, $\dfrac{60}{60 - 15} = \dfrac{60}{45} = 1 \cdot 333$, the quotient arising m dividing the versed sine of the base by the differ-:e between the said versed sine and the difference the diameters; and $\sqrt{1.333} = 1.154$, the square

root of the said quotient; then $.492029 \times 421875 \times 1.333 \times 1.154 = 207574.734375 \times 1.333 \times 1.154 = 276697.120921875 \times 1.154 = 319308.477543$, the reserved product.

Again, $90^3 = 729000$, the cube of the bottom diameter; and $\frac{60}{90} = .666\frac{2}{3}$, the quotient of the versed sine of the base divided by the bottom diameter. The *Area Seg.* answering to this quotient, is .556226; then $.556226 \times 729000 = 405488.754$, the product arising from multiplying the last *Area Seg.* by the cube of the bottom diameter.

Now, $405488.754 - 319308.477543 = 86180.276457$, the difference between the last product and the reserved product; also $81 \div 3 = 27$, one-third of the perpendicular height of the ungula; then $86180.276457 \times \frac{27}{15} = 86180.276457 \times \frac{9}{5} = \frac{775622.488113}{5} = 155124.4976226$, the content of the ungula, in cubic inches; and $155124.497 \div 282 = 550.086$, the content in ale gallons.

Note. In order to find the *Area Seg.* answering to $.666\frac{2}{3}$, we have $1 - .666\frac{2}{3} = .333\frac{1}{3}$. The *Area Seg.* answering to .333, is .229801, and that answering to .333, is .228858; their difference is .000943; one-third of which is .000314 This being added to .228858, gives .229172, the *Area Seg.* corresponding to $.333\frac{1}{3}$; then $.785398 - .229172 = .556226$, the *Area Seg.* answering to $.666\frac{2}{3}$. (See Notes under Rule II., Prob. XV., Part IV.)

Exam. 69.

By Prob. XXXI., Part V., we have $\frac{55 - 40 + 18}{55} = \frac{15 + 18}{55} = \frac{33}{55} = .6$; the *Area Seg.* answering to this quotient is .492029; and $.492029 \times 55^3 = .492029 \times 166375 = 81861.324875$, the first number.

Again, $\frac{18}{15 + 18} = \frac{18}{33} = .545$; and $\sqrt{.545^3} = \sqrt{.161878625} = .4023$, the second number.

Again, $\frac{18}{40} = .45$; the *Area Seg.* answering to this quotient, is .342782; and $.342782 \times 40^3 = .342782 \times 64000 = 21938.048$, the third number.

Again, $\frac{21}{15 \times 3} = \frac{21}{45} = .4666$, the fourth number.

Now, $81861.324875 \times .4023 = 32932.8109972125$, the product of the first and second numbers; then $(32932.810997 - 21938.048) \times .4666 = 10994.762997 \times .4666 = 5130.1564144002$, the content of the ungula in cubic inches; and $5130.156 \div 282 = 18.192$, the content in ale gallons.

EXAM. 70.

By Prob. XXXII., Part V., we have $50 - 37 = 13$, the versed sine of the base of the ungula; and $\frac{13}{50} = .26$. The *Area Seg.* answering to this quotient, is .162263; then $.162263 \times 50^2 = .162263 \times 2500 = 405.6575$, the area of the ungula's base; and $405.6575 \times \frac{50}{13} = \frac{20282.875}{13} = 1560.221$, the reserved quotient.

Again, $\sqrt{13 \times 37} \times 37 \times \frac{4}{3} = \sqrt{481} \times \frac{148}{3} = 21.93 \times \frac{148}{3} = \frac{3245.64}{3} = 1081.88$.

Then, $(1560.221 - 1081.88) \times \frac{17.4}{3} = 478.341 \times 5.8 = 2774.3778$, the content of the ungula, in cubic inches; and $2774.3778 \div 231 = 12.0102$, the content in wine gallons.

EXAM. 71.

By Prob. XIV., Part V., we have $32 \times 32 \times 2 = 1024 \times 2 = 2048$, twice the square of the bung diameter; and $26 \times 26 = 676$, the square of the head

diameter; then $(2048 + 676) \times 45 = 2724 \times 45 = 122580$; and $122580 \div 1077.15 = 113.800$; the content in ale gallons; also, $122580 \div 882.36 = 138.922$, the content in wine gallons.

EXAM. 72.

By Prob. XVI., Part V., we have $32 \times 32 \times 8 = 1024 \times 8 = 8192$, eight times the square of the bung diameter; $26 \times 26 \times 3 = 676 \times 3 = 2028$, three times the square of the head diameter; and $32 \times 26 \times 4 = 832 \times 4 = 3328$, four times the product of the diameters; then $(8192 + 2028 + 3328) \times 45 = 13548 \times 45 = 609660$; and $609660 \div 5385.75 = 113.198$, the content in ale gallons; also $609660 \div 4411.8 = 138.187$, the content in wine gallons.

EXAM. 73.

By Prob. XVIII., Part V., we have $32 \times 32 = 1024$, the square of the bung diameter; $26 \times 26 = 676$, the square of the head diameter; and $(1024 + 676) \times 45 = 1700 \times 45 = 76500$, the sum of the squares of the two diameters, multiplied by the length; then $76500 \div 718.1 = 106.530$, the content in ale gallons; and $76500 \div 588.24 = 130.048$, the content in wine gallons.

EXAM. 74.

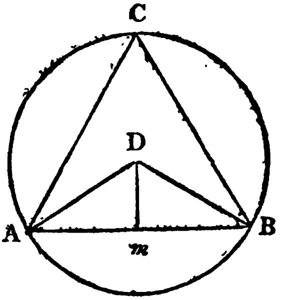
By Rule I., Prob. VIII., Part V., we have $32 \times 26 \times 3 = 832 \times 3 = 2496$, three times the product of the bung and head diameters; and $\overline{32 - 26}|^2 = 6^2 = 36$, the square of their difference; then $(2496 + 36) \times 45 = 2532 \times 45 = 113940$; and $113940 \div 1077.15 = 105.779$, the content in ale gallons; also $113940 \div 882.36 = 129.130$, the content in wine gallons.

EXAM. 75.

By Prob. X., Part IV., the square of the side of any regular polygon, multiplied by its proper factor, will give the area in square inches; consequently, if the area of any polygon be divided by its proper factor, the quotient will be the square of the side; hence

we have $282 \times 45 \div .433013 = 12690 \div .433013 = 29306.279488$, the square of the side; and $\sqrt{29306.279488} = 171.190$ inches, the side required.

Exam. 76.



Let the triangle ABC represent the base of the prismatic vessel; and the circle ABC that of the cylindrical vessel; D being the centre of the triangle, and also that of its circumscribing circle; and AD = DB, the radius of the said circle.

Now, by Prob. X., Part IV., we have $.433013 \times 50^2 = .433013 \times 2500 = 1082.5325$ square inches, the area of the triangle ABC; and $\frac{1082.5325}{3} = 360.844$, the area of the triangle ABD; and this area divided by half the base AB, will give the perpendicular D m; that is $360.844 \div 25 = 14.433$ inches $=$ D m.

By Prob. IV., Part I., we have $Am^2 + Dm^2 = 25^2 + 14.433^2 = 625 + 208.311489 = 833.311489$, the square of the radius AD; and $833.311489 \times 3.1416 = 2617.9313738424$ square inches, the area of the circle ABC.

Now, $1082.5325 \times 40 = 43301.3$ cubic inches, the content of the prismatic vessel; and $2617.93137 \times 40 = 104717.2548$ cubic inches, the content of the cylindrical vessel; then $104717.2548 - 43301.3 = 61415.9548$, the difference of their contents; and $61415.9548 \div 282 = 217.787$ ale gallons, the answer required.

Note. Here it may be proper to observe that no attention has beed paid to the thickness of the sides of the prismatic vessel.

Exam. 77.

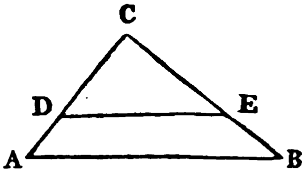
By Prob. II., Part IV., the length of a rectangle, multiplied by its breadth, will give the area; consequently, if the area of a rectangle be divided by one of its dimensions, the quotient will be the other dimension; hence we have $282 \times 12 \div 60 = 3384 \div 60 = 56.4$ inches; and $80 - 56.4 = 23.6$ inches; therefore, the length of each vessel is 60 inches; the breadth of the greater 56.4 inches, and that of the less 23.6 inches.

Exam. 78.

By Prob. II., Part V., it is evident that if we divide the content of a parallelopipedon by one of its dimensions, the quotient will be the product of the other two dimensions; hence, we have $282 \times 380 = 107160$ cubic inches, the content of the less cistern; and $107160 \div 48 = 2232.5$ the area of the base; then by Prob. II., Part IV., we have $2232.5 \div 64 = 34.882$; and $86 - 34.882 = 51.118$; consequently, the length of each cistern is 64 inches; the breadth of the greater 51.118 inches, and that of the less 34.882 inches.

Exam. 79.

Let A B C denote the triangular cooler; then by the question, $AC = 60$, $BC = 70$, and $AB = 80$ inches.



By Prob. V., Part IV., we have $\frac{60 + 70 + 80}{2} = \frac{210}{2} = 105$, half the sum of the three sides; then $105 - 60 = 45$, the first remainder; $105 - 70 = 35$, the

second remainder; and $105 - 80 = 25$, the third remainder; then $\sqrt{105 \times 45 \times 35 \times 25} = \sqrt{4134375}$ $= 2033.316$ square inches, the area of the triangle ABC.

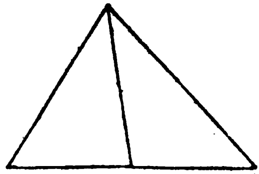
Now, $282 \times 3 = 846$ square inches, the area of the trapezoid ABED; the line DE being the dividing plane; and $2033.316 - 846 = 1187.316$ square inches, the area of the triangle DEC.

Then by Theo. 13, Part III., as the area of the triangle ABC is to the square of any of the sides, so is the area of the triangle DEC, to the square of any similar side; hence, as $2033.316 : 60^2 :: 1187.316 : 2102.151165 = DC^2$; and $\sqrt{2102.151165} = 45.849$ inches $= DC$.

Again, as $2033.316 : 70^2 :: 1187.316 : 2861.261309$ $= EC^2$; and $\sqrt{2861.261309} = 53.490$ inches, $= EC$.

Lastly, as $2033.316 : 80^2 :: 1187.316 : 3737.157628$ $= DE^2$; and $\sqrt{3737.157628} = 61.132$ inches $=$ DE; hence all the sides of the triangle DEC are determined.

EXAM. 80.



Let ABC represent the triangular cooler; and CD the dividing plane; then by the question, $AC = 60$, $BC = 70$, $AB = 80$, and $AD = DB = 40$ inches.

By Theo. 14, Part III., we have $\frac{AC^2 + BC^2 - 2AD^2}{2}$ $= CD^2$: that is $\frac{3600 + 4900 - 3200}{2} =$

$\frac{8500 - 3200}{2} = \frac{5300}{2} = 2650 = CD^2$; and $\sqrt{2650}$ $= 51.478$ inches $= CD$.

By Prob. V., Part IV., we have $\frac{60 + 40 + 51.478}{2}$ $= \frac{151.478}{2} = 75.739$, half the sum of the three sides; then $75.739 - 60 = 15.739$, the first remainder: $75.739 - 40 = 35.739$, the second remainder; and $75.739 - 51.478 = 24.261$, the third remainder; and by multiplying the half sum and the three remainders continually together, we obtain 1033588.804259, the square root of which is 1016,655, the area of the triangle ADC in square inches; hence $1016.655 \div 282 = 3.605$, the area in ale gallons.

Now, by the solution to the last example, the area of the triangle ABC is 2033.316 square inches; consequently, $2033.316 - 1016.655 = 1016.661$ square inches, the area of the triangle BCD; and $1016.661 \div 282 = 3.605$, the area in ale gallons; hence it appears that the line DC divides the triangle ABC into two equal parts.

Note. By Theo. 6, Part III., triangles upon the same base, (or of equal bases,) and between the same parallels are equal; hence the last question might have been solved on this principle, by dividing the area of the given triangle by two.

Exam. 81.

Here $16 \times 8 \times 2150.42 = 128 \times 2150.42 = 275253.76$ cubic inches, the content of the cistern, whose depth is required; then by the Note under Prob. II., Part V., we have $\frac{275253.76}{94 \times 72} = \frac{275253.76}{6768} =$ 40.669 inches, the depth required.

EXAM. 82.

By Prob. V., Part I., we have $\sqrt{180^2 - 108^2} = \sqrt{32400 - 11664} = \sqrt{20736} = 144$ inches, the length of the cistern.

Now, by Prob. II., Part V., we have $144 \times 108 \times 42 = 15552 \times 42 = 653184$, the content in cubic inches; and $653184 \div 5 = 130636.8$ cubic inches, the allowance for the swell of the grain; then $\frac{653184 - 130636.8}{2150.42} = \frac{522547.2}{2150.42} = 242.997$ bushels, the answer required.

EXAM. 83.

Here $9743 \times 282 = 2747526$, the content in cubic inches; and by Note 1, Prob. IV., Part V., we have $147 \times 147 \times .7854 = 21609 \times .7854 = 16971.7086$, the area of the base; then by Note 2, of the same Prob. we have $2747526 \div 16971.7086 = 161.888$ inches, the depth required.

EXAM. 84.

By Prob. IV., Part I., we have $\sqrt{36^2 + 36^2} = \sqrt{1296 + 1296} = \sqrt{2592}$ = the side of the given vessel; then by Prob. I., Part V., we have $\sqrt{2592} \times \sqrt{2592} \times \sqrt{2592} = \sqrt{2592 \times 2592 \times 2592} = \sqrt{17414258688} = 131963.095$, the content in cubic inches; and $131963.095 \div 231 = 571.268$, the content in wine gallons.

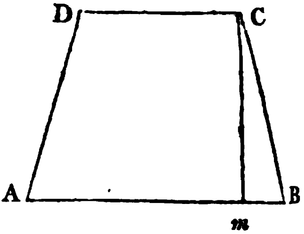
EXAM. 85.

By Note 1, Prob. IV., Part V., we have $18.5 \times 18.5 \times 7854 \times 8 = 342.25 \times .7854 \times 8 = 268.80315 \times 8 = 2150.4252$, the content of the Winchester bushel, in cubic inches; and $2150.4252 \div 9 = 238.93613$ square inches, the area of the base of the bushel,

whose depth is 9 inches; then $\sqrt{238.93613 \div .7854} = \sqrt{304.222222} = 17.441$ inches, the diameter required.

Exam. 86.

Let ABCD represent the middle perpendicular section of the given vessel; then by the question, AB = 150, CD = 86, and BC = 68 inches; consequently $Bm = \frac{150 - 86}{2} = \frac{64}{2} = 32$ inches.



By Prob. V., Part I., or Theo. 7, Part III., we have $\sqrt{68^2 - 32^2} = \sqrt{4624 - 1024} = \sqrt{3600} = 60$ inches, the perpendicular height Cm.

By Rule II., Prob.VIII., Part V., we have $\overline{150 + 86}|^2 = 236^2 = 55696$, the square of the sum of the diameters; and $150 \times 86 = 12900$, the product of the diameters; then $(55696 - 12900) \times 60 = 42796 \times 60 = 2567760$; and $2567760 \div 1077.15 = 2383.846$, the content in ale gallons.

Exam. 87.

(See the last Figure.)

Let ABCD represent the middle perpendicular section of the vessel; then by the question, AB = 195, BC = 85, and $Cm = 75$; then by Prob. V., Part I., we have $Bm = \sqrt{85^2 - 75^2} = \sqrt{7225 - 5625} = \sqrt{1600} = 40$ inches, half the difference between the sides; hence $195 - 40 \times 2 = 195 - 80 = 115 =$ CD, the side of the top of the given vessel.

By Rule II., Prob. VI., Part V., we have $195 \times 195 = 38025$, the square of the side of the base; $115 \times 115 = 13225$, the square of the side of the top; and $195 \times 115 = 22425$, the product of the sides; then $(38025 + 13225 + 22425) \times 75 \div 3 = 73675 \times 25 = 1841875$, the content in cubic inches. Hence $1841875 \times .000465 = 856.471875$, the content in malt bushels.

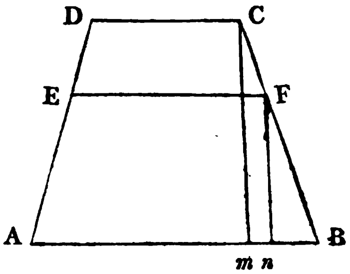
EXAM. 88.

Here $3834 \times 231 = 885654$, the content in cubic inches; and by Prob. X., Part IV., we have $74 \times 74 \times 1.720477 = 5476 \times 1.720477 = 9421.332052$ square inches, the area of the base; then by Note 3, Prob. III., Part V., we have $885654 \div 9421.332052 = 94.005$ inches, the depth required.

EXAM. 89.

By Prob. II., Part V., we have $182 \times 112 \times 74 = 20384 \times 74 = 1508416$, the content of the reservoir in cubic inches; and $26 \times 3 = 78$ pints $= 2749.5$ cubic inches; the quantity of water, which the reservoir receives in one minute; then $\frac{1508416}{2749.5} = 548.614$ minutes $= 9$ hours, 8.614 minutes, the time required.

EXAM. 90.



Let ABCD represent the middle section of the given vessel; then by the question, $AB = 86$, $CD = 62$, $Cm = 58$, $Fn = 40$; and $Bm = \frac{86 - 62}{2} = \frac{24}{2} = 12$ inches.

By Theo. 11, Part III., the triangle BnF is similar to the triangle BmC; hence, as Cm : Bm :: Fn : Bn; that is, as 58 : 12 :: 40 : 8.275 = Bn; then AB — 2 Bn = 86 — 16.55 = 69.45 inches = EF, the diameter at the surface of the liquor.

By Rule I., Prob. VIII., Part V., we have 69.45 × 86 × 3 = 5972.7 × 3 = 17918.1, three times the product of the top and bottom diameters; and $\overline{86 - 69.45}^2 = 16.55^2 = 273.9025$, the square of their difference; then (17918.1 + 273.9025) × 40 = 18192.0025 × 40 = 727680.1; and 727680.1 ÷ 1077.15 = 675.560, the content in ale gallons.

EXAM. 91.

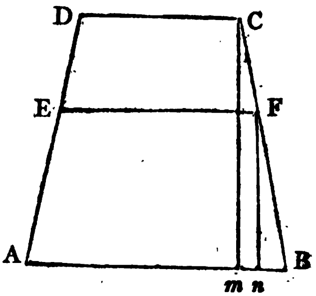
(See the last Figure.)

Let ABCD represent the middle section of the given vessel; then by the question, AB = 94, CD = 68, Cm = 65; Fn = 45, and Bm = $\frac{94 - 68}{2} = \frac{26}{2} =$ 13 inches.

By Theo. 11, Part III., the triangle BnF is similar to the triangle BmC; hence, as Cm : Bm :: Fn : Bn; that is, as 65 : 13 :: 45 : 9 = Bn; then AB — 2 Bn = 94 — 18 = 76 inches = EF, the side of the vessel, at the surface of the liquor.

By Rule II., Prob. VI., Part V., we have 94 × 94 = 8836, the square of a side of the base; 76 × 76 = 5776, the square of the side at the surface of the liquor; and 94 × 76 = 7144, the product of the sides; then (8836 + 5776 + 7144) × 45 ÷ 3 = 21756 × 15 = 326340, the content in cubic inches; hence 326340 × .004329 = 1412.725, the content in wine gallons.

Exam. 92.



Let ABCD represent the middle, perpendicular section, made through the transverse diameters of the given vessel; then by the question, AB $= 86$, CD $= 62$, C$m = 65$, F$n = 42$, and B$m = \frac{86 - 62}{2} = \frac{24}{2} = 12$ inches.

By Theo. 11, Part III., the triangle BnF is similar to the triangle BmC; hence, as Cm : Bm :: Fn : Bn; that is, as $65 : 12 :: 42 : 7.753 =$ Bn; then AB $- 2$ B$n = 86 - 15.506 = 70.494$ inches $=$ EF, the transverse diameter at the surface of the liquor.

Again, let ABCD represent the middle perpendicular section, made through the conjugate diameters of the given vessel; then by the question, AB $= 58$, CD $= 42$, C$m = 65$, F$n = 42$, and B$m = \frac{58 - 42}{2} = \frac{16}{2} = 8$ inches.

By similar triangles, as $65 : 8 :: 42 : 5.169 =$ Bn; and AB $- 2$ B$n = 58 - 10.338 = 47.662$ inches $=$ EF, the conjugate diameter at the surface of the liquor.

By Prob. IX., Part V., we have $\frac{86 + 70.494}{2} = \frac{6.494}{2} = 78.247$, the transverse diameter of the

[illegible] section: and $\frac{[illegible]}{2} = \frac{136.[illegible]}{2} =$ [illegible] the [illegible] diameter of the middle section.

[illegible] = 4288 [illegible] = 3?[illegible] [illegible] × [illegible] [illegible] four times the area of the middle section. and [illegible] × [illegible] × .7854 = [illegible], the area of the [illegible].

[illegible] + 3658.855) × [illegible] = [illegible], the content [illegible] ÷ 282 = 485.1[illegible] [illegible].

Remark.

The content of a cylindroid, or the frustum of an elliptical cone, may easily be obtained by the following rule: [illegible] one sum, the product of the diameters of the base, the product of the diameters of the top, and four times the product of the diameters of the middle section. Multiply this sum by the perpendicular height, divide the product by 2154.3, [illegible], and [illegible]; and the respective quotients will be the content in ale and wine gallons, and malt bushels.

Solution of the last Example by this Rule.

Here (4288 + 13335.459028 + 3359.885028) × 42 = [illegible] × 42 = 1045100.870352; and 1045100.870352 ÷ 2154.3 = 485.12318, the content in ale gallons.

Exam. 93.

By Rule I., Prob. VII., Part V., we have 64 × 64 × 28 = 4096 × 28 = 115784, the square of the diameter

of the base multiplied by one-third of the altitude; and $118784 \div 359.05 = 330.828$, the content of the given vessel, in ale gallons.

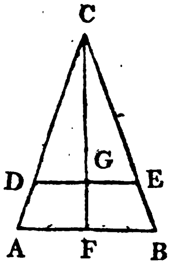
Then by Proportion, as $3 : 2 :: 330.828 : 220.552$, the content of the frustum; and as $3 : 1 :: 330.828 : 110.276$, the content of the segment, in ale gallons.

EXAM. 94.

By Rule II., Prob VI., Part V., we have $60 \times 60 = 3600$, the square of the side of the base; $40 \times 40 = 1600$, the square of the side of the top; and $60 \times$ $0 = 2400$, the product of the sides; then $(3600 +$ $600 + 2400) \times \frac{84}{3} = 7600 \times 28 = 212800$, the ontent in cubic inches; hence $212800 \times .004329 =$ 21.2112, the content in wine gallons.

Then by Proportion, as $9 : 5 :: 921.2112 :$ 11.784, the content of the greater vessel; and as 9 $4 :: 921.2112 : 409.4272$, the content of the less ssel, in wine gallons.

EXAM. 95.



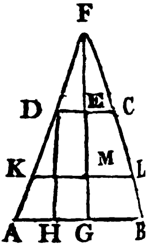
By the question, AB $= 60$, and $F = 72$; then by Note 1, Prob. VII., rt V., we have $60 \times 60 \times .7854 =$ $00 \times .7854 = 2827.44$, the area of e base in square inches; and $27.44 \times 24 = 67858.56$, the con-it of the cone ABC, in cubic inches; o, $67858.56 \div 2 = 33929.28 =$ content of the segment or cone C.

Now, by similar solids, Theo. 20, Part III., as $358.56 : 60^3 :: 33929.28 : 108000 = DE^3$; and $\overline{08000} = 47.622$ inches $=$ DE, the diameter of

the top of the frustum, and also that of the base of the segment.

Again, as $67858.56 : 72^3 :: 33929.28 : 186624 =$ CG^3; and $\sqrt[3]{186624} = 57.146$ inches $=$ CG, the altitude of the segment; and $72 - 57.146 = 14.854$ inches $=$ FG, the altitude of the frustum.

Exam. 96.



Let ABCD represent the given vessel; and AFB, the pyramid when completed; then by the question AB $= 60$, CD $= 40$, DH $= 50$, AG $=$ 30, and AH $= 10$ inches.

Now, by similar triangles, Theo. 11, Part III., as AH : HD :: AG : GF; that is, as $10 : 50 :: 30 :$ $150 =$ GF; and $150 - 50 = 100$ inches $=$ EF.

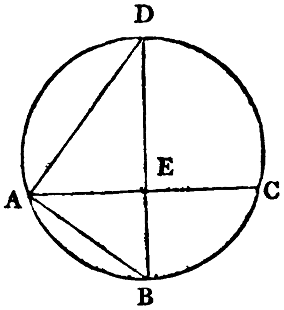
By Prob. V., Part V., we have $40 \times 40 \times \frac{100}{3} =$ $1600 \times \frac{100}{3} = \frac{160000}{3}$, the content of the pyramid DFC.

Also, by Rule II., Prob. VI., Part V., we have $(60 \times 60 + 40 \times 40 + 60 \times 40) \times \frac{50}{3} = \frac{380000}{3} =$ the content of the given vessel ABCD; and this being divided by 2, we obtain $\frac{190000}{3}$, the content of the frustum KLCD; the line KL being the dividing plane; then $\frac{160000}{3} + \frac{190000}{3} = \frac{350000}{3}$, the content of the pyramid KFL.

Now, by similar solids, Theo. 20, Part III., as $\frac{160000}{3} : 40^3 :: \frac{350000}{3} : 140000 = KL^3$; and $\sqrt[3]{140000} = 51.924$ inches $=$ KL, the top diameter of the vessel ABLK, and the bottom diameter of the vessel KLCD.

Again, by similar solids, as $\frac{160000}{3} : 100^3 :: \frac{350000}{3} : 2187500 = FM^3$; and $\sqrt[3]{2187500} = 129.812 =$ FM; then FM $-$ FE $= 129.812 - 100 = 29.812$ inches $=$ EM, the altitude of the vessel KLCD; and DH $-$ EM $= 50 - 29.812 = 20.188$ inches $=$ MG, the altitude of the vessel ABLK.

Exam. 97.



Let ABCD denote the given spherical vessel; then by the question BD $= 74$, BE $= 25$, and ED $= 74 - 25 = 49$ inches.

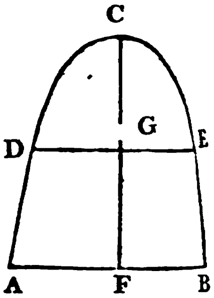
Now, by Theo. 9, Part III., the angle BAD, in the semi-circle, is a right angle; and by Theo 12, BE $\times$ ED $=$ AE^2; that is, $49 \times 25 = 1225 =$ the square of the radius AE.

By Rule II., Prob. X., Part V., we have $(74 \times 74 \times 74) \div 538.58 = 5476 \times 74 \div 538.58 = 405224 \div 538.58 = 752.393$ ale gallons, the content of the given vessel ABCD.

By Rule I., Prob. XI., Part V., we have $1225 \times 3 = 3675$, three times the square of the radius AE; and $25 \times 25 = 625$, the square of the depth BE; then $(3675 + 625) \times 25 \times .5236 = 4300 \times 25 \times .5236 = 107500 \times .5236 = 56287$, the content in cubic inches; hence $56287 \div 282 = 199.599$ ale gallons, the content of the less segment ABC; consequently, $752.393 - 199.599 = 552.794$ ale gallons, the content of the greater segment ADC.

EXAM. 98.

Let ABC represent the given semi-spheroidal vessel; then by the question, AB = 50, FC = 60, and FG = 30; the line DE being the dividing plane.



By the Note at the end of Prob. XIII., Part V., we have $60 + 30 = 90$, the altitude of the given vessel ABC, added to that of the frustum ABED; $60 - 30 = 30$, the difference between their altitudes; and $2\sqrt{90 \times 30} = 2\sqrt{2700} = 51.9615 \times 2 = 103.923$, twice the square root of the product of the sum and difference.

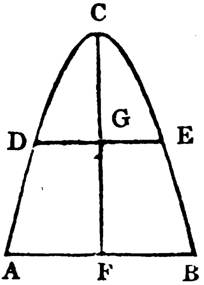
Then, as $120 : 50 :: 103.923 : 43.30125$ inches, the diameter DE.

Now, by the Remark at the end of Prob. XIII., Part V., we have $50 \times 50 \times 60 = 2500 \times 60 = 150000$, the square of the diameter AB, multiplied by the perpendicular FC; then $150000 \div 441.18 =$

339.997 wine gallons, the content of the given vessel ABC.

Again, by the Remark at the end of Prob. XIV., Part V., we have $2500 \times 2 + 43.30125^2 = 5000 + 1874.9982515625 = 6874.9982515625$, twice the square of the diameter AB, added to the square of the diameter DE; then $6874.99825 \times 30 \div 882.36 = 206249.9475 \div 882.36 = 233.748$ wine gallons, the content of the frustum ABED; and $389.997 - 233.748 = 106.249$ wine gallons, the content of the segment DEC.

EXAM. 99.



Let ABC represent the given parabolic vessel; then by the question, AB = 50, FC = 60, and FG = 30; the line DE being the dividing plane.

By Note 4, Prob. XVII., Part V., we have $60 - 30 = 30$, the reserved remainder; then, as $60 : 50^2 :: 30 : 1250$, the square of the diameter DE.

By Prob XVII., Part V., we have $50 \times 50 \times 30 = 2500 \times 30 = 75000$, the square of the diameter AB, multiplied by half the altitude FC; and $75000 \div 294.12 = 254.997$ wine gallons, the content of the given vessel ABC.

Again, by Prob. XVIII., Part V., we have $(2500 + 1250) \times 30 = 3750 \times 30 = 112500$, the sum of the squares of AB and DE multiplied by FG; and $112500 \div 588.24 = 191.248$ wine gallons, the content of the frustum ABED; and $254.997 - 191.248$

= 63.749 wine gallons, the content of the segment DEC.

EXAM. 100.

By Prob. XVIII., Part V., we have $(300^2 + 312^2) \times 336 = (90000 + 97344) \times 336 = 187344 \times 336 = 62947584$, the sum of the squares of the head and bung diameters multiplied by the height; then, $62947584 \div 588.24 = 107010$ wine gallons, the content of this enormous cask.

PART VI.

The Method of Gauging and Fixing Victuallers' Utensils; of Gauging and Inching Common Brewers' Utensils; and of Gauging and Ullaging Casks. Also, the Method of Gauging and Fixing Maltsters' Utensils; and of Gauging and Inching a Still, and a Distiller's Wash-Back. Likewise, the Method of Gauging and Fixing the Utensils of Soap-Makers, Starch-Makers, and Glass-Makers, as practised in the EXCISE.

SECTION I.

THE METHOD OF GAUGING AND FIXING VICTUALLERS' UTENSILS, AS PRACTISED IN THE EXCISE.

PROBLEM I.

To gauge and fix a mash tun in the form of the frustum of a cone.

EXAM. 2.

BY THE PEN.

To find the area.

Inches.

```
         73.4 }
         72.6 } cross diameters.
       2)146.0
          73   mean diameter.
          73   ditto.
         219
        511
Divisor 289)5329(18.439 mean area in gallons.
        289
        2439
        2312
         1270
         1156
          1140
           867
           2730
           2601
            129
```

To find the content.

Mash Tun gallons.

```
     18.44
      50.5
     -----
      9220
    9220
    -------
   931.220  content.
```

BY THE SLIDING RULE.

	On D.		On C.		On D.		On C.	
As	17.07	:	1	::	73	:	18.44	area.

Again,

As	17.07	:	50.5	::	73	:	931.22	content.

PROBLEM II.

To gauge and fix a mash tun in the form of the frustum of a square pyramid.

EXAM. 2.

BY THE PEN.

```
      Inches.
        47.8  mean side.
        47.8  ditto.
      ------
        3824
      3346
     1912
227)2284.84(10.065 mean area in gallons.
     227
     ----
        1484
        1362
        ----
         1220
         1135
         ----
           85
```

To find the content.

Mash Tun gallons.

```
   10.07 area.
    63.6 depth.
   ------
   6042
  3021
 6042
 -------
 640.452 content in gallons.
 =======
```

BY THE SLIDING RULE.

On A.		On B.		On A.		On B.
As 227	:	47.8	::	47.8	:	10.07 area.
				Again,		
As 1	:	10.07	::	63.6	:	640.45 content.

Note. If the vessel be elliptical, divide the product of the mean transverse and conjugate diameters by 289, and the quotient will be the mean area in mash-tun gallons.

PROBLEM III.

To gauge and fix a copper with a falling crown.

EXAM. 3.

BY RULE II.

To find the area.

Here $48.4 \times 48.4 \div 359.05 = 2342.56 \div 359.05 = 6.52$, the area of the first section, in ale gallons; $54.6 \times 54.6 \div 359.05 = 2981.16 \div 359.05 = 8.30$, the area of the second section; $57.6 \times 57.6 \div 359.05 = 3317.76 \div 359.05 = 9.24$, the area of the third section; $59.2 \times 59.2 \div 359.05 = 3504.64 \div 359.05 = 9.76$, the area of the fourth section; $56.8 \times 56.8 \div 359.05 = 3226.24 \div 359.05 = 8.98$, the area of the fifth section; and $54.4 \times 54.4 \div 359.05 = 2959.36 \div 359.05 = 8.24$, the area of the sixth section.

BY THE SLIDING RULE.

On A.	On B.		On A.		On B.
As 359.05 :	48.4 54.6 57.6 59.2 56.8 54.4	::	48.4 54.6 57.6 59.2 56.8 54.4	:	6.52 first section. 8.30 second section. 9.24 third section. 9.76 fourth section. 8.98 fifth section. 8.24 sixth section.

Having found the areas of the different sections, both by the *Pen* and the *Sliding Rule,* in order to *prove* the work; the dimensions and areas will appear in the Note Book, as below.

NOTE BOOK.

A. B.'s Copper, No. 2, gauged Jan. 23, 1821.

Divisions in Inches.	Depths from the Bottom.	Mean Diameters.	Areas.
8.4	34.2	54.4	8.24
6	27	56.8	8.98
6	21	59.2	9.76
6	15	57.6	9.24
6	9	54.6	8.30
6	3	48.4	6.52

To find the content.

Here $6.52 \times 6 = 39.12$ ale gallons, the content of the first 6 inches from the bottom; $8.30 \times 6 = 49.80$, the content of the second 6 inches; $9.24 \times 6 = 55.44$, the content of third 6 inches; $9.76 \times 6 = 58.56$, the content of the fourth 6 inches; $8.98 \times$

$6 = 53.88$, the content of the fifth 6 inches; and $8.24 \times 8.4 = 69.216$, the content of the last 8.4 inches of the depth; hence $39.12 + 49.8 + 55.44 + 58.56 + 53.88 + 69.216 = 326.016$ ale gallons, the whole content of the copper.

PROBLEM IV.

To gauge and fix an under-back.

EXAM. 2.

BY THE PEN.

Here $38.6 \times 38.6 = 1489.96$, the square of the mean diameter; and $1489.96 \div 359.05 = 4.149$, the area in ale gallons; then $4.15 \times 32.4 = 134.46$, the content in ale gallons.

BY THE SLIDING RULE.

To find the area.

	On A.		On B.		On A.		On B.	
As	359.05	:	38.6	::	38.6	:	4.15	area.

To find the content.

	On A.		On B.		On A.		On B.	
As	1	:	4.15	::	32.4	:	134.46	content.

EXAM. 3.

BY THE PEN.

To find the area.

```
   Inches.
     58.6 transverse diameter.
     40.4 conjugate ditto.
    -----
    2344
  2344
  -------
359.05)2367.44(6.593 ale gallons.
       215430
       ------
        213140
        179525
        ------
         336150
         323145
         ------
          130050
          107715
          ------
           22335
```

To find the content.

Ale Gallons.

```
     6.59 area.
     28.6 depth.
    -----
     3954
    5272
   1318
  -------
  188.474 content
```

BY THE SLIDING RULE.

To find the area.

	On A.		On B.		On A.		On B.	
As	359.05	:	58.6	::	40.4	:	6.59	area.

To find the content.

	On A.		On B.		On A.		On B.	
As	1	:	6.59	::	28.6	:	188.47	content.

PROBLEM V.

To gauge and fix a rectangular back or cooler.

EXAM. 2.

BY THE PEN.

Ale gallons.

```
    28.25 area.
      6.4 depth.
   ------
   11300
  16950
  -------
  180.800 content.
```

BY THE SLIDING RULE.

	On A.		On B.		On A.		On B.
As	1	:	28.25	::	6.4	:	180.8 ale gallons.

EXAM. 3.

BY THE PEN.

Here 98.3 × 74.8 = 7352.84, the area in inches; and 7352.84 ÷ 282 = 26.073, the area in ale gallons; then 26.07 × 7.2 = 187.704, the content in ale gallons.

BY THE SLIDING RULE.

	On A.		On B.		On A.		On B.
As	282	:	98.3	::	74.8	:	26.07 area.

And,

	On A.		On B.		On A.		On B.
As	1	:	26.07	::	7.2	:	187.70 content.

PROBLEM VI.

To gauge and fix a guile-tun in the form of the frustum of a cone.

EXAM. 2.

Contents.

Ale gallons.

113.0 first 10 inches.
122.1 second 10 inches.
131.4 third 10 inches.
13.14 × 2.4 = 31.536 upper 2.4 inches.
398.036 *Ans.*

Exam. 3.

Areas.

Having found the area corresponding to each diameter, in the Table of Ale Areas, the dimensions and areas will appear in the Note Book, as below.

NOTE BOOK.

A. B.'s Guile Tun, No. 2, gauged Feb. 20th, 1821.			
Divisions in Inches.	Depths from the Bottom.	Mean Diameters.	Areas.
8	34	57.2	9.11
10	25	55.3	8.52
10	15	53.2	7.88
10	5	51.1	7.27

BY THE SLIDING RULE.

On D.	On C.	On D.	On C.
As 18.95 :	1 ::	57.2 :	9.11 fourth section.
		55.3	8.52 third section.
		53.2	7.88 second section.
		51.1	7.27 first section.

To find the content.

Contents.

Ale gallons.

72.7 first 10 inches.
78.8 second 10 inches.
85.2 third 10 inches.
$9.11 \times 8 =$ 72.88 upper 8 inches.
309.58 whole content.

PROBLEM VII.

To gauge and fix a circular guile-tun, with curved sides.

EXAM. 2.

Contents.

	Ale gallons.	
	47.76	first 8 inches.
	54.80	second 8 inches.
	63.04	third 8 inches.
$8.24 \times 6.4 =$	52.736	upper 6.4 inches.
	218.336	whole content.

EXAM. 3.

Having found the area corresponding to each diameter, in the Table of Ale Areas, the dimensions and areas will appear in the Note Book, as below.

NOTE BOOK.

A.E.'s Guile Tun, No. 2, gauged March 20th, 1821.

Divisions in Inches.	Depths from the Bottom.	Mean Diameters.	Areas.
10	37	29.8	2.47
8	28	31.5	2.76
8	20	32.6	2.96
8	12	31.2	2.71
8	4	28.4	2.25

BY THE SLIDING RULE.

$$\text{As } 359.05 : \left\{\begin{matrix} 29.8 \\ 31.5 \\ 32.6 \\ 31.2 \\ 28.4 \end{matrix}\right\} :: \left\{\begin{matrix} 29.8 \\ 31.5 \\ 32.6 \\ 31.2 \\ 28.4 \end{matrix}\right\} : \left\{\begin{matrix} 2.47 \text{ fifth section.} \\ 2.76 \text{ fourth section.} \\ 2.96 \text{ third section.} \\ 2.71 \text{ second section.} \\ 2.25 \text{ first section.} \end{matrix}\right.$$

On A. On B. On A. On B.

BY THE PEN.

To find the content.

Areas.		Depths.		Contents.	
Ale gallons.		*Inches.*		*Ale gallons.*	
2.25	×	8	=	18.00	first division.
2.71	×	8	=	21.68	second division.
2.96	×	8	=	23.68	third division.
2.76	×	8	=	22.08	fourth division.
2.47	×	10	=	24.70	fifth division.
				110.14	whole content.

BY THE SLIDING RULE.

On A. On B. On A. On B.

$$\text{As } 1 : \left\{\begin{matrix} 2.25 \\ 2.71 \\ 2.96 \\ 2.76 \\ 2.47 \end{matrix}\right\} :: \left\{\begin{matrix} 8 \\ 8 \\ 8 \\ 8 \\ 10 \end{matrix}\right\} : \left\{\begin{matrix} 18.00 \text{ first division.} \\ 21.68 \text{ second division.} \\ 23.68 \text{ third division.} \\ 22.08 \text{ fourth division.} \\ 24.70 \text{ fifth division.} \end{matrix}\right.$$

110.14 whole content.

PROBLEM VIII.

To gauge and fix a guile-tun in the form of the frustum of an elliptical cone.

EXAM. 2.

Contents.

Ale gallons.

	24.1 first 10 inches.
	33.4 second 10 inches.
$4.37 \times 3.4 =$	14.858 upper 3.4 inches.
	72.358 whole content.

EXAM. 3.

BY THE PEN.

To find the area.

Here $49.6 \times 37.8 \div 359.05 = 1874.88 \div 359.05 = 5.221$ ale gallons, the area of the first section; $55.4 \times 41.7 \div 359.05 = 2310.18 \div 359.05 = 6.434$ ale gallons, the area of the second section; $61.6 \times 46.2 \div 359.05 = 2845.92 \div 359.05 = 7.926$ ale gallons, the area of the third section; and $67.3 \times 50.5 \div 359.05 = 3398.65 \div 359.05 = 9.465$ ale gallons, the area of the fourth section.

BY THE SLIDING RULE.

	On A.		On B.		On A.		On B.
As	359.05 :		49.6, 55.4, 61.6, 67.3	::	37.8, 41.7, 46.2, 50.5	:	5.22 first section. 6.43 second section. 7.93 third section. 9.47 fourth section.

The dimensions and areas will appear in the Note Book, as below.

NOTE BOOK.

A. F.'s Guile Tun, No. 2. gauged April 12th, 1821.

Divisions in Inches.	Depths from the Bottom.	Transverse and Conjugate Diameters.	Areas.
10	35	T. 67.3 C. 50.5	9.47
10	25	T. 61.6 C. 46.2	7.93
10	15	T. 55.4 C. 41.7	6.43
10	5	T. 49.6 C. 37.8	5.22

To find the content.

Contents.

Ale gallons.

52.2 first division.
64.3 second division.
79.3 third division.
94.7 fourth division.
290.5 whole content.

PROBLEM IX.

To gauge and fix a guile-tun with an elliptical base and a circular top, generally called a cylindroid.

EXAM. 2.

Contents.

Ale gallons.

46.4 first 10 inches.
39.8 second 10 inches.
$3.35 \times 6.7 =$ 22.445 upper 6.7 inches.
108.645 whole content.

EXAM. 3.

BY THE PEN.

To find the area.

Here 62.7 × 55.4 ÷ 359.05 = 3473.58 ÷ 359.05 = 9.674 ale gallons, the area of the first section; 58.8 × 53.6 ÷ 359.05 = 3151.68 ÷ 359.05 = 8.777 ale gallons, the area of the second section; 54.9 × 51.8 ÷ 359.05 = 2843.82 ÷ 359.05 = 7.920 ale gallons, the area of the third section; and 50.8 × 49.7 ÷ 359.05 = 2524.76 ÷ 359.05 = 7.031 ale gallons, the area of the fourth section.

BY THE SLIDING RULE.

$$\begin{array}{lllll} \text{On A.} & \text{On B.} & \text{On A.} & \text{On B.} & \\ \text{As } 359.05 : & \left\{\begin{array}{l} 62.7 \\ 58.8 \\ 54.9 \\ 50.8 \end{array}\right\} :: & \left\{\begin{array}{l} 55.4 \\ 53.6 \\ 51.8 \\ 49.7 \end{array}\right\} : & \left\{\begin{array}{l} 9.67 \text{ first section.} \\ 8.78 \text{ second section.} \\ 7.92 \text{ third section.} \\ 7.03 \text{ fourth section.} \end{array}\right. \end{array}$$

The dimensions and areas will appear in the Note Book, as below.

NOTE BOOK.

A. G.'s Guile Tun, No. 2, gauged April 18th, 1821.

Divisions in Inches.	Depths from the Bottom.	Transverse and Conjugate Diameters.	Areas.
10	35	T. 50.8 C. 49.7	7.03
10	25	T. 54.9 C. 51.8	7.92
10	15	T. 58.8 C. 53.6	8.78
10	5	T. 62.7 C. 55.4	9.67

To find the content.

Contents.

Ale gallons.

96.7 first division.
87.8 second division.
79.2 third division.
70.3 fourth division.
334.0 whole content.

PROBLEM X.

To gauge and fix a guile-tun in the form of the frustum of a square pyramid.

EXAM. 2.

Contents.

Ale gallons.

39.1 first 10 inches.
57.9 second 10 inches.
$8.38 \times 7.8 =$ 65.364 upper 7.8 inches.
162.364 *Ans.*

EXAM. 3.

BY THE PEN.

To find the area.

Here $49.2 \times 49.2 \div 282 = 2420.64 \div 282 =$ 8.583 ale gallons, the area of the first section; $52.3 \times 52.3 \div 282 = 2735.29 \div 282 = 9.699$ ale gallons, the area of the second section; $55.2 \times 55.2 \div 282 = 3047.04 \div 282 = 10.805$ ale gallons, the area of the third section; $58.4 \times 58.4 \div 282 = 3410.56 \div 282 = 12.094$ ale gallons, the area of the fourth section; and $61.2 \times 61.2 \div 282 = 3745.44 \div 282 = 13.281$ ale gallons, the area of the fifth section.

BY THE SLIDING RULE.

	On D.	On C.		On D.		On C.
As	16.79 :	1 ::		49.2	:	8.58 first section.
				52.3		9.70 second section.
				55.2		10.81 third section.
				58.4		12.09 fourth section.
				61.2		13.28 fifth section.

The dimensions and areas will appear in the Note Book, as below.

NOTE BOOK.

A. C's. Guile Tun, No. 2, gauged March 6th, 1821.			
Divisions in Inches.	Depths from the Bottom.	Mean Sides.	Areas.
12	46	61.2	13.28
10	35	58.4	12.09
10	25	55.2	10.81
10	15	52.3	9.70
10	5	49.2	8.58

To find the content.

Contents.

Ale gallons.

	85.8 first 10 inches.
	97.0 second 10 inches.
	108.1 third 10 inches.
	120.9 fourth 10 inches.
$13.28 \times 12 =$	159.36 upper 12 inches.
	571.16 whole content.

PROBLEM XII.

The method of guaging by tubs.

EXAM. 3.

BY THE PEN.

By Rule II.

```
              Inches.
               25.6  diameter.
               25.6  ditto.
              -----
              1536
             1280
             512
             ------
             655.36  square of diameter.
               22.7  depth.
             ------
            458752
          131072
         131072
         ---------
Divisor 359)14876.672(41.439 Ans.
         1436
         ----
           516
           359
           ----
           1576
           1436
           ----
            1407
            1077
            ----
             3302
             3231
             ----
               71
```

BY THE SLIDING RULE.

By Rule II.

	On D.		On C.		On D.		On C.
As	18.95	:	22.7	::	25.6	:	41.44 content.

EXAM. 4.

BY THE PEN.

By Rule III.

```
        Inches.
         45.3  transverse diameter.
         36.8  conjugate ditto.
        ------
        3624
       2718
      1359
      -------
       1667.04 product.
          12.7 depth.
      ---------
      1166928
      333408
     166704
     ----------
Divisor 359)21171.408(58.973 Ans.
            1795
            ----
             3221
             2872
             ----
              3494
              3231
              ----
               2630
               2513
               ----
                1178
                1077
                ----
                 101
```

BY THE SLIDING RULE,

By Rule III.

	On A.		On B.		On A.		On B.	
As	359	:	45.3	::	36.8	:	4.64	area.

And,

As	1	:	4.64	::	12.7	:	58.97	content.

PROBLEM XIII.

To deduct the heat out of victuallers' warm wort.

CASE I.

When the number of warm gallons are known.

EXAM. 2.

By Subtraction.	*By Multiplication.*
Gallons.	*Gallons.*
225.6	225.6
22.56	.9
203.04 *Ans.*	203.04 *Ans.*

BY THE SLIDING RULE.

	On A.		On B.		On A.		On B.
As	1.11	:	1	::	225.6	:	203 *gallons.*

CASE II.

When the area of the fixed utensil, and the depth of the liquor are known, but not the number of warm gallons.

EXAM. 2.

BY THE PEN.

```
                  Ale gallons.
                  26.45 area.
                      6.8 depth.
                  -------
                  21160
                 15870
                 -------
Divisor 1.11)179.860(162.036 Ans.
             111
             ---
              688
              666
              ---
               226
               222
               ---
                400
                333
                ---
                 670
                 666
                 ---
                   4
```

BY THE SLIDING RULE.

	On A.		On B.		On A.		On B.
As	1.11	:	26.45	::	6.8	:	162.04 *Ans.*

EXAM. 3.

BY THE PEN.

Here $4.9 \times 8 = 39.2$, the area of the first section multiplied by its depth; and $5.25 \times 7.4 = 38.85$, the area of the second section multiplied by its depth; then $39.2 + 38.85 \div 1.11 = 78.05 \div 1.11 = 70.315$ ale gallons, the answer required.

BY THE SLIDING RULE.

CASE III.

When the warm wort is in a circular by-tub.

EXAM. 2.

BY THE SLIDING RULE.

	On D.		On C.		On D.		On C.
As	20	:	23.6	::	27.8	:	45.6 *ale gallons.*

PROBLEM XIV.

Given the top and bottom diameters, and the perpendicular depth of a vessel in the form of the frustum of a cone, to determine the intermediate diameters, at any assigned depth.

EXAM. 2.

Here $\frac{58 - 42}{35} = \frac{16}{35} = .457$, the common factor or multiplier; then $58 - 5 \times .457 = 58 - 2.285 = 55.715$, the diameter at 5 inches from the bottom; $58 - 10 \times .457 = 58 - 4.570 = 53.430$, the diameter at 10 inches from the bottom; $58 - 15 \times .457 = 58 - 6.855 = 51.145$, the diameter at 15 inches from the bottom; $58 - 20 \times .457 = 58 - 9.140 = 48.860$, the diameter at 20 inches from the bottom; $58 - 25 \times .457 = 58 - 11.425 = 46.575$, the diameter at 25 inches from the bottom; and $58 - 30 \times .457 = 58 - 13.710 = 44.290$, the diameter at 30 inches from the bottom.

PROBLEM XV.

Given the top and bottom diameters, and the perpendicular depth of a vessel in the form of the frustum of a semi-sphere, to determine the intermediate diameters, at any assigned depth from the top of the vessel.

EXAM. 2.

To find the diameter of the sphere.

By Rule I., we have $\frac{18^2 - (12^2 + 8^2)}{8 \times 2} = \frac{324 - (144 + 64)}{16} = \frac{324 - 208}{16} = \frac{116}{16} = 7.25$ inches, the distance of the bottom diameter of the frustum, from the centre of the sphere.

Again, $2\sqrt{7.25^2 + 18^2} = 2\sqrt{52.5625 + 324} = 2\sqrt{376.5625} = 19.4 \times 2 = 38.8$ inches, the diameter of the sphere.

To find the height of the segment.

By Rule II., we have $19.4 - (8 + 7.25) = 19.4 - 15.25 = 4.15$ inches, the height of the segment.

To find the intermediate diameters.

By Rule III., we have $2\sqrt{(38.8 - 5.15) \times 5.15}$ $2\sqrt{33.65 \times 5.15} = 2\sqrt{173.2975} = 13.16 \times 2 = 26.32$ inches, the diameter at 1 inch from the top of the given vessel.

Again, $2\sqrt{(38.8 - 6.15) \times 6.15} = 2\sqrt{32.65 \times 6.15} = 2\sqrt{200.7975} = 14.17 \times 2 = 28.34$ inches, the diameter at 2 inches from the top of the vessel.

Again, $2\sqrt{(38.8-7.15)\times 7.15} = 2\sqrt{31.65\times 7.15}$ $= 2\sqrt{226.2975} = 15.04\times 2 = 30.08$ inches, the diameter at 3 inches from the top of the vessel.

By a similar process, we find the diameter at *four* inches, to be 31.60; at *five*, 32.94; at six, 34.10; and at seven inches from the top of the vessel, the diameter is found to be 35.[illegible] inches.

EXAM. 3.

To find the diameter of the sphere.

By Rule I., we have $\dfrac{27.8^2-(14^2+12^2)}{12\times 2} =$ $\dfrac{772.84-(196+144)}{24} = \dfrac{772.84-340}{24} = \dfrac{432.84}{24} =$ 18.03 inches, the distance of the bottom diameter of the frustum from the centre of the sphere.

Again, $2\sqrt{18.03^2+27.8^2} = 2\sqrt{325.0809+772.84} = 2\sqrt{1097.9209} = 33.13\times 2 = 66.26$ inches, the diameter of the sphere.

To find the height of the segment.

By Rule II., we have $33.13-(18.03+12) = 33.13-30.03 = 3.1$ inches, the height of the segment.

To find the intermediate diameters.

By Rule III., we have $2\sqrt{(66.26-4.1)\times 4.1} = 2\sqrt{62.16\times 4.1} = 2\sqrt{254.856} = 15.96\times 2 = 31.92$ inches, the diameter at 1 inch from the top of the given vessel.

Again, $2\sqrt{(66.26-5.1)\times 5.1} = 2\sqrt{61.16\times 5.1} = 2\sqrt{311.916} = 17.66\times 2 = 35.32$ inches, the diameter at 2 inches from top.

Again, $2\sqrt{(66.26-6.1)\times 6.1} = 2\sqrt{60.16\times 6.1} = \sqrt{366.976} = 19.15 \times 2 = 38.30$ inches, the diameter at 3 inches from the top.

By a similar process we find the fourth intermediate diameter to be 40.98, the fifth 43.40, the sixth 45.60, the seventh 47.62, the eighth 49.48, the ninth 51.18, the tenth 52.76, and the eleventh 54.22 inches.

SECTION II.

THE METHOD OF GAUGING AND INCHING COMMON BREWERS' UTENSILS, AS PRACTISED IN THE EXCISE.

PROBLEM I.

To guage and inch a mash-tun in the form of the frustum of a cone, standing upon its greater base.

EXAM. 2.

BY THE PEN.

To find the area.

Inches.		*Inches.*		*Divisor.*		*M. T. gallons.*
90.4	×	90.4	÷	289	=	28.277 area of the 1st section.
88.2	×	88.2	÷	289	=	26.917 area of the 2nd section.
86.1	×	86.1	÷	289	=	25.651 area of the 3rd section.
83.8	×	83.8	÷	289	=	24.299 area of the 4th section.
81.6	×	81.6	÷	289	=	23.040 area of the 5th section.

To find the content.

Contents.

Mash Tun gallons.

282.77 first division.
269.17 second division.
256.51 third division.
242.99 fourth division.
$23.04 \times 12 =$ 276.48 fifth division.
8)1327.92 whole content.
8)165 bu. 7.92 gallons.
20 qrs. 5 bu. 7.92 gallons.

DIMENSION BOOK.

From the question, and from the foregoing areas and contents, we form the Dimension Book, as below.

A.B.'s Round Mash Tun, No. 2. gauged July 20, 1821.

Divisions in Inches.	Depth from the Bottom.	Mean Diameters.	Areas in Gallons.	Contents in Gallons.	Areas in Q.	B.	G.	Contents in Q.	B.	G.
12	46	81.6	23.040	276.48	0.	2.	7.040	4	2	4.48
10	35	83.8	24.299	242.99	0.	3.	0.299	3	6	2.99
10	25	86.1	25.651	256.51	0.	3.	1.651	4	0	0.51
10	15	88.2	26.917	269.17	0.	3.	2.917	4	1	5.17
10	5	90.4	28.277	282.77	0.	3.	4.277	4	3	2.77
52	Depth.	Whole Content.		1327.92				20	5	7.92

A TABLE

Shewing the Method of Inching the Mash Tun, given in the foregoing Example.

Wet Inches.	Contents in Q.	B.	G.	Wet Inches.	Contents in Q.	B.	G.	Wet Inches.	Contents in Q.	B.	G.
1	0	3	4.277	15	6	4	1.355	29	12	1	6.799
	0	3	4.277		0	3	2.917		0	3	1.651
2	0	7	0.554	16	6	7	4.272	30	12	5	0·450
	0	3	4.277		0	3	2.917		0	3	0.299
3	1	2	4.831	17	7	2	7.189	31	13	0	0.749
	0	3	4.277		0	3	2.917		0	3	0.299
4	1	6	1.108	18	7	6	2.106	32	13	3	1.048
	0	3	4.277		0	3	2.917		0	3	0.299
5	2	1	5.385	19	8	1	5.023	33	13	6	1.347
	0	3	4.277		0	3	2.917		0	3	0.299
6	2	5	1.662	20	8	4	7·940	34	14	1	1.646
	0	3	4.277		0	3	1.651		0	3	0.299
7	3	0	5.939	21	9	0	1.591	35	14	4	1.945
	0	3	4.277		0	3	1.651		0	3	0.299
8	3	4	2.216	22	9	3	3.242	36	14	7	2.244
	0	3	4.277		0	3	1.651		0	3	0.299
9	3	7	6.493	23	9	6	4.893	37	15	2	2.543
	0	3	4.277		0	3	1.651		0	3	0.299
10	4	3	2.770	24	10	1	6.544	38	15	5	2.842
	0	3	2.917		0	3	1.651		0	3	0.299
11	4	6	5.687	25	10	5	0.195	39	16	0	3.141
	0	3	2.917		0	3	1.651		0	3	0.299
12	5	2	0.604	26	11	0	1.846	40	16	3	3.440
	0	3	2:917		0	3	1.651		0	2	7.040
13	5	5	3.521	27	11	3	3.497	41	16	6	2.480
	0	3	2.917		0	3	1.651		0	2	7.040
14	6	0	6.438	28	11	6	5.148	42	17	1	1.520
	0	3	2.917		0	3	1.651		0	2	7.040

TABLE CONTINUED.

Wet Inches.	Contents in Q.	B.	G.	Wet Inches.	Contents in Q.	B.	G.	Wet Inches.	Contents in Q.	B.	G.
43	17 0	4 2	0.560 7.040	47	18 0	7 2	4.720 7.040	50	20 0	0 2	1.840 7.040
44	17 0	6 2	7.600 7.040	48	19 0	2 2	3.760 7.040	51	20 0	3 2	0.880 7.040
45	18 0	1 2	6.640 7.040	49	19 0	5 2	2.800 7.040	52	20	5	7.920
46	18 0	4 2	5.680 7.040								

COMMON BREWERS' TABLE BOOK.

A. B.'s Round Mash Tun, No. 2.

Wet Inches.	Contents in Q.	B.	G.	Wet Inches.	Contents in Q.	B.	G.	Wet Inches.	Contents in Q.	B.	G.	Wet Inches.	Contents in Q.	B.	G.
1	0	3	4	14	6	0	6	27	11	3	3	40	16	3	3
2	0	7	1	15	6	4	1	28	11	6	5	41	16	6	2
3	1	2	5	16	6	7	4	29	12	1	7	42	17	1	2
4	1	6	1	17	7	2	7	30	12	5	0	43	17	4	1
5	2	1	5	18	7	6	2	31	13	0	1	44	17	7	0
6	2	5	2	19	8	1	5	32	13	3	1	45	18	1	7
7	3	0	6	20	8	5	0	33	13	6	1	46	18	4	6
8	3	4	2	21	9	0	2	34	14	1	2	47	18	7	5
9	3	7	6	22	9	3	3	35	14	4	2	48	19	2	4
10	4	3	3	23	9	6	5	36	14	7	2	49	19	5	3
11	4	6	6	24	10	1	7	37	15	2	3	50	20	0	2
12	5	2	1	25	10	5	0	38	15	5	3	51	20	3	1
13	5	5	4	26	11	0	2	39	16	0	3	52	20	6	0

PROBLEM II.

To gauge and inch a copper with a rising crown.

EXAM. 2.

A TABLE

wing the Method of Inching the Copper given in this Example.

Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.
Full	22	0	8.0898	10	16	1	1.3678	20	11	1	1.9709
	0	2	3.3722		0	2	3.3722		0	1	8.5583
1	21	2	4.7176	11	15	2	6.9956	21	10	3	2.4126
	0	2	3.3722		0	1	8.5583		0	1	6.0458
2	21	0	1.3454	12	15	0	7.4373	22	10	1	5.3668
	0	2	3.3722		0	1	8.5583		0	1	6.0458
3	20	1	6.9732	13	14	2	7.8790	23	9	3	8.3210
	0	2	3.3722		0	1	8.5583		0	1	6.0458
4	19	3	3.6010	14	14	0	8.3207	24	9	2	2.2752
	0	2	3.3722		0	1	8.5583		0	1	6.0458
5	19	1	0.2288	15	13	2	8.7624	25	9	0	5.2294
	0	2	3.3722		0	1	8.5583		0	1	6.0458
6	18	2	5.8566	16	13	1	0.2041	26	8	2	8.1836
	0	2	3.3722		0	1	8.5583		0	1	6.0458
7	18	0	2.4844	17	12	3	0.6458	27	8	1	2.1378
	0	2	3.3722		0	1	8.5583		0	1	6.0458
8	17	1	8.1122	18	12	1	1.0875	28	7	3	5.0920
	0	2	3.3722		0	1	8.5583		0	1	6.0458
9	16	3	4.7400	19	11	3	1.5292	29	7	1	8.0462
	0	2	3.3722		0	1	8.5583		0	1	6.0458

TABLE CONTINUED.

Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.
30	7	0	2.0004	39	4	0	0.2378	48	1	2	8.0660
	0	1	6.0458	*	0	1	0.3046		0	0	7.7350
31	6	2	4.9546	40	3	2	8.9332	49	1	2	0.3310
*	0	1	2.8396		0	1	0.3046		0	0	7.7350
32	6	1	2.1150	41	3	1	8.6286	50	1	1	1.5960
	0	1	2.8396		0	1	0.3046		0	0	7.7350
33	5	3	8.2754	42	3	0	8.3240	51	1	0	2.8610
	0	1	2.8396		0	1	0.3046		0	0	7.7350
34	5	2	5.4358	43	2	3	8.0194	52	0	3	4.1260
	0	1	2.8396		0	1	0.3046		0	0	7.7350
35	5	1	2.5962	44	2	2	7.7148	53	0	2	5.3910
	0	1	2.8396		0	1	0.3046		0	0	4.6410
36	4	3	8.7566	45	2	1	7.4202	53.6	0	2	0.7500
	0	1	2.8396		0	1	0.3046	crown	0	2	0.7500
37	4	2	5.9170	46	2	0	7.1056				
	0	1	2.8396		0	1	0.3046				
38	4	1	3.0774	47	1	3	6.8010				
	0	1	2.8396	*	0	0	7.7350				

Note. The number 4.6410 gallons, in the third line from the end of the above Table, is obtained by taking $\frac{6}{10}$ of 7.7350 gallons, the area of the sixth section. (See the Dimension Book, in the Gauging.)

COMMON BREWER'S TABLE BOOK.

A. B.'s Copper, No. 2.

Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.
Full	22	0	8	15	13	3	0	30	7	0	2	45	2	1	7
1	21	2	5	16	13	1	0	31	6	2	5	46	2	0	7
2	21	0	1	17	12	3	1	32	6	1	2	47	1	3	7
3	20	1	7	18	12	1	1	33	5	3	8	48	1	2	8
4	19	3	4	19	11	3	2	34	5	2	5	49	1	2	0
5	19	1	0	20	11	1	2	35	5	1	3	50	1	1	2
6	18	2	6	21	10	3	2	36	5	0	0	51	1	0	3
7	18	0	2	22	10	1	5	37	4	2	6	52	0	3	4
8	17	1	8	23	9	3	8	38	4	1	3	53	0	2	5
9	16	3	5	24	9	2	2	39	4	0	0	53.6	0	2	1
10	16	1	1	25	9	0	5	40	3	3	0	crown	0	2	1
11	15	2	7	26	8	2	8	41	3	2	0				
12	15	0	7	27	8	1	2	42	3	0	8				
13	14	2	8	28	7	3	5	43	2	3	8				
14	14	0	8	29	7	1	8	44	2	2	8				

PROBLEM III.

To gauge and inch a Copper with a falling crown.

EXAMPLE.

BY THE PEN.

ind the content of the crown, considering it as the segment of a sphere.

y Rule I., Prob. XI., Part V., we have $29.2 \times 29.2 = 852.64 \times 3 = 2557.92$, three times the re of the radius of the top; also $8.2 \times 8.2 = 76.24$, quare of the depth; then $(2557.92 + 76.24) \times 8.2$ $236 = 2634.16 \times 8.2 \times .5236 = 21600.112 \times$ $5 = 11039.818$, the content in cubic inches; and $9.818 \div 282 = 40.105$, the content in ale ns. (See the Dimension Book, in the Gauging.)

R

A TABLE

Shewing the Method of Inching the Copper given in this Example.

Wet Inches.	B.	F.	G.	Wet Inches.	B.	F.	G.	Wet Inches.	B.	F.	G.
crown	1	0	4.110	13	5	1	7.906	26	10	2	6.110
	0	1	2.515		0	1	4.882		0	1	6.005
1	1	1	6.625	14	5	3	3.788	27	11	0	3.115
	0	1	2.515		0	1	4.882		0	1	6.005
2	1	3	0.140	15	6	0	8.670	28	11	2	0.120
	0	1	2.515		0	1	4.882		0	1	6.005
3	2	0	2.655	16	6	2	4.552	29	11	3	6.125
	0	1	2.515		0	1	4.882		0	1	6.005
4	2	1	5.170	17	7	0	0.434	30 •	12	1	3.130
	0	1	2.515		0	1	4.882		0	1	7.002
5	2	2	7.685	18	7	1	5.316	31	12	3	1.132
	0	1	2.515		0	1	4.882		0	1	7.002
6	3	0	1.200	19	7	3	1.198	32	13	0	8.134
	0	1	2.515		0	1	4.882		0	1	7.002
7	3	1	3.715	20 •	8	0	6.080	33	13	2	6.136
	0	1	2.515		0	1	6.005		0	1	7.002
8	3	2	6.230	21	8	2	3.085	34	14	0	4.138
	0	1	2.515		0	1	6.005		0	1	7.002
9	3	3	8.745	22	9	0	0.090	35	14	2	2.140
	0	1	2.515		0	1	6.005		0	1	7.002
10 •	4	1	2.260	23	9	1	6.095	36	15	0	0.142
	0	1	4.882		0	1	6.005		0	1	7.002
11	4	2	7.142	24	9	3	3.100	37	15	1	7.144
	0	1	4.882		0	1	6.005		0	1	7.002
12	5	0	3.024	25	10	1	0.105	38	15	3	5.146
	0	1	4.882		0	1	6.005		0	1	7.002

TABLE CONTINUED.

Wet Inches	Contents in B.	F.	G.	Wet Inches	Contents in B.	F.	G.	Wet Inches	Contents in B.	F.	G.
39	16	1	3.148	43	18	0	7.246	47	20	0	3.374
	0	1	7.002		0	1	8.032		0	1	8.032
40	16	3	1.150	44	18	2	6.278	48	20	2	2.406
	0	1	8.032		0	1	8.032		0	1	8.032
41	17	1	0.182	45	19	0	5.310	49	21	0	1.438
	0	1	8.032		0	1	8.032		0	1	8.032
42	17	2	8.214	46	19	2	4.342	50	21	2	0.470
	0	1	8.032		0	1	8.032				

COMMON BREWER'S TABLE BOOK.

A. B.'s Copper, No. 9.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Content in B.	F.	G.
crown	1	0	4	13	5	1	8	26	10	2	6	39	16	1	3
1	1	1	7	14	5	3	4	27	11	0	3	40	16	3	1
2	1	3	0	15	6	1	0	28	11	2	0	41	17	1	0
3	2	0	3	16	6	2	5	29	11	3	6	42	17	2	8
4	2	1	5	17	7	0	0	30	12	1	3	43	18	0	7
5	2	2	8	18	7	1	5	31	12	3	1	44	18	2	6
6	3	0	1	19	7	3	1	32	13	0	8	45	19	0	5
7	3	1	4	20	8	0	6	33	13	2	6	46	19	2	4
8	3	2	6	21	8	2	3	34	14	0	4	47	20	0	3
9	4	0	0	22	9	0	0	35	14	2	2	48	20	2	2
10	4	1	2	23	9	1	6	36	15	0	0	49	21	0	1
11	4	2	7	24	9	3	3	37	15	1	7	50	21	2	0
12	5	0	3	25	10	1	0	38	15	3	5				

PROBLEM IV.

To gauge and inch a Copper Back.

EXAMPLE.

A TABLE

Shewing the Method of Inching the Copper Back given in this Example.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
1	1	3	8 878	12	23	3	7.536	23	45	3	6.194
	1	3	8.878		1	3	8.878		1	3	8.878
2	3	3	8.756	13	25	3	7.414	24	47	3	6.072
	1	3	8.878		1	3	8.878		1	3	8.878
3	5	3	8.634	14	27	3	7.292	25	49	3	5.950
	1	3	8.878		1	3	8.878		1	3	8.878
4	7	3	8.512	15	29	3	7.170	26	51	3	5.828
	1	3	8.878		1	3	8.878		1	3	8.878
5	9	3	8.390	16	31	3	7.048	27	53	3	5.706
	1	3	8.878		1	3	8 878		1	3	8.878
6	11	3	8.268	17	33	3	6.926	28	55	3	5.584
	1	3	8.878		1	3	8.878		1	3	8.878
7	13	3	8.146	18	35	3	6.804	29	57	3	5.462
	1	3	8.878		1	3	8.878		1	3	8.878
8	15	3	8.024	19	37	3	6.682	30	59	3	5.340
	1	3	8.878		1	3	8.878		1	3	8.879
9	17	3	7.902	20	39	3	6.560	31	61	3	5.218
	1	3	8.878		1	3	8 878		1	3	8.878
10	19	3	7.780	21	41	3	6.438	32	63	3	5.096
	1	3	8.878		1	3	8.878				
11	21	3	7.658	22	43	3	6.316				
	1	3	8.878		1	3	8.878				

COMMON BREWER'S TABLE BOOK.

A. B.'s Copper Back.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
1	2	0	0	9	17	3	8	17	33	3	7	25	49	3	6
2	4	0	0	10	19	3	8	18	35	3	7	26	51	3	6
3	6	0	0	11	21	3	8	19	37	3	7	27	53	3	6
4	8	0	0	12	23	3	8	20	39	3	7	28	55	3	6
5	9	3	8	13	25	3	7	21	41	3	6	29	57	3	5
6	11	3	8	14	27	3	7	22	43	3	6	30	59	3	5
7	13	3	8	15	29	3	7	23	45	3	6	31	61	3	5
8	15	3	8	16	31	3	7	24	47	3	6	32	63	3	5

PROBLEM V.

To gauge and inch an Under Back in the form of a cylinder.

EXAMPLE.

A TABLE

Shewing the Method of Inching the Under Back given in this Example.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
1	0	1	1.95	6	1	3	2.70	11	3	1	3.45
	0	1	1.95		0	1	1.95		0	1	1.95
2	0	2	3.90	7	2	0	4.65	12	3	2	5.40
	0	1	1.95		0	1	1.95		0	1	1.95
3	0	3	5.85	8	2	1	6.60	13	3	3	7.35
	0	1	1.95		0	1	1.95		0	1	1.95
4	1	0	7.80	9	2	2	8.55	14	4	1	0.30
	0	1	1.95		0	1	1.95		0	1	1.95
5	1	2	0.75	10	3	0	1.50	15	4	2	2.25
	0	1	1.95		0	1	1.95		0	1	1.95

TABLE CONTINUED.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
16	4 0	3 1	4.20 1.95	28	8 0	2 1	0.60 1.95	40	12 0	0 1	6.00 1.95
17	5 0	0 1	6.15 1.95	29	8 0	3 1	2.55 1.95	41	12 0	1 1	7.95 1.95
18	5 0	1 1	8.10 1.95	30	9 0	0 1	4.50 1.95	42	12 0	3 1	0.90 1.95
19	5 0	3 1	1.05 1.95	31	9 0	1 1	6.45 1.95	43	13 0	0 1	2.85 1.95
20	6 0	0 1	3.00 1.95	32	9 0	2 1	8.40 1.95	44	13 0	1 1	4.80 1.95
21	6 0	1 1	4.95 1.95	33	10 0	0 1	1.35 1.95	45	13 0	2 1	6.75 1.95
22	6 0	2 1	6.90 1.95	34	10 0	1 1	3.30 1.95	46	13 0	3 1	8.70 1.95
23	6 0	3 1	8.85 1.95	35	10 0	2 1	5.25 1.95	47	14 0	1 1	1.65 1.95
24	7 0	1 1	1.80 1.95	36	10 0	3 1	7.20 1.95	48	14 0	2 1	3.60 1.95
25	7 0	2 1	3.75 1.95	37	11 0	1 1	0.15 1.95	49	14 0	3 1	5.55 1.95
26	7 0	3 1	5.70 1.95	38	11 0	2 1	2.10 1.95	50	15	0	7.50
27	8 0	0 1	7.65 1.95	39	11 0	3 1	4.05 1.95				

COMMON BREWER'S TABLE BOOK.

A. B.'s Under-Back.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
1	0	1	2	14	4	1	0	27	8	0	8	40	12	0	6
2	0	2	4	15	4	2	2	28	8	2	1	41	12	1	8
3	0	3	6	16	4	3	4	29	8	3	3	42	12	3	1
4	1	0	8	17	5	0	6	30	9	0	5	43	13	0	3
5	1	2	1	18	5	1	8	31	9	1	6	44	13	1	5
6	1	3	3	19	5	3	1	32	9	2	8	45	13	2	7
7	2	0	5	20	6	0	3	33	10	0	1	46	14	0	0
8	2	1	7	21	6	1	5	34	10	1	3	47	14	1	2
9	2	3	0	22	6	2	7	85	10	2	5	48	14	2	4
10	3	0	2	23	7	0	0	36	10	3	7	49	14	3	6
11	3	1	3	24	7	1	2	37	11	1	0	50	15	0	8
12	3	2	5	25	7	2	4	38	11	2	2				
13	3	3	7	26	7	3	6	39	11	3	4				

PROBLEM VI.

To gauge and inch a rectangular Hop Back.

EXAMPLE.

A TABLE

Shewing the Method of Inching the Hop Back given in this Example.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
1	1	0	1.72	4	4	0	6.88	7	7	1	3.04
	1	0	1.72		1	0	1.72		1	0	1.72
2	2	0	3.44	5	5	0	8.60	8	8	1	4.76
	1	0	1.72		1	0	1.72		1	0	1.72
3	3	0	5.16	6	6	1	1.32	9	9	1	6.48
	1	0	1 72		1	0	1.72		1	0	1.72

TABLE CONTINUED.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
10	10	1	8.20	16	16	3	0.52	22	23	0	1.84
	1	0	1.72		1	0	1.72		1	0	1.72
11	11	2	0.92	17	17	3	2.24	23	24	0	3.56
	1	0	1.72		1	0	1.72		1	0	1.72
12	12	2	2.64	18	18	3	3.96	24	25	0	5.28
	1	0	1.72		1	0	1.72		1	0	1.72
13	13	2	4.36	19	19	3	5.68	25	26	0	7.00
	1	0	1.72		1	0	1.72		1	0	1.72
14	14	2	6.08	20	20	3	7.40	26	27	0	8.72
	1	0	1.72		1	0	1.72				
15	15	2	7.80	21	22	0	0.12				
	1	0	1.72		1	0	1.72				

COMMON BREWER'S TABLE BOOK.

A. B.'s Hop-Back.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
1	1	0	2	8	8	1	5	15	15	2	8	22	23	0	2
2	2	0	3	9	9	1	6	16	16	3	1	23	24	0	4
3	3	0	5	10	10	1	8	17	17	3	2	24	25	0	5
4	4	0	7	11	11	2	1	18	18	3	4	25	26	0	7
5	5	1	0	12	12	2	3	19	19	3	6	26	27	1	0
6	6	1	1	13	13	2	4	20	20	3	7				
7	7	1	3	14	14	2	6	21	22	0	0				

To find the depth to be allowed for hops, false bottom, &c.

EXAMPLE.

SOLUTION BY THE PEN.

Here $114 \times 84 = 9576$ square inches, the area of the base of the hop-back; and $9576 \times 15.5 = 148428$ cubic inches, the content of the liquor, hops, and false bottom, in the hop-back.

Again, $152 \times 126 = 19152$ square inches, the area of the base of the cooler; and $19152 \times 7 = 134064$ cubic inches, the content of the liquor when drawn off into the cooler.

Now, $148428 - 134064 = 14364$ cubic inches, the content of the hops and false bottom; then $14364 \div 9576 = 1.5$ inches, the deduction that must be made in the depth, when a gauge of wort is taken in the hop-back.

PROBLEM VIII.

To gauge and inch a Guile Tun, in the form of a parallelopipedon.

EXAMPLE.

A TABLE

Shewing the Method of Inching the Guile Tun, given this Example.

Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.
Full	55 0	3 3	7.344 0.977	10	48 0	0 3	6.574 0.977	20	40 0	1 3	5.804 0.977
1	55 0	0 3	6.367 0.977	11	47 0	1 3	5.597 0.977	21	39 0	2 3	4.827 0.977
2	54 0	1 3	5.390 0.977	12	46 0	2 3	4.620 0.977	22	38 0	3 3	3.850 0.977
3	53 0	2 3	4.413 0.977	13	45 0	3 3	3.643 0.977	23	38 0	0 3	2.873 0.977
4	52 0	3 3	3.436 0.977	14	45 0	0 3	2.666 0.977	24	37 0	1 3	1.896 0.977
5	52 0	0 3	2.459 0.977	15	44 0	1 3	1.689 0.977	25	36 0	2 3	0.919 0.977
6	51 0	1 3	1.482 0.977	16	43 0	2 3	0.712 0.977	26	35 0	2 3	8.942 0.977
7	50 0	2 3	0.505 0.977	17	42 0	2 3	8.735 0.977	27	34 0	3 3	7.965 0.977
8	49 0	2 3	8.528 0.977	18	41 0	3 3	7.758 0.977	28	34 0	0 3	6.988 0.977
9	48 0	3 3	7.551 0.977	19	41 0	0 3	6.781 0.977	29	33 0	1	6.011 0.977

TABLE CONTINUED.

Dry Inches.	B.	F.	G.	Dry Inches.	B.	F.	G.	Dry Inches.	B.	F.	G.
30	32 0	2 3	5.034 0.977	45	20 0	3 3	8.379 0.977	60	9 0	1 3	2.724 0.977
31	31 0	3 3	4.057 0.977	46	20 0	0 3	7.402 0.977	61	8 0	2 3	1.747 0.977
32	31 0	0 3	3.080 0.977	47	19 0	1 3	6.425 0.977	62	7 0	3 3	0.770 0.977
33	30 0	1 3	2.103 0.977	48	18 0	2 3	5.448 0.977	63	6 0	3 3	8.793 0.977
4	29 0	2 3	1.126 0.977	49	17 0	3 3	4.471 0.977	64	6 0	0 3	7.816 0.977
5	28 0	3 3	0.149 0.977	50	17 0	0 3	3.494 0.977	65	5 0	1 3	6.839 0.977
6	27 0	3 3	8.172 0.977	51	16 0	1 3	2.517 0.977	66	4 0	2 3	5.862 0.977
7	27 0	0 3	7.195 0.977	52	15 0	2 3	1.540 0.977	67	3 0	3 3	4.885 0.977
	26 0	1 3	6.218 0.977	53	14 0	3 3	0.563 0.977	68	3 0	0 3	3.908 0.977
	25 0	2 3	5.241 0.977	54	13 0	3 3	8.586 0.977	69	2 0	1 3	2.931 0.977
	24 0	3 3	4.264 0.977	55	13 0	0 3	7.609 0.977	70	1 0	2 3	1.954 0.977
	24 0	0 3	3.287 0.977	56	12 0	1 3	6.632 0.977	71	0 0	3 3	0.977 0.977
	23 0	1 3	2.310 0.977	57	11 0	2 3	5.655 0.977	72	0	0	0.000
	22 0	2 3	1.333 0.977	58	10 0	3 3	4.678 0.977				
	21 0	3 3	0.356 0.977	59	10 0	0 3	3.701 0.977				

[illegible] BREWERS' TABLE BOOK.

[illegible] No. 1.

Inches	B.	F.	G.	Inches	B.	F.	G.	Inches	B.	F.	G.	Inches	B.	F.	G.
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	54	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	55	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	56	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	57	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	58	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	59	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	60	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	61	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	62	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	63	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	64	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	65	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	66	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	67	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	68	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	69	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	70	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	71	[illegible]	[illegible]	[illegible]
												72	[illegible]	[illegible]	[illegible]

PROBLEM IX.

To gauge and inch a Guile Tun in the form of the frustrum of a cone, and make an allowance for the drip or fall.

EXAM. 1.

A TABLE

Shewing the Method of tabulating the Guile Tun given in this Example, for Dry Inches.

Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.
Full	45	0	7.2670	10	38	2	3.9960	20	31	3	2.4730
	0	2	5.7271		0	2	6.4523		0	2	7.6140
1	44	2	1.5399	11	37	3	6.5437	21	31	0	3.8590
	0	2	5.7271		0	2	6.4523		0	2	7.6140
2	43	3	4.8128	12	37	1	0.0914	22	30	1	5.2450
	0	2	5.7271		0	2	6.4523		0	2	7.6140
3	43	0	8.0857	13	36	2	2.6391	23	29	2	6.6310
	0	2	5.7271		0	2	6.4523		0	2	7.6140
4	42	2	2.3586	14	35	3	5.1868	24	28	3	8.0170
	0	2	5.7271		0	2	6.4523		0	2	7.6140
5	41	3	5.6315	15	35	0	7.7345	25	28	1	0.4030
	0	2	5.7271		0	2	6.4523		0	2	7.6140
6	41	0	8.9044	16	34	2	1.2822	26	27	2	1.7890
	0	2	5.7271		0	2	6.4523		0	2	7.6140
7	40	2	3.1773	17	33	3	3.8299	27	26	3	3.1750
	0	2	5.7271		0	2	6.4523		0	2	7.6140
8	39	3	6.4502	18	33	0	6.3776	28	26	0	4.5610
	0	2	5.7271		0	2	6.4523		0	2	7.6140
9	39	1	0.7231	19	32	1	8.9253	29	25	1	5.9470
	0	2	5.7271		0	2	6.4523		0	2	7.6140

COMMON BREWER'S TABLE BOOK.

A. B.'s Square Guile Tun, No. 1.

Dry Inches	Contents in B.	F.	G.	Dry Inches	Contents in B.	F.	G.	Dry Inches	Contents in B.	F.	G.	Dry Inches	Contents in B.	F.	G.
Full	55	3	7	18	41	3	8	36	27	3	8	54	14	[illegible]	[illegible]
1	55	0	6	19	41	0	7	37	27	0	7	55	13	[illegible]	[illegible]
2	54	1	5	20	40	1	6	38	26	1	6	56	12	1	[illegible]
3	53	2	4	21	39	2	5	39	25	2	5	57	11	2	5
4	52	3	3	22	38	3	4	40	24	3	4	58	10	[illegible]	[illegible]
5	52	0	2	23	38	0	3	41	24	0	3	59	10	[illegible]	4
6	51	1	1	24	37	1	2	42	23	1	2	60	9	1	3
7	50	2	1	25	36	2	1	43	22	2	1	61	8	2	[illegible]
8	49	3	0	26	35	3	0	44	21	3	0	62	7	3	[illegible]
9	48	3	8	27	34	3	8	45	20	3	8	63	7	0	[illegible]
10	48	0	7	28	34	0	7	46	20	0	7	64	6	0	[illegible]
11	47	1	6	29	33	1	6	47	19	1	6	65	5	1	[illegible]
12	46	2	5	30	32	2	5	48	18	2	5	66	4	2	6
13	45	3	4	31	31	3	4	49	17	3	4	67	3	3	5
14	45	0	3	32	31	0	3	50	17	0	3	68	3	0	4
15	44	1	2	33	30	1	2	51	16	1	3	69	2	1	3
16	43	2	1	34	29	2	1	52	15	2	2	70	1	2	2
17	42	3	0	35	28	3	0	53	14	3	1	71	0	3	1
												72	0	0	0

COMMON BREWER'S TABLE BOOK.

A. B.'s Round Guile Tun, No. 1.

ry In-es	Contents in B.	F.	G.	Dry In ches.	Contents in B.	F.	G.	Dry In-ches.	Contents in B.	F.	G.	Dry In-ches.	Contents in B.	F.	G.
ill	45	0	7	17	33	3	4	33	22	1	7	49	9	3	5
1	44	2	2	18	33	0	6	34	21	2	7	50	9	0	3
2	43	3	5	19	32	2	0	35	20	3	7	51	8	0	8
3	43	0	8	20	31	3	2	36	20	0	7	52	7	1	4
4	42	2	2	21	31	0	4	37	19	1	7	53	6	2	0
5	41	3	6	22	30	1	5	38	18	2	7	54	5	2	5
	41	1	0	23	29	2	7	39	17	3	7	55	4	3	1
	40	2	3	24	28	3	8	40	17	0	7	56	3	3	6
	39	3	6	25	28	1	0	41	16	1	5	57	3	0	2
	39	1	1	26	27	2	2	42	15	2	3	58	2	0	7
	38	2	4	27	26	3	3	43	14	3	1	59	1	1	3
	37	3	7	28	26	0	5	44	13	3	7	60	0	1	8
	37	1	0	29	25	1	6	45	13	0	5	Drip	0	1	8
	36	2	3	30	24	2	7	46	12	1	3				
	35	3	5	31	23	3	7	47	11	2	1				
	35	0	8	32	23	0	7	48	10	2	7				
	34	2	1	32											

Exam. 2.

thod of tabulating the Guile Tun given in this Example, for Wet Inches.

Contents in B.	F.	G.	Wet In-ches.	Contents in B.	F.	G.	Wet In-ches.	Contents in B.	F.	G.
0	1	7.5000	7	4	3	0.7885	12	9	0	3.0770
0	3	4.0577		0	3	4.0577		0	3	2.2039
1	1	2.5577	8	5	2	4.8462	13	9	3	5.2809
0	3	4.0577		0	3	4.0577		0	3	2 2039
2	0	6.6154	9	6	1	8.9039	14	10	2	7.4848
0	3	4.0577		0	3	4.0577		0	3	2-2039
3	0	1.6731	10	7	1	3.9616	15	11	2	0.6887
0	3	4.0577		0	3	4.0577		0	3	2.2039
3	3	5.7308	11	8	0	8.0193	16	12	1	2.8926
0	3	4.0577		0	3	4.0577		0	3	2.2039

TABLE CONTINUED.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
17	13	0	5.0965	33	25	1	5.9470	49	36	2	2.6391
	0	3	2.2039		0	2	7.6140		0	2	6.4523
18	13	3	7.3004	34	26	0	4.5610	50	37	1	0.0914
	0	3	2.2039		0	2	7.6140		0	2	6.4523
19	14	3	0.5043	35	26	3	3.1750	51	37	3	6.5437
	0	3	2.2039		0	2	7.6140		0	2	6.4523
20	15	2	2.7082	36	27	2	1.7890	52 •	38	2	3.9960
	0	3	2.2039		0	2	7.6140		0	2	5.7271
21	16	1	4.9121	37	28	1	0.4030	53	38	1	0.7231
	0	3	2.2039		0	2	7.6140		0	2	5.7271
22 •	17	0	7.1160	38	28	3	8.0170	54	39	3	6.4502
	0	3	0.0217		0	2	7.6140		0	2	5.7271
23	17	3	7.1377	39	29	2	6.6310	55	40	2	3.1773
	0	3	0.0217		0	2	7.6140		0	2	5.7271
24	18	2	7.1594	40	30	1	5.2450	56	41	0	8.9044
	0	3	0.0217		0	2	7.6140		0	2	5.7271
25	19	1	7.1811	41	31	0	3.8590	57	41	3	5.6315
	0	3	0.0217		0	2	7.6140		0	2	5.7271
26	20	0	7.2028	42 •	31	3	2.4730	58	42	2	2.3586
	0	3	0.0217		0	2	6.4523		0	2	5.7271
27	20	3	7.2245	43	32	1	8.9253	59	43	0	8.0857
	0	3	0.0217		0	2	6.4523		0	2	5.7271
28	21	2	7.2462	44	33	0	6.3776	60	43	3	4.8128
	0	3	0.0217		0	2	6.4523		0	2	5.7271
29	22	1	7.2679	45	33	3	3.8299	61	44	2	1.5399
	0	3	0.0217		0	2	6.4523		0	2	5.7271
30	23	0	7.2896	46	34	2	1.2822	62	45	0	7.2670
	0	3	0.0217		0	2	6.4523				
31	23	3	7.3113	47	35	0	7.7345				
	0	3	0.0217		0	2	6.4523				
32 •	24	2	7.3330	48	35	3	5.1868				
	0	2	7.6140		0	2	6.4523				

COMMON BREWER'S TABLE BOOK.

A. B.'s Round Guile Tun, No. 1.															
Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
Drip	0	1	8	19	14	3	1	36	27	2	2	53	39	1	1
3	1	1	3	20	15	2	3	37	28	1	0	54	39	3	6
4	2	0	7	21	16	1	5	38	28	3	8	55	40	2	3
5	3	0	2	22	17	0	7	39	29	2	7	56	41	1	0
6	3	3	6	23	17	3	7	40	30	1	5	57	41	3	6
7	4	3	1	24	18	2	7	41	31	0	4	58	42	2	2
8	5	2	5	25	19	1	7	42	31	3	2	59	43	0	8
9	6	2	0	26	20	0	7	43	32	2	0	60	43	3	5
10	7	1	4	27	20	3	7	44	33	0	6	61	44	2	2
11	8	0	8	28	21	2	7	45	33	3	4	62	45	0	7
12	9	0	3	29	22	1	7	46	34	2	1				
13	9	3	5	30	23	0	7	47	35	0	8				
14	10	2	7	31	23	3	7	48	35	3	5				
15	11	2	1	32	24	2	7	49	36	2	3				
16	12	1	3	33	25	1	6	50	37	1	0				
17	13	0	5	34	26	0	5	51	37	3	7				
18	13	3	7	35	26	3	3	52	38	2	4				

PROBLEM X.

To gauge and inch a circular Guile Tun with curved sides, and make an allowance for the drip or fall.

Exam. 1.

The Method of tabulating the Guile Tun given in this Example, for Wet Inches.

Wet In-ches.	Contents in			Wet In-ches.	Contents in			Wet In-ches.	Contents in		
	B.	F.	G.		B.	F.	G.		B.	F.	G.
Drip	0	0	5.250	12	3	1	3.002	23	7	0	3.552
	0	1	1.093		0	1	2.336		0	1	4.145
2	0	1	6.343	13	3	2	5.338	24	7	1	7.697
	0	1	1.093		0	1	2.336		0	1	4.145
3	0	2	7.436	14	3	3	7.674	25	7	3	2.842
	0	1	1.093		0	1	2.336	•	0	1	5.558
4	0	3	8.529	15	4	1	1.010	26	8	0	8.400
	0	1	1.093		0	1	2.336		0	1	5.558
5	1	1	0.622	16	4	2	3.346	27	8	2	4.958
	0	1	1.093		0	1	2.336		0	1	5.558
6	1	2	1.715	17	4	3	5.682	28	9	0	1.516
	0	1	1.093	•	0	1	4.145		0	1	5.558
7	1	3	2.808	18	5	1	0.827	29	9	1	7.074
	0	1	1.093		0	1	4.145		0	1	5.558
8	2	0	3.901	19	5	2	4.972	30	9	3	3.632
	0	1	1.093		0	1	4.145		0	1	5.558
9	2	1	4.994	20	6	0	0.117	31	10	1	0.190
•	0	1	2.336		0	1	4.145		0	1	5.558
10	2	2	7.330	21	6	1	4.262	32	10	2	5.748
	0	1	2.336		0	1	4.145		0	1	5.558
11	3	0	0.666	22	6	2	8.407	33	11	0	2.306
	0	1	2.336		0	1	4.145	•	0	1	4.803

TABLE CONTINUED.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
34	11	1	7.109	42	14	2	0.183	50	17	1	8.308
	0	1	4.803		0	1	4.453		0	1	3.954
35	11	3	2.912	43	14	3	4.636	51	17	3	3.262
	0	1	4.803		0	1	4.453		0	1	3.954
36	12	0	7.715	44	15	1	0.089	52	18	0	7.216
	0	1	4.803		0	1	4.453		0	1	3.954
37	12	2	3.518	45	15	2	4.542	53	18	2	2.170
	0	1	4.803		0	1	4.453		0	1	3.954
38	12	3	8.321	46	15	3	8.995	54	18	3	6.124
	0	1	4.803		0	1	4.453		0	1	3.954
9	13	1	4.124	47	16	1	4.448	55	19	1	1.078
	0	1	4.803		0	1	4.453		0	1	3.954
0	13	2	8.927	48	16	2	8.901	56	19	2	5.032
	0	1	4.803		0	1	4.453		0	1	3.954
1	14	0	4.730	49	17	0	4.354	57	19	3	8.986
	0	1	4.453		0	1	3.954				

COMMON BREWER'S TABLE BOOK.

A. B.'s Round Guile Tun, No. 2.

Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
0	0	5	16	4	2	3	31	10	1	0	46	16	0	0
0	1	6	17	4	3	6	32	10	2	6	47	16	1	4
0	2	7	18	5	1	1	33	11	0	2	48	16	3	0
1	0	0	19	5	2	5	34	11	1	7	49	17	0	4
1	1	1	20	6	0	0	35	11	3	3	50	17	1	8
1	2	2	21	6	1	4	36	12	0	8	51	17	3	3
1	3	3	22	6	2	8	37	12	2	4	52	18	0	7
2	0	4	23	7	0	4	38	12	3	8	53	18	2	2
2	1	5	24	7	1	8	39	13	1	4	54	18	3	6
2	2	7	25	7	3	3	40	13	3	0	55	19	1	1
3	0	1	26	8	0	8	41	14	0	5	56	19	2	5
3	1	3	27	8	2	5	42	14	2	0	57	20	0	0
3	2	5	28	9	0	2	43	14	3	5				
3	3	8	29	9	1	7	44	15	1	0				
4	1	1	30	9	3	4	45	15	2	5				

EXAM. 2.

The Method of tabulating the Guide Tun given in this Example, for Dry Inches.

Dry Inches	B.	F.	G.	Dry Inches	B.	F.	G.	Dry Inches	B.	F.	G.
Full	29	3	8.986	12	25	2	4.542	24	11	0	2.306
	0	1	3.454		0	1	4.453		0	1	5.558
1	29	2	5.532	13	25	1	0.089	25	10	2	5.746
	0	1	3.454		0	1	4.453		0	1	5.558
2	29	1	1.078	14	24	3	4.636	26	10	1	0.190
	0	1	3.454		0	1	4.453		0	1	5.558
3	28	3	6.124	15	24	2	0.183	27	9	3	3.632
	0	1	3.454		0	1	4.453		0	1	5.558
4	28	2	2.770	16	24	0	4.730	28	9	0	7.074
	0	1	3.454		0	1	4.503		0	1	5.558
5	28	1	7.316	17	23	2	8.927	29	9	0	1.516
	0	1	3.464		0	1	4.503		0	1	5.558
6	27	3	3.862	18	23	1	4.124	30	8	2	4.958
	0	1	3.954		0	1	4.503		0	1	5.558
7	27	2	8.398	19	22	3	8.321	31	8	0	8.400
	0	1	3.464		0	1	4.503		0	1	5.558
8	27	1	4.354	20	22	2	3.818	32	7	3	2.842
	0	1	4.453		0	1	4.503		0	1	4.145
9	26	3	8.904	21	22	0	7.715	33	7	1	7.697
	0	1	4.453		0	1	4.503		0	1	4.145
10	26	1	4.446	22	21	3	2.912	34	7	0	3.552
	0	1	4.453		0	1	4.503		0	1	4.145
11	25	3	8.996	23	21	1	7.109	35	6	2	6.407
	0	1	4.453		0	1	4.503		0	1	4.145

TABLE CONTINUED.

Dry In-ches.	Contents in B.	F.	G.	Dry In-ches.	Contents in B.	F.	G.	Dry In-ches.	Contents in B.	F.	G.
36	6	1	4.262	44	3	2	5.338	52	1	1	0.622
	0	1	4.145		0	1	2.336		0	1	1.093
37	6	0	0.117	45	3	1	3.002	53	0	3	8.529
	0	1	4.145		0	1	2.336		0	1	1.093
38	5	2	4.972	46	3	0	0.666	54	0	2	7.436
	0	1	4.145		0	1	2.336		0	1	1.093
39	5	1	0.827	47	2	2	7.330	55	0	1	6.343
	0	1	4.145		0	1	2.336		0	1	1.093
0	4	3	5.682	48	2	1	4.994	56	0	0	5.250
	0	1	2.336		0	1	1.093		0	0	5.250
1	4	2	3.346	49	2	0	3.901	Drip	0	0	0.000
	0	1	2.336		0	1	1.093				
2	4	1	1.010	50	1	3	2.808				
	0	1	2.336		0	1	1.093				
3	3	3	7.674	51	1	2	1.715				
	0	1	2.336		0	1	1.093				

COMMON BREWER'S TABLE BOOK.

A. B.'s Round Guile Tun, No. 2.

Contents in B.	F.	G.	Dry In-ches.	Contents in B.	F.	G.	Dry In-ches.	Contents in B.	F.	G.	Dry In-ches.	Contents in B.	F.	G.
20	0	0	15	14	2	0	30	8	2	5	45	3	1	3
19	2	5	16	14	0	5	31	8	0	8	46	3	0	1
19	1	1	17	13	3	0	32	7	3	3	47	2	2	7
18	3	6	18	13	1	4	33	7	1	8	48	2	1	5
18	2	2	19	12	3	8	34	7	0	4	49	2	0	4
18	0	7	20	12	2	4	35	6	2	8	50	1	3	3
17	3	3	21	12	0	8	36	6	1	4	51	1	2	2
17	1	8	22	11	3	3	37	6	0	0	52	1	1	1
7	0	4	23	11	1	7	38	5	2	5	53	1	0	0
6	3	0	24	11	0	2	39	5	1	1	54	0	2	7
6	1	4	25	10	2	6	40	4	3	6	55	0	1	6
6	0	0	26	10	1	0	41	4	2	3	56	0	0	5
5	2	5	27	9	3	4	42	4	1	1	Drip	0	0	5
5	1	0	28	9	0	7	43	3	3	8				
4	3	5	29	9	0	2	44	3	2	5				

PROBLEM XI.

To gauge and inch an elliptical Guile Tun, and make an allowance for the fall or drip.

Exam. 1.

The Method of tabulating the Guile Tun given in this Example, for Dry Inches.

Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.	Dry Inches.	Contents in B.	F.	G.
Full	16 0	2 1	2.602 0.782	10	13 0	3 1	3.782 1.710	20	10 0	3 1	4.682 2.706
1	16 0	1 1	1.820 0.782	11	13 0	2 1	2.072 1.710	21	10 0	2 1	1.976 2.706
2	16 0	0 1	1.038 0.782	12	13 0	1 1	0.362 1.710	22	10 0	0 1	8.270 2.706
3	15 0	3 1	0.256 0.782	13	12 0	3 1	7.652 1.710	23	9 0	3 1	5.564 2.706
4	15 0	1 1	8.474 0.782	14	12 0	2 1	5.942 1.710	24	9 0	2 1	2.858 2.706
5	15 0	0 1	7.692 0.782	15	12 0	1 1	4.232 1.710	25	9 0	1 1	0.152 2.706
6	14 0	3 1	6.910 0.782	16	12 0	0 1	2.522 1.710	26	8 0	3 1	6.446 2.706
7	14 0	2 1	6.128 0.782	17	11 0	3 1	0.812 1.710	27	8 0	2 1	3.740 2.706
8	14 0	1 1	5.346 0.782	18	11 0	1 1	8.102 1.710	28	8 0	1 1	1.034 2.706
9	14 0	0 1	4.564 0.782	19	11 0	0 1	6.392 1.710	29	7 0	3 1	7.328 2.706

TABLE CONTINUED.

Dry Inches.	B.	F.	G.	Dry Inches.	B.	F.	G.	Dry Inches.	B.	F.	G.
30	7	2	4.622	37	5	0	4.917	44	2	2	1.040
•	0	1	3.815		0	1	3.815		0	1	4.858
31	7	1	0.807	38	4	3	1.102	45	2	0	5.182
	0	1	3.815		0	1	3.815		0	1	4.858
32	6	3	5.992	39	4	1	6.287	46	1	3	0.324
	0	1	3.815		0	1	3.815		0	1	4.858
33	6	2	2.177	40	4	0	2.472	47	1	1	4.466
	0	1	3.815	•	0	1	4.858		0	1	4.858
34	6	0	7.362	41	3	2	6.614	48	0	3	8.608
	0	1	3.815		0	1	4.858		0	1	4.858
35	5	3	3.547	42	3	1	1.756	49	0	2	3.750
	0	1	3.815		0	1	4.858	Drip	0	2	3.750
36	5	1	8.732	43	2	3	5.898		0	0	0.000
	0	1	3.815		0	1	4.858				

COMMON BREWER'S TABLE BOOK.

A. B.'s Round Guile Tun, No. 3.

Contents in B.	F.	G.	Dry Inches.	B.	F.	G.	Dry Inches.	B.	F.	G.	Dry Inches.	B.	F.	G.
16	2	3	13	12	3	8	26	8	3	6	39	4	1	6
16	1	2	14	12	2	6	27	8	2	4	40	4	0	2
16	0	1	15	12	1	4	28	8	1	1	41	3	2	7
15	3	0	16	12	0	3	29	7	3	7	42	3	1	2
15	1	8	17	11	3	1	30	7	2	5	43	2	3	6
15	0	8	18	11	1	8	31	7	1	1	44	2	2	1
14	3	7	19	11	0	6	32	6	3	6	45	2	0	5
14	2	6	20	10	3	5	33	6	2	2	46	1	3	0
14	1	5	21	10	2	2	34	6	0	7	47	1	1	4
14	0	5	22	10	0	8	35	5	3	4	48	1	0	0
13	3	4	23	9	3	6	36	5	2	0	49	0	2	4
13	2	2	24	9	2	3	37	5	0	5	Drip	0	2	4
13	1	0	25	9	1	0	38	4	3	1				

[illegible] 2.

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	27	9 1 [illegible] 9 1 [illegible]
[illegible]	[illegible]	[illegible]	[illegible]	28	9 2 [illegible] 9 1 [illegible]
[illegible]	[illegible]	[illegible]	[illegible]	29	9 3 [illegible] 9 1 2.70[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	30	[illegible] [illegible]
[illegible]	[illegible]	[illegible]	[illegible]	31	[illegible] 2 1.576 3 1 2.706
[illegible]	[illegible]	[illegible]	[illegible] 3 [illegible] [illegible] 1 [illegible]	32	[illegible] 3 4.6[illegible] 0 1 1.710
[illegible]	[illegible]	[illegible]	[illegible] [illegible]	33	11 0 6.5[illegible] 0 1 1.710
[illegible]	[illegible]	[illegible]	[illegible] 2 4.6[illegible] [illegible] 1 2.706	34	11 1 8.10[illegible] 0 1 1.710
[illegible]	[illegible]	[illegible]	[illegible] 3 [illegible] [illegible] 1 2.706	35	11 3 0.912 0 1 1.710
[illegible]	[illegible]	[illegible]	[illegible] 1.0[illegible] [illegible] 2.706	36	12 0 2.522 0 1 1.710
[illegible]	[illegible]	[illegible]	[illegible] 3.7[illegible] [illegible] 2.706	37	12 1 4.2[illegible] 0 1 1.710
[illegible]	[illegible]	[illegible]	9 3 6.446 0 [illegible] 2.706	38	12 2 5.9[illegible] 0 1 1.710

TABLE CONTINUED.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.
39	12	3	7.652	44	14	1	5.346	49	15	3	0.256
	0	1	1.710		0	1	0.782		0	1	0.782
40	13	1	0.362	45	14	2	6.128	50	16	0	1.038
	0	1	1.710		0	1	0.782		0	1	0.782
41	13	2	2.072	46	14	3	6.910	51	16	1	1.820
	0	1	1.710		0	1	0.782		0	1	0.782
42	13	3	3.782	47	15	0	7.692	52	16	2	2.602
	0	1	0.782		0	1	0.782				
43	14	0	4.564	48	15	1	8.474				
	0	1	0.782		0	1	0.782				

COMMON BREWER'S TABLE BOOK.

A. B.'s Round Guile Tun, No. 3.

Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G.	Wet Inches.	Contents in B.	F.	G
Drip	0	2	4	16	5	2	0	29	9	3	6	42	13	3	4
4	1	0	0	17	5	3	4	30	10	0	8	43	14	0	5
5	1	1	4	18	6	0	7	31	10	2	2	44	14	1	5
6	1	3	0	19	6	2	2	32	10	3	5	45	14	2	6
7	2	0	5	20	6	3	6	33	11	0	6	46	14	3	7
8	2	2	1	21	7	1	1	34	11	1	8	47	15	0	8
9	2	3	6	22	7	2	5	35	11	3	1	48	15	1	8
10	3	1	2	23	7	3	7	36	12	0	3	49	15	3	0
11	3	2	7	24	8	1	1	37	12	1	4	50	16	0	1
12	4	0	2	25	8	2	4	38	12	2	6	51	16	1	2
13	4	1	6	26	8	3	6	39	12	3	8	52	16	2	3
14	4	3	1	27	9	1	0	40	13	1	0				
15	5	0	5	28	9	2	3	41	13	2	2				

EXAM. 1.

BY THE PEN.

To find the area of the sections.

Here $64.8 \times 54.2 \div 359.05 = 3512.16 \div 359.05 = 9.7818$, the area of the first section; $67.7 \times 56.8 \div 359.05 = 3845.36 \div 359.05 = 10.7098$, the area of the second section; $70.4 \times 59.7 \div 359.05 = 4202.88 \div 359.05 = 11.7055$, the area of the third section; $73.5 \times 62.6 \div 359.05 = 4601.10 \div 359.05 = 12.8146$, the area of the fourth section; and $76.2 \times 65.3 \div 359.05 = 4975.86 \div 359.05 = 13.8584$ ale gallons, the area of the fifth section.

PROBLEM XII.

To deduct the heat from Common Brewers' warm wort.

EXAM. 2.

	B.	*F.*	*G.*	
10)	72	1	2	warm gauge.
	7	0	8.5	deduction.
	65	0	2.5	neat gauge.

Note. Here the decimal part of a gallon, in the deduction, is only .3; but it is called .5, according to Note 2, in the Gauging.

EXAM. 3.

	B.	F.	G.	
10)	29	3	8	warm gauge.
	3	0	0	deduction.
	26	3	8	Ans.

Note. According to the observation in Note 2, of the Gauging, the deduction, in the last Example, becomes exactly 3 barrels.

SECTION III.

THE METHOD OF GAUGING AND ULLAGING CASKS, AS PRACTISED IN THE EXCISE.

CASK GAUGING.

PROBLEM III.

To guage and fix a cask of the first variety, as practised in the Excise.

EXAM. 2.

By Rule I.

Bung diameter	30.9 inches.
Head diameter	24.6 inches.
Difference	6.3 inches.
Multiplier	.7
Product	4.41
Head diameter	24.6
Mean diameter	29.01 inches.
Ditto	29.01 inches.
	2901
	26109
	5802
	841.5801
	46.7
	58910607
	50494806
	33663204

Divisor 359.05)39301.79067(109.460 ale gallons.

Also, $39301.79067 \div 294.12 = 133.625$, the content in wine gallons.

BY THE SLIDING RULE.

The difference between the head and bung diameters is 6.3 inches; against this number on the line of inches, we have 4.4 on the line marked *spheroid*, which being added to 24.6, the head diameter, gives 29 inches, for the mean diameter. Then,

$$\begin{array}{lcccccc} & \text{On D.} & & \text{On C.} & & \text{On D.} & & \text{On C.} \\ \text{As} & \left\{\begin{matrix}18.95\\17.15\end{matrix}\right\} & : & 46.7 & :: & 29.0 & : & \left\{\begin{matrix}\text{109.4 ale gallons.}\\\text{123.6 wine gallons.}\end{matrix}\right. \end{array}$$

EXAM. 3.

By Rule II.

Bung diameter	26.2 inches.
Head diameter	21.4 inches.
Difference	4.8 inches.
Multiplier	.68
	384
	288
Product	3.264
Head diameter	21.4
Mean diameter	24.664 inches.

The areas answering to the mean diameter 24.7 inches, are 1.6991 ale, and 2.0743 wine gallons; then $1.6991 \times 32.5 = 55.220$, the content in ale gallons; and $2.0743 \times 32.5 = 67.414$, the content in wine gallons.

BY THE SLIDING RULE.

The difference of the diameters is 4.8 inches;

against this number on the line of inches, we find 3.34, on the line marked *spheroid*, which being added to 21.4, the head diameter, gives 24.74 inches, for the mean diameter. Then,

	On D.		On C.		On D.		On C.
As	18.95 17.15	:	32.5	::	24.74	:	55.2 ale gallons. 67.4 wine gallons.

EXAM. 4.

By Rule II.

Here 23.4 − 19.6 = 3.8 inches, the difference of the diameters; and 3.8 × .68 = 2.584; then 19.6 + 2.584 = 22.184 inches, the mean diameter. The areas corresponding to 22.2 inches, are 1.3726 ale, and 1.6757 wine gallons; then 1.3726 × 27.7 = 38.021, the content in ale gallons; and 1.6757 × 27.7 = 46.416, the content in wine gallons.

BY THE SLIDING RULE.

Here the difference of the diameters is 3.8 inches: against this number on the line of inches, we have 2.65 on the line marked *spheroid*; and 19.6 + 2.65 = 22.25 inches, the mean diameter. Then,

	On D.		On C.		On D.		On C.
As	18.95 17.15	:	27.7	::	22.25	:	38.0 ale gallons. 46.4 wine gallons.

PROBLEM IV.

To guage and fix a cask of the second variety, as practised in the Excise.

Exam. 2.

By Rule I.

Bung diameter	30.9 inches.
Head diameter	24.6 inches.
Difference	6.3 inches.
Multiplier	.64
	252
	378
Product	4.032
Head diameter	24.600
Mean diameter	28.632 inches.

Then $(28.632 \times 28.632 \times 46.7) \div 359.05 = (819.791424 \times 46.7) \div 359.05 = 38284.2595008 \div 359.05 = 106.62654$, the content in ale gallons; and $38284.2595008 \div 294.12 = 130.16544$, the content in wine gallons.

BY THE SLIDING RULE.

Here the difference of the diameters is 6.3 inches; against this number on the line of inches, we find 4.0 on the line marked *2nd Variety;* and $24.6 + 4.0 = 28.6$ inches, the mean diameter. Then,

	On D.		On C.		On D.		On C.
As	18.95 / 17.15	:	46.7	::	28.6	:	106.6 ale gallons. / 130.1 wine gallons.

EXAM. 3.

By Rule II.

Here 26.2 − 21.4 = 4.8 inches, the difference of the diameters; and 4.8 × .62 = 2.976; then 21.4 + 2.976 = 24.376 inches, the mean diameter. The areas answering to 24.4 inches, are 1.6581 ale, and 2.0242 wine gallons; then 1.6581 × 32.5 = 53.88825, the content in ale gallons; and 2.0242 × 32.5 = 65.7865, the content in wine gallons.

BY THE SLIDING RULE.

The difference of the diameters is 4.8 inches; against this number on the line of inches, we have 3.04 on the line marked *2nd Variety;* and 21.4 + 3.04 = 24.44 inches, the mean diameter. Then,

	On D.		On C.		On D.		On C.
As	18.95 / 17.15	:	32.5	::	24.44	:	53.9 ale gallons / 65.8 wine gallons

EXAM. 4.

By Rule II.

Here 23.4 − 19.6 = 3.8 inches, the difference of the diameters; and 3.8 × .62 = 2.356; then 19.6 + 2.356 = 21.956 inches, the mean diameter. The areas corresponding to 22.0 inches, are 1.3480 ale, and 1.6456 wine gallons; then 1.348 × 27.7 = 37.3396, the content in ale gallons; and 1.6456 × 27.7 = 45.58312, the content in wine gallons.

BY THE SLIDING RULE.

The difference of the diameters is 3.8 inches; opposite this number on the line of inches, we find 2.4 on the line marked *2nd Variety;* and 19.6 + 2.4 = 22.0 inches, the mean diameter. Then,

	On D.		On C.		On D.		On C.
As	18.95 17.15	:	27.7	::	22.0	:	37.3 ale gallons. 45.6 wine gallons.

PROBLEM V.

To gauge and fix a cask of the third variety, as practised in the Excise.

Exam. 2.

By Rule 1.

Here $30.9 - 24.6 = 6.3$ inches, the difference of the diameters; and $6.3 \times .57 = 3.591$; then $24.6 + 3.591 = 28.191$ inches, the mean diameter. Now, $(28.191 \times 28.191 \times 46.7) \div 359.05 = (794.732481 \times 46.7) \div 359.05 = 37114.0068627 \div 359.05 = 103.36723$, the content in ale gallons; and $37114.0068627 \div 294.12 = 126.18661$, the content in wine gallons.

Note. In solving these questions, the Rule given in the Problem is called Rule I.; and the method by the Tables of Ale and Wine Areas, is denominated Rule II.

BY THE SLIDING RULE.

As the third variety of casks is not placed on any of the slides, we must make use of the mean diameter found by the multiplier; Hence,

	On D.		On C.		On D.		On C.
As	18.95 17.15	:	46.7	::	28.2	:	103.3 ale gallons. 126.2 wine gallons.

EXAM. 3.

By Rule II.

Here $26.2 - 21.4 = 4.8$ inches, the difference of the diameters; and $4.8 \times .55 = 2.64$; then $21.4 + 2.64 = 24.04$ inches, the mean diameter. The areas corresponding to 24.0 inches, are 1.6042 ale, and 1.9584 wine gallons; then $1.6042 \times 32.5 = 52.1365$, the content in ale gallons; and $1.9584 \times 32.5 = 63.648$, the content in wine gallons.

BY THE SLIDING RULE.

$$\begin{array}{lcccc} & \text{On D.} & \text{On C.} & \text{On D.} & \text{On C.} \\ \text{As} & \left\{\begin{matrix}18.95\\17.15\end{matrix}\right\} & : 32.5 :: & 24.04 : & \left\{\begin{matrix}\text{52.1 ale gallons.}\\\text{63.6 wine gallons.}\end{matrix}\right. \end{array}$$

EXAM. 4.

By Rule II.

Here $23.4 - 19.6 = 3.8$ inches, the difference of the diameters; and $3.8 \times .55 = 2.09$; then $19.6 + 2.09 = 21.69$ inches, the mean diameter. The areas answering to 21.7 inches, are 1.3114 ale, and 1.6010 wine gallons; then $1.3114 \times 27.7 = 36.32578$, the content in ale gallons; and $1.601 \times 27.7 = 44.3477$, the content in wine gallons.

BY THE SLIDING RULE.

$$\begin{array}{lcccc} & \text{On D.} & \text{On C.} & \text{On D.} & \text{On C.} \\ \text{As} & \left\{\begin{matrix}18.95\\17.15\end{matrix}\right\} & : 27.7 :: & 21.69 : & \left\{\begin{matrix}\text{36.3 ale gallons.}\\\text{44.3 wine gallons.}\end{matrix}\right. \end{array}$$

PROBLEM VI.

To gauge and fix a cask of the fourth variety, as practised in the Excise.

EXAM. 2.

By Rule I.

Bung diameter	30.9 inches.
Head diameter	24.6 inches.
Difference	6.3 inches.
Multiplier	.52
	126
	315
Product	3.276
Head diameter	24.6
Mean diameter	27.876 inches.

Then $(27.876 \times 27.876 \times 46.7) \div 359.05 = 777.071376 \times 46.7) \div 359.05 = 36289.2332592 \div 359.05 = 101.07018$, the content in ale gallons; and $36289.2332592 \div 294.12 = 123.3824$, the content in wine gallons.

BY THE SLIDING RULE.

	On D.		On C.		On D.		On C.
As	18.95 / 17.15	:	46.7	::	27.87	:	101.1 ale gallons. / 123.4 wine gallons.

EXAM. 3.

By Rule II.

Here $26.2 - 21.4 = 4.8$ inches, the difference of the diameters; and $4.8 \times .5 = 2.40$; then $21.4 +$

2.4 = 23.8 inches, the mean diameter. The areas corresponding to this diameter, are 1.5775 ale, and 1.9259 wine gallons; then 1.5775 × 32.5 = 51.26875, the content in ale gallons; and 1.9259 × 32.5 = 62.59175, the content in wine gallons.

BY THE SLIDING RULE.

	On D.		On C.		On D.		On C.
As	18.95 / 17.15	:	32.5	::	23.8	:	51.3 ale gallons. / 62.6 wine gallons.

EXAM. 4.

By Rule II.

Here 23.4 − 19.6 = 3.8 inches, the difference of the diameters; and 3.8 × .5 = 1.90; then 19.6 + 1.9 = 21.5 inches, the mean diameter. The areas answering to this diameter, are 1.2874 ale, and 1.5717 wine gallons: then 1.2874 × 27.7 = 35.66098, the content in ale gallons; and 1.5717 × 27.7 = 43.53609, the content in wine gallons.

BY THE SLIDING RULE.

	On D.		On C.		On D.		On C.
As	18.95 / 17.15	:	27.7	::	21.5	:	35.6 ale gallons. / 43.5 wine gallons.

PROBLEM VII.

To find the content of a cask, by the Diagonal Rod, without paying any regard to its variety.

EXAM. 2.

BY THE DIAGONAL ROD.

Opposite 36.2 inches, the given diagonal, we find 106.0 ale gallons, and 129.2 wine gallons, the contents required.

BY THE SLIDING RULE.

$$\begin{array}{lcccc} & \text{On D.} & \text{On C.} & \text{On D.} & \text{On C.} \\ \text{As} & \left\{\begin{array}{l}21.2\\19.2\end{array}\right\} : 36.2 :: & 36.2 : & \left\{\begin{array}{l}106.0 \text{ ale gallons.}\\129.2 \text{ wine gallons.}\end{array}\right. \end{array}$$

By Note I.

Here $36.2 \times 36.2 \times 36.2 = 1310.44 \times 36.2 = 47437.928$, the cube of the diagonal; then $47437.928 \times .002228 = 105.691703584$, the content in ale gallons; and $47437.928 \times .00272 = 129.03116416$ the content in wine gallons.

EXAM. 3.

BY THE DIAGONAL ROD.

Opposite 28.6 inches, the given diagonal, we find 52.2 ale gallons, and 63.8 wine gallons, the contents required.

BY THE SLIDING RULE.

	On D.		On C.		On D.		On C.
As	$\begin{Bmatrix}21.2\\19.2\end{Bmatrix}$	:	28.6	::	28.6	:	$\begin{cases}52.2 \text{ ale gall.}\\63.5 \text{ wine gall.}\end{cases}$

By Note I.

Here 28.6 × 28.6 × 28.6 = 817.96 × 28.6 = 23393.656, the cube of the diagonal; then 23393.656 × .002228 = 52.121065568, the content in ale gallons; and 23393.656 × .00272 = 63.63074432, the content in wine gallons.

EXAM. 4.

BY THE DIAGONAL ROD.

Opposite 25.5 inches, the given diagonal, we find 37.0 ale gallons, and 45.1 wine gallons, the contents required.

BY THE SLIDING RULE.

	On D.		On C.		On D.		On C.
As	$\begin{Bmatrix}21.2\\19.2\end{Bmatrix}$	:	25.5	::	25.5	:	$\begin{cases}36.9 \text{ ale gall.}\\44.9 \text{ wine gall.}\end{cases}$

By Note I.

Here 25.5 × 25.5 × 25.5 = 650.25 × 25.5 = 16581.375, the cube of the diagonal; then 16581.375 × .002228 = 36.9433035, the content in ale gallons; and 16581.375 × .00272 = 45.10134, the content in wine gallons.

PROBLEM VIII.

To find the contents of casks in general, from the head diameter, bung diameter, and length.

EXAM. 2.

By Rule I.

Here 30.9 × 30.9 × 39 = 954.81 × 39 = 37237.59, thirty-nine times the square of the bung diameter; 24.6 × 24.6 × 25 = 605.16 × 25 = 15129.00, twenty-five times the square of the head diameter; 30.9 × 24.6 × 26 = 760.14 × 26 = 19763.64, twenty-six times the product of the diameters; then (37237.59 + 15129 + 19763.64) × 46.7 × .00034 = 72130.23 × 46.7 × .00034 = 3368481.741 × .00034 = 1145.28379194; hence $\frac{1145.2837}{11}$ = 104.1167, the content in ale gallons; and $\frac{1145.2837}{9}$ = 127.2537, the content in wine gallons.

BY THE SLIDING RULE.

Here 24.6 ÷ 30.9 = .79, the quotient of the head, divided by the bung diameter. Opposite to this number, in the table, we find 20.74, and 18.77, the gauge-points for ale and wine gallons; then,

$$\begin{array}{lccccccc} & \text{On D.} & & \text{On C.} & & \text{On D.} & & \text{On C.} \\ \text{As} & \left\{\begin{matrix}20.74\\18.77\end{matrix}\right\} & : & 46.7 & :: & 30.9 & : & \left\{\begin{matrix}104.1 \text{ ale gall.}\\127.2 \text{ wine gall.}\end{matrix}\right. \end{array}$$

EXAM. 3.

By Rule II.

Here $\frac{21.4}{26.2} = .81\ \frac{178}{262}$, the quotient of the head divided by the bung diameter. The multiplier corresponding to this quotient, is .00238 for ale, and .0029056 for wine gallons; then .00238 × 26.2² × 32.5 = .00238 × 686.44 × 32.5 = 1.6337272 × 32.5 = 53.0961340, the content in ale gallons;

and $.0029056 \times 686.44 \times 32.5 = 1.994520064 \times 32.5 = 64.82190208$, the content in wine gallons.

Note. In order to find the multiplier answering to $.81\frac{178}{262}$, we proceed in the following manner, as directed in Note I. of the Gauging: The multiplier opposite to .81 is .002366, and that opposite to .82 is .0023867; then $(.0023867 - .002366) \times \frac{178}{262} = .0000207 \times \frac{178}{262} = \frac{.0036846}{262} = .000014$; and $.0023867 + .000014 = .00238$, the multiplier for ale gallons, corresponding to $.81\frac{178}{262}$. In the same manner we find .0029056 to be the multiplier corresponding to $.81\frac{178}{262}$, for wine gallons.

BY THE SLIDING RULE.

The gauge-points opposite to .81, in the table, are 20.56, and 18.61, for ale and wine gallons; then,

$$\begin{array}{ccccccc} & \text{On D.} & & \text{On C.} & \text{On D.} & & \text{On C.} \\ \text{As} \left\{\begin{array}{c}20.56\\18.61\end{array}\right\} & & : & 32.5 \;::\; & 26.2 & : & \left\{\begin{array}{l}53.1 \text{ ale gall.}\\64.8 \text{ wine gall.}\end{array}\right. \end{array}$$

EXAM. 4.

By Rule II.

Here $\frac{19.6}{23.4} = .83\frac{178}{234} = .83\frac{3}{4}$ *nearly*, the quotient of the head, divided by the bung diameter. The multiplier answering to this quotient is .0024232 for ale, and .0029583 for wine gallons; then $.0024232 \times 23.4^2 \times 27.7 = .0024232 \times 547.56 \times 27.7 = 1.326847392 \times 27.7 = 36.7536727584$, the content in ale gallons; and $.0029583 \times 547.56 \times 27.7 = 1.619846748 \times 27.7 = 44.8697549196$, the content in wine gallons.

Note. The multiplier answering to $.83\frac{3}{4}$ is found as directed in Note 2, of the Gauging.

BY THE SLIDING RULE.

As the quotient of the head by the bung diameter, is nearer .84 than .83, we take the gauge points oppo-

site to .84; which are 20.29, and 18.36 for ale and wine gallons; then,

	On D.		On C.		On D.		On C.
As	$\begin{Bmatrix} 20.29 \\ 18.36 \end{Bmatrix}$	:	27.7	::	23.4	:	$\begin{Bmatrix} 36.8 \text{ ale gall.} \\ 44.9 \text{ wine gall.} \end{Bmatrix}$

PROBLEM IX.

To find the true content of any cask without paying regard to its variety, by means of four dimensions; namely, the length, the bung and head diameters, and the middle diameter, between the bung and head.

EXAM. 2.

By Rule I.

Here $29 \times 29 \times 4 = 841 \times 4 = 3364$, four times the square of the middle diameter; and $30.9^2 + 24.6^2 = 954.81 + 605.16 = 1559.97$, the sum of the squares of the bung and head diameters; then $(3364 + 1559.97) \times 46.7 = 4923.97 \times 46.7 = 229949.399$; and $229949.399 \div 2154.3 = 106.739$, the content in ale gallons; also, $229949.399 \div 1764.72 = 130.303$, the content in wine gallons.

By Rule II.

Ale Measure.

	Inches.	*Area.*		*Areas.*
Middle diameters	29.0	2.3423 × 4	=	9.3692
Bung diameter	30.9	area	=	2.6592
Head diameter	24.6	area	=	1.6854
Sum of the areas			=	13.7138
Multiply by the length			=	46.7
				959966
				822828
				548552
Divide by				6)640.43446
Content in ale gallons				106 73907
By Note 2, Multiply by				11
Divide by				9)1174.12977
Content in wine gallons				130.45886

Wine Measure.

	Inches.	*Areas.*		*Areas.*
Middle diameter	29.0	2.8594 × 4	=	11.4376
Bung diameter	30.9	area	=	3.2464
Head diameter	24.6	area	=	2.0575
Sum of the areas			=	16.7415
Multiply by the length			=	40.7
				117.1905
				100.4490
				669660
Divide by				6)681.37905
Content in wine gallons				130.304[illegible]
By Note 2, Multiply by				9
Divide by				11)1172.74[illegible]
Content in ale gallons				106.6129

BY THE SLIDING RULE.

Ale Measure.

On C.	On D.	On D.	On C.		Ale Gal.
As 40.7 :	46.4 ::	24.6 :	13.1	=	13.1
		30.9	20.6	=	20.6
		29.0	18.2 × 4	=	72.8
Content in ale gallons					106.5 sum.

Wine Measure.

On C.	On D.	On D.	On C.		Wine Gal.
As 40.7 :	42.0 ::	24.6 :	15.9	=	15.9
		30.9	25.2	=	25.2
		29.0	22.3 × 4	=	89.2
Content in wine gallons					130.3 sum.

Exam. 3.

By Rule I.

Here $24.8 \times 24.8 \times 4 = 615.04 \times 4 = 2460.16$, four times the square of the middle diameter; and $26.2^2 + 21.4^2 = 686.44 + 457.96 = 1144.4$, the sum

of the squares of the bung and head diameters; then (2460.16 + 1144.4) × 32.5 = 3604.56 × 32.5 = 117148.2; and 117148.2 ÷ 2154.3 = 54.378, the content in ale gallons; also, 117148.2 ÷ 1764.72 = 66.383, the content in wine gallons.

BY THE SLIDING RULE.

Ale Measure.

On C.	On D.	On D.	On C.	Ale Gal.
As 32.5 :	46.4 ::	21.4 :	6.9...... =	6.9
		26.2	10.3...... =	10.3
		24.8	9.3 × 4 =	37.2

Content in wine gallons......54.4 sum.

Wine Measure.

On C.	On D.	On D.	On C.	Wine Gal.
As 32.5 :	42.0 ::	21.4 :	8.4...... =	8.4
		26.2	12.6...... =	12.6
		24.8	11.3 × 4 =	45.2

Content in wine gallons......66.2 sum.

EXAM. 4.

By Rule II.

Ale Measure.

	Inches.	*Area*	*Areas.*
Middle diameter	22.1 ...	1.3602 × 4 =	5.4408
Bung diameter	23.4 ...	area ...... =	1.5250
Head diameter	19.6 ...	area ...... =	1.0699

Sum of the areas .... = 8.0357
Multiply by the length......... .... = 27.7

562499
562499
160714

Divide by.................................. 6)222.58889

Content in ale gallons.................. 37.09814

Multiply by............................... 11

Divide by .... 9)408.07954

Content in wine gallons,.............. 45.34217

Wine Measure.

	Inches.	Area.	Areas.
Middle diameter	22.1	1.6606 × 4	= 6.6424
Bung diameter	23.4	area	= 1.8617
Head diameter	19.6	area	= 1.3061

Sum of the areas .... = 9.8102
Multiply by the length .... = 27.7

686714
686714
196204

Divide by .... 6)271.74254
Content in wine gallons..................... 45.29042
Multiply by 9
Divide by.....................................11)407.61378
Content in ale gallons....................... 37.05579

BY THE SLIDING RULE.

Ale Measure.

On C.	On D.	On D.	On C.	Ale Gal.

$$\text{As } 27.7 : 46.4 :: \left\{\begin{array}{l}19.6\\23.4\\22.1\end{array}\right\} : \left\{\begin{array}{l}4.9\ldots\ldots = 4.9\\7.0\ldots\ldots = 7.0\\6.3 \times 4 = 25.2\end{array}\right.$$

Content in ale gallons .... 37.1 sum.

Wine Measure.

On C.	On D.	On D.	On C.	Wine Gal.

$$\text{As } 27.7 : 42.0 :: \left\{\begin{array}{l}19.6\\23.4\\22.1\end{array}\right\} : \left\{\begin{array}{l}6.0\ldots\ldots = 6.0\\8.6\ldots\ldots = 8.6\\7.7 \times 4 = 30.8\end{array}\right.$$

Content in wine gallons .... 45.4 sum

PROBLEM X.

To ullage a lying cask, having given the bung diameter, wet inches, and content.

EXAM. 2.

BY THE PEN.

Here 10.4 ÷ 32 = .325, which is less than .500 by .175, one-fourth part of which is = .04375; .325 − .04375 = .28125, the multiplier; hence 115.323 × .28125 = 32.43479375 wine gallons, the ullage required.

BY THE SLIDING RULE.

	On C.		OnS.L.		On C.		On S. L.
As	32	:	100	::	10.4	:	26.0, the fourth number.

And,

	On A.		On B.		On A.		On B.
As	100	:	115.3	::	26.0	:	30.0 wine gallons.

Or,

As 100 : 26.0 :: 115.3 : 30.0, the ullage required.

BY THE ULLAGE RULE.

	On B.		On S. L.		On B.		On S. L.
As	32	:	100	::	10.4	:	26.0, the fourth number.

And,

	On A.		On B.		On A.		On B.
As	100	:	115.3	::	26.0	:	30.0 wine gallons.

Or,

As 100 : 26.0 :: 115.3 : 30.0, the ullage required.

PROBLEM XI.

To ullage a standing cask, having given the length, the wet inches, and the content.

Exam. 2.

BY THE PEN.

Here $8.8 \div 40 = .220$, which is less than .500 by .280, one-tenth part of which is $= .028$; then $.220 - .028 = .192$, the multiplier; hence $115.323 \times .192 = 22.142016$ wine gallons, the ullage required.

BY THE SLIDING RULE.

	On C.		On S. S.		On C.		On S. S.
As	40	:	100	::	8.8	:	20.0, the fourth number.

And,

	On A.		On B.		On A.		On B.
As	100	:	115.3	::	20.0	:	23.0 wine gallons.

Or,

As 100 : 20.0 :: 115.3 : 23.0, the ullage required.

BY THE ULLAGE RULE.

	On C.		On S. S.		On C.		On S. S.
As	40	:	100	::	8.8	:	20.0, the fourth number.

And,

	On A.		On C.		On A.		On C.
As	100	:	115.3	::	20.0	:	23.0 wine gallons.

Or,

As 100 : 20.0 :: 115.3 : 23.0, the ullage required.

PROBLEM XII.

To inch a standing cask of the second or third variety.

EXAMPLE OF THE THIRD VARIETY.

Operation.

	Inches.		
Bung diameter	30	....	3.0600
Head diameter	24	....	1.9584
Difference		....	1.1016

Then $(1.1016 \times \frac{4}{9}) \div 20^2 = .4896 \div 400 =$ .001224, multiplier.

For the first inch from the bung.

From the area of the bung diameter	3.060000
Subtract $.001224 \times 1 \times 1 =$	.001224
Content of one inch from the bung	3.058776

For the second inch from the bung.

From the area of the bung diameter	3.060000
Subtract $.001224 \times 2 \times 2 =$	.004896
Difference	3.055104
Distance from the bung	2
Content at two inches from the bung	6.110208

For the third inch from the bung.

From the area of the bung diameter	3.060000
Subtract $.001224 \times 3 \times 3 =$	.011016
Difference	3.048984
Distance from the bung	3
Content of three inches from the bung	9.146952

For the fourth inch from the bung.

From the area of the bung diameter	3.060000
Subtract .001224 × 4 × 4 = ...	.019584
Difference	3.040416
Distance from the bung	4
Content of four inches from the bung	12.161664

A TABLE

Shewing the method of obtaining three series of differences; and the content of the cask at every inch, from the bung to the head.

Inches from the bung.	Contents.	First differences.	Second differences.	Third differences.
1	3.0588	3.0514	.0146	.0075
2	6.1102	3.0368	.0221	.0075
3	9.1470	3.0147	.0296	.0075
4	12.1617	2.9851	.0371	.0075
5	15.1468	2.9480	.0446	.0075
6	18.0948	2.9034	.0521	.0075
7	20.9982	2.8513	.0596	.0075
8	23.8495	2.7917	.0671	.0075
9	26.6412	2.7246	.0746	.0075
10	29.3658	2.6500	.0821	.0075
11	32.0158	2.5679	.0896	.0075
12	34.5837	2.4783	.0971	.0075
13	37.0620	2.3812	.1046	.0075
14	39.4432	2.2766	.1121	.0075
15	41.7198	2.1645	.1196	.0075
16	43.8843	2.0449	.1271	.0075
17	45.9292	1.9178	.1346	.0075
18	47.8470	1.7832	.1421	
19	49.6303	1.6411		
20	51.2713			

A TABLE

Shewing the Method of Inching the Cask, given in the foregoing Example.

Wet Inches.	Contents.	Wet Inches.	Contents.	Wet Inches.	Contents.	Wet Inches.	Contents.
1	1.6411 1.7832	11	24.6301 2.7917	21	54.3301 3.0514	31	83.2871 2.5679
2	3.4243 1.9178	12	27.4218 2.8513	22	57.3815 3.0368	32	85.8550 2.4783
3	5.3421 2.0449	13	30.2731 2.9034	23	60.4183 3.0147	33	88.3333 2.3812
4	7.3870 2.1645	14	33.1765 2.9480	24	63.4330 2.9851	34	90.7145 2.2766
5	9.5515 2.2766	15	36.1245 2.9851	25	66.4181 2.9480	35	92.9911 2.1645
6	11.8281 2.3812	16	39.1096 3.0147	26	69.3661 2.9034	36	95.1556 2.0449
7	14.2093 2.4783	17	42.1243 3.0368	27	72.2695 2.8513	37	97.2005 1.9178
8	16.6876 2.5679	18	45.1611 3.0514	28	75.1208 2.7917	38	99.1183 1.7832
9	19.2555 2.6500	19	48.2125 3.0588	29	77.9125 2.7246	39	100.9015 1.6411
10	21.9055 2.7246	20	51.2713 3.0588	30	80.6371 2.6500	40	102.5426
11	24.6301	21	54.3301	31	83.2871		

The Method of Gauging Casks by the Calipers, as practised in the Ports of Great Britain and Ireland by the Port Gaugers of the Excise and Customs.

Exam. 2.

To find the true dimensions.

	Inches
External horizontal bung diameter	37.5
Twice the thickness of the staff	3.2
Internal horizontal bung diameter	34.3
Internal perpendicular bung diameter	34.2
Divide by	2)68.5
True bung diameter	34.3
Whole length	39.5
Bung diameters 34.3 — 28.2 — allowance	[illegible]
Corrected or true length	30.3
Back head diameter	28.[illegible]
Ditto	28.1
Divide by	2)56.[illegible]
Mean back head diameter	28.3
Front head diameter	28.5
Ditto	28.4
Divide by	2)57.0
Mean front head diameter	28.5
Mean back head diameter	28.3
Divide by	2)56.8
True head diameter	28.4

	Inches.
Length	36.4
Ditto	36.3
Ditto	36.5
Divide by	3)109.2
Mean length	36.4
Deduction for the heads, 2.4 − 2.0	0.4
Difference	36.0
Deduction for the variety	0.8
True length	35.2

True dimensions.	*Inches.*
Length	35.2
Head diameter	28.4
Bung diameter	34.5
Wet Inches	30.8

To find the mean diameter.

Set the brass index on the slide, to the head diameter 28.4, on the lower line of the stock; then against the bung diameter 34.5, on the same line of the stock, we have 4.2 nearly on the line marked spheroid: and opposite this number on the lower line of the slide, we have 32.6, the mean diameter, on the lower line of the stock.

To find the content.

Set the guage-point on D, to the length 35.2 on C; then against the mean diameter 32.6 on D, is 127, *very nearly*, on C, the content in wine gallons.

To find the ullage.

Set the bung diameter 34.5 on N, to 100 on S. L.; then against the wet inches 30.8 on N, we have 95 the fourth number on S. L.

Again, set 95 on B, to 100 on A; then against 127,

the content of the cask on A, is 121 nearly, the content of the ullage on B.

EXAM. 3.

In casting the contents of the casks given in this Example, let Sl. denote the Slide; B. I. the Brass Index on the Slide; St. the Stock; Sp. Spheroid; and G P. the Guage-Point on the Slide D. By this means we shall be able to abridge the method of expressing the operations of finding the mean diameters, contents and ullages.

No. 1.

On Sl. On St. On St. On Sp.
As B. I. : 22.4 :: 27.3 : 3.4; then opposite this number on the lower line of the slide, we have 25.8 the mean diameter, on the lower line of the stock.

On D. On C. On D. On C.
As G. P. : 47.5 :: 25.8 : 107, the content of the cask

On N. On S. L. On N. On S. L.
As 27.3 : 100 :: 24.7 : 96, the fourth number.

On A. On B. On A. On B.
As 100 : 96 :: 107 : 103, the ullage required

No. 2.

On Sl. On St. On St. On Sp.
As B. I. : 21.3 :: 27.9 : 4.6; then opposite this number on the lower line of the slide, we have 25.9 the mean diameter on the lower line of the stock.

On D. On C. On D. On C.
As G. P. : 46.8 :: 25.9 : 106, the content of the cask

On N. On S.L. On N. On S. L.
As 27.9 : 100 :: 25.9 : 97.6, fourth number.

On A. On B. On A. On B.
As 100 : 97.6 :: 106 : 103, the ullage required

No. 3.

	On Sl.		On St.		On St.		On Sp.
As	B. I.	:	21.7	::	27.8	:	4.2;

then opposite this number on the lower line of the slide we have 25.9 the mean diameter on the lower line of the stock.

	On D.		On C.		On D.		On C.
As	G. P.	:	48.4	::	25.9	:	110, the content of the cask.

	On N.		On S.L.		On N.		On S.L.
As	27.8	:	100	::	26.0	:	98, the fourth number.

	On A.		On B.		On A.		On B.
As	100	:	98	::	110	:	108, the ullage required.

No. 4.

	On Sl.		On St.		On St.		On Sp.
As	B. I.	:	22.3	::	27.8	:	3.8;

then opposite this number on the lower line of the slide, we have 26.1, the mean diameter on the lower line of the stock.

	On D.		On C.		On D.		On C.
As	G. P.	:	47.2	::	26.1	:	109, the content of the cask.

	On N.		On S.L.		On N.		On S.L.
As	27.8	:	100	::	25.7	:	97, the fourth number.

	On A.		On B.		On A.		On B.
As	100	:	97	::	109	:	106, the ullage required.

No. 5.

	On Sl.		On St.		On St.		On Sp.
As	B. I.	:	22.3	::	27.5	:	3.6;

then opposite this number on the lower line of the slide, we have 25.9, the mean diameter on the lower line of the stock.

	On D.		On C.		On D.		On C.
As	G. P.	:	47.1	::	25.9	:	107, the content of the cask.

	On N.		On S.L.		On N.		On S.L.
As	100	:	27.5	::	25.9	:	98, the fourth number.

	On A.		On B.		On A.		On B.
As	100	:	98	::	107	:	105, the ullage required.

No. 6.

On Sl. On St. On St. On Sp.
As B. 1. : 22.3 :: 27.5 : 3.6; then opposite this number on the lower line of the slide, we have 25.9, the mean diameter on the lower line of the stock.

On D. On C. On D. On C.
As G.P. : 48.3 :: 25.9 : 110, the content of the cask.

On N. On S.L. On N. On S.L.
As 100 : 27.5 :: 25.7 : 98, the fourth number.

On A. On B. On A. On B.
As 100 : 98 :: 109 : 108, the ullage required.

SECTION IV.

ALT GAUGING, OR THE METHOD OF GAUGING AND FIXING MALTSTERS' UTENSILS, AS PRACTISED IN THE EXCISE.

PROBLEM I.

) gauge and fix a Maltster's Cistern, in the form of a parallelopipedon.

EXAM. 1.

ie Method of tabulating the Cistern, given in this Example.

Inches.	Contents.	Inches.	Contents.	Inches.	Contents.
20.0	57.200 .286	20.8	59.488 .286	21.6	61.776 .286
20.1	57.486 .286	20.9	59.774 .286	21.7	62.062 .286
20.2	57.772 .286	21.0	60.060 .286	21.8	62.348 .286
20.3	58.058 .286	21.1	60.346 .286	21.9	62.634 .286
20.4	58.344 .286	21.2	60.632 .286	22.0	62.920 .286
20.5	58.630 .286	21.3	60.918 .286	22.1	63.206 .286
20.6	58.916 .286	21.4	61.204 .286	22.2	63.492 .286
20.7	59.202 .286	21.5	61.490 .286	22.3	63.778 .286

TABLE CONTINUED.

Inches.	Contents.	Inches.	Contents.	Inches.	Contents.
22.4	64.064 .286	23.3	66.638 .286	24.2	69.212 .286
22.5	64.350 .286	23.4	66.924 .286	24.3	69.498 .286
22.6	64.636 .286	23.5	67.210 .286	24.4	69.784 .286
22.7	64.922 .286	23.6	67.496 .286	24.5	70.070 .286
22.8	65.208 .286	23.7	67.782 .286	24.6	70.356 .286
22.9	65.494 .286	23.8	68.068 .286	24.7	70.642 .286
23.0	65.780 .286	23.9	68.354 .286	24.8	70.928 .286
23.1	66.066 .286	24.0	68.640 .286	24.9	71.214 .286
23.2	66.352 .286	24.1	68.926 .286	25.0	71.500

See the Gauging, in which a Table Book is formed from the above Table of Inches and Contents.

Exam. 2.

BY THE PEN.

To find the area and content.

Here 128.6 × 85.4 ÷ 2150.42 = 10982.44 ÷ 2150.42 = 5.107 bushels, the area of the cistern and 5.107 × 52.8 = 269.6496 bushels, the content required.

BY THE SLIDING RULE.

M. B. *Length.* *Breadth.* *Area.*
As 2150.42 on A : 128.6 on B :: 85.4 on A : 5.11 on B.

And,

Unity. *Area.* *Depth.* *Content.*
As 1 on A : 5.11 on B :: 52.8 on A : 269.65 on B.

Or,

Length. *Depth.* *Breadth.* *Content.*
As 128.6 on B : 52.8 on MD :: 85.4 on A : 269.65 on B.

EXAM. 3.

Here $5.107 \times 43.7 = 223.1759$ bushels, the content required.

PROBLEM II.

To gauge a couch of malt contained in a rectangular frame.

EXAM. 2.

BY THE PEN.

To find the area and content.

Here $148 \times 125 \div 2150.42 = 18500 \div 2150.42 = 8.602$ bushels, the area; and $8.602 \times 26.8 = 30.5336$ bushels, the content required.

BY THE SLIDING RULE.

M. B. *Length.* *Breadth.* *Area.*
As 2150.42 on A : 148 on B :: 125 on A : 8.6 on B.

And,

Unity.	*Area.*	*Depth.*	*Content.*

As 1 on A : 8.6 on B :: 26.8 on A : 230.53 on B.

Or,

Length.	*Depth.*	*Breadth.*	*Content.*

As 148 on B : 26.8 on MD :: 125 on A : 230.53 on B.

EXAM. 3.

Here 8.602 × 18.7 = 160.8574 bushels, the content required.

PROBLEM III.

To gauge a couch of malt, not in a frame, but laid upon the floor, in a square or rectangular form, with one or more of its sides slanting.

EXAM. 2.

BY THE PEN.

To find the area and content.

Here 138 × 113 ÷ 2150.42 = 15594 ÷ 2150.42 = 7.251 bushels, the area of the couch; and 7.251 × 28.6 = 207.3786 bushels, the content required.

BY THE SLIDING RULE.

Length.	*Depth.*	*Breadth.*	*Content.*

As 138 on B : 28.6 on MD :: 113 on A : 207.37 on B.

PROBLEM IV.

To gauge a couch of malt either in the form of a cone, or a conical frustum.

EXAM. 2.

BY THE PEN.

To find the content.

Here $\frac{144 + 64}{2} = \frac{208}{2} = 104$, the mean diameter, when the frustum is reduced to a cylinder; then, $104 \times 104 \times 28.2 = 10816 \times 28.2 = 305011.2$; and $305011.2 \div 2738 = 111.399$, the content in malt bushels.

BY THE SLIDING RULE.

	On D.		On C.		On D.		On C.	
As	52.32	:	28.2	::	104	:	111.39,	content.

PROBLEM V.

To gauge a rectangular floor of malt.

EXAM. 2.

BY THE PEN.

To find the content.

```
      Inches.
        435 length.
        218 breadth.
       ----
       3480
       435
      870
      -----
      94830 product.
         5.2 depth.
     ------
     189660
    474150
    -------
Divisor 2150.42)493116.0(229.311 bushels.
```

BY THE SLIDING RULE.

Length.	*Depth.*	*Breadth.*	*Content.*

As 465 on B : 5.2 on MD :: 218 on A : 229.31 on B.

EXAM. 3.

BY THE PEN.

Here 485 × 224 × 3.4 = 108640 × 3.4 = 369376 and 369376 ÷ 2150.42 = 171.769, the content in malt bushels.

BY THE SLIDING RULE.

Breadth.	*Depth.*	*Length.*	*Content.*

As 224 on B : 3.4 on MD :: 485 on A : 171.7 on B.

PROBLEM VI.

To find the content of a rectangular floor of malt by another method.

EXAM. 2.

BY THE RULE.

```
*Inches.*
  435 length.
  109 half the breadth.
 ----
 3915
435
-----
47.415 product.
   5.2 depth.
------
 94830
237075
--------
246.5580 product.
    .930 factor.
---------
 739674
2219022
----------
229.29894 content.
```

By the first Note.

Here $435 \times 109 \times 5.2 = 246.558$ false con.

And $\frac{246.558 \times 7}{100} = \frac{1725.906}{100} = 17.259$ deduction.

229.299 true con.

EXAM. 3.

By the third Note.

	Bushels.
Here	18.6, the area of the floor.
Multiply by	6.7, the depth of the grain.
	1302
	1116
	124.62 content.

EXAM. 4.

By the fourth Note.

Here half the length of the floor is 50; then if we multiply 250 by 50, and cut off three figures, as decimals, we obtain 12.5 for the *false* area. This being multiplied by 6, gives 75 bushels, for the *false* content. Lastly, if we multiply 75 by 7, and cut off two figures for a decimal, we obtain 5.25 bushels for the deduction; hence the true content of the floor is 69.75 bushels.

PROBLEM VII.

To gauge and fix a rectangular malt-kiln.

Exam. 2.

BY THE PEN.

Bushels.

```
  20.012 area.
     4.7 depth.
  ------
 140084
 80048
 -------
 94.0564 content.
```

BY THE SLIDING RULE.

Unity.	*Area.*	*Depth.*	*Content.*
As 1 on A :	20.01 on B ::	4.7 on A :	94.05 on B.

Exam. 3.

BY THE PEN.

Here 50 × 30 = 1500, the area of the well-hole; and 1500 ÷ 212 = 7 inches, the number to be deducted from the length, in order to make an allowance for the well-hole; hence, we have 236 − 7 = 229 inches, the true length of the kiln; then 229 × 212 ÷ 2150.42 = 48548 ÷ 2150.42 = 22.576 malt bushels, the area required.

PROOF.

Here 236 × 212 = 50032, the area of both the kiln and the well-hole; and 50032 − 1500 = 48532, the area of the kiln, in square inches; then 48532 ÷ 2150.42 = 22.568, its area in malt bushels, nearly as before.

PROBLEM VIII.

To find whether the duty will arise from the cistern, the couch, or the floor.

EXAM. 2.

Bushels.	
69.8	couch-gauge.
1.6	multiplier.
418.8	
698	
111.68	product.

Here the number of bushels obtained by multiplying the couch-guage by 1.6, is less than the number of floor bushels; consequently, the charge will arise from the floor.

Bushels.	
112.6	floor bushels.
.5	multiplier.
56.30	product.

Here the neat bushels are 56.3, the number upon which the duty must be charged.

SECTION V.

THE METHOD OF GAUGING AND INCHING DISTILLERS' UTENSILS.

PROBLEM I.

To gauge and inch a Wash Still.

Exam. 1.

The Method of tabulating the Wash Still, given in this Example.

Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.
Full	644.698 4.910	7	603.098 7.320	14	533.434 11.636	21	450.800 12.818
1	639.788 4.910	8 *	595.778 9.768	15	521.798 11.636	22	437.982 12.818
2	634.878 4.910	9	586.010 9.768	16	510.162 11.636	23	425.164 12.818
3	629.968 4.910	10	576.242 9.768	17	498.526 11.636	24	412.346 12.818
4 *	625.058 7.320	11	566.474 9.768	18	486.890 11.636	25	399.528 12.818
5	617.738 7.320	12 *	556.706 11.636	19	475.254 11.636	26	386.710 12.818
6	610.418 7.320	13	545.070 11.636	20 *	463.618 12.818	27	373.892 12.818

TABLE CONTINUED.

Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.
28	361.074	36	253.802	44	150.586	51	71.570
•	13.409	*	12.902	•	11.517		9.914
29	347.665	37	240.900	45	139.069	52	61.656
	13.409		12.902		11.517		9.914
30	334.256	38	227.998	46	127.552	53	51.742
	13.409		12.902		11.517		9.914
31	320.847	39	215.096	47	116.035	54	41.828
	13.409		12.902		11.517		9.914
32	307.438	40	202.194	48	104.518	55	31.914
	13.409		12.902		11.517		9.914
33	294.029	41	189.292	49	93.001	56	22.000
	13.409		12.902		11.517	crown	22.000
34	280.620	42	176.390	50	81.484		00.000
	13.409		12.902	•	9.914		
35	267.211	43	163.488				
	13.409		12.902				

A DISTILLER'S TABLE BOOK.

A. B.'s Wash Still, No. 1.							
Dry Inches.	Contents.	Dry Inches.	Contents.	Dry Inches.	Contents.	Dry Inches.	Contents.
Full	645	18	533	32	361	46	176
5	640	19	522	33	348	47	163
6	635	20	510	34	334	48	151
7	630	21	499	35	321	49	139
8	625	22	487	36	307	50	128
9	618	23	475	37	294	51	116
10	610	24	464	38	281	52	105
11	603	25	451	39	267	53	93
12	596	26	438	40	254	54	81
13	586	27	425	41	241	55	72
14	576	28	412	42	228	56	62
15	566	29	400	43	215	57	52
16	557	30	387	44	202	58	42
17	545	31	374	45	189	59	32
						crown	22
							22

A TABLE

Shewing the Method of Inching the Still given in this Example, when the upper part is considered as the frustum of a sphere.

Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.
Full	639.465 3.040	5	616.596 6.733	10	576.686 9.732	15	521.798 11.636
1	636.425 3.838	6	609.863 7.383	11	566.954 10.248	16	510.162 11.636
2	632.587 4.604	7	602.480 7.998	12 *	556.706 11.636	17	498.526 11.636
3	627.983 5.332	8	594.482 8.602	13	545.070 11.636	18	486.890 11.636
4	622.651 6.055	9	585.880 9.194	14	533.434 11.636	19	475.254 11.636

TABLE CONTINUED.

Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.
20 *	463.618 12.818	30	334.256 13.409	40	202.194 12.902	49	93.001 11.517
21	450.800 12.818	31	320.847 13.409	41	189.292 12.902	50 *	81.484 9.914
22	437.982 12.818	32	307.438 13.409	42	176.390 12.902	51	71.570 9.914
23	425.164 12.818	33	294.029 13.409	43	163.488 12.902	52	61.656 9.914
24	412.346 12.818	34	280.620 13.409	44 *	150.586 11.517	53	51.742 9.914
25	399.528 12.818	35	267.211 13.409	45	139.069 11.517	54	41.828 9.914
26	386.710 12.818	36 *	253.802 12.902	46	127.552 11.517	55	31.914 9.914
27	373.892 12.818	37	240.900 12.902	47	116.035 11.517	56 crown	22.000 22.000
28 *	361.074 13.409	38	227.998 12.902	48	104.518 11.517		00.000
29	347.665 13.409	39	215.096 12.902				

Exam. 2.

The Method of tabulating the Wash Still, given in this Example.

Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.
Full	1549 53 7.74	13	1405.10 17.79	26	1136.11 24.77	39	809.95 26.42
1	1541.79 7.74	14	1387.31 17.79	27	1111.34 24.77	40 *	783.53 26.60
2	1534.05 7.74	15	1369.52 17.79	28	1086.57 24.77	41	756.93 26.60
3	1526.31 7.74	16 *	1351.73 20.76	29	1061.80 24.77	42	730.33 26.60
4 *	1518.57 10.33	17	1330.97 20.76	30	1037.03 24.77	43	703.73 26.60
5	1508.24 10.33	18	1310.21 20.76	31	1012.26 24.77	44	677.13 26.60
6	1497.91 10.33	19	1289.45 20.76	32 *	987.49 24.57	45	650.53 26.60
7	1487.58 10.33	20	1268.69 20.76	33	962.92 24.57	46	623.93 26.60
8 *	1477.25 13.59	21	1247.93 20.76	34	938.35 24.57	47	597.33 26.60
9	1463.66 13.59	22	1227.17 20.76	35	913.78 24.57	48 *	570.73 25.88
10	1450.07 13 59	23	1206.41 20.76	36 *	889.21 26.42	49	544.85 25.88
11	1436.48 13.59	24 *	1185.65 24.77	37	862.79 26.42	50	518.97 25.88
12 *	1422.89 17.79	25	1160.88 24.77	38	836.37 26.42	51	493.09 25.88

TABLE CONTINUED.

Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.	Dry Inches.	Contents in Gallons.
52	467.21	57	340.07	62	221.97	67	113.41
	25.88		23.62		23.62	*	20.47
53	441.33	58	316.45	63	198.35	68	92.94
	25.88		23.62		23.62		20.47
54	415.45	59	292.83	64	17.473	69	72.47
	25.88		23.62	*	20.44		20.47
55	389.57	60	269.21	65	154.29	70	52.00
	25.88		23.62		20.44	crown	52.00
56	363.69	61	245.59	66	133.85		00.00
*	23.62		23.62		20.44		

PROBLEM II.

To find the content of a still-head.

EXAM. 2.

To find the content.

Here $2.515 + 4.261 = 6.776$, the sum of the top and bottom areas; also, $8.843 \times 2 = 17.686$, twice the area of the middle section; and $6.703 + 7.866 \times 4 = 14.569 \times 4 = 58.276$, four times the sum of the areas, at one fourth, and three-fourths, of the depth; then,

$$(6.776 + 17.686 + 58.276)\frac{40}{12} = \frac{82.738 \times 40}{12} = \frac{3309.52}{12} = 275.793$$ wine gallons, the content of the head.

Again, $4.261 \times 6.2 = 26.418$ wine gallons, the content of the cylindrical collar; then, $275.793 + 26.418 = 302.211$ wine gallons, the whole content required.

PROBLEM III.

To gauge and inch a distiller's oval wash-back, by the method of equi-distant ordinates.

Exam. 1.

To find the area of the second Section.

Here $33.6 + 33.6 = 67.2$, the sum of the extreme ordinates; $(59.6 + 79.4 + 79.4 + 59.6 \times 4 = 278 \times 4 = 1112.0$, four times the sum of all the even ordinates; and $(72.6 + 81.6 + 72.6) \times 2 = 226.8 \times 2 = 453.6$, twice the sum of all the ordinates; then, $(67.2 + 1112.0 + 453.6) \times 4 = 1632.8 \times 4 = 6531.2$ square inches, the area of that part of the section circumscribed by the sides of the vessel and the two extreme ordinates.

Again $\dfrac{67.2 \times 9.4}{3} = \dfrac{631.68}{3} = 210.56$ square inches, the area of the two segments; and $6531.2 + 210.56 = 6741.76$ square inches, the area of the whole section; then $6741.76 \div 231 = 29.185$, the area in wine gallons.

To find the area of the third Section.

Here $27.0 + 27.0 = 54.0$, the sum of the extreme ordinates; $(56.1 + 76.8 + 76.8 + 56.1) \times 4 = 265.8 \times 4 = 1063.2$, four times the sum of all the even ordinates: and $69.7 + 79.0 + 69.7) \times 2 = 218.4 \times 2 = 436.8$, twice the sum of all the odd ordinates; then

(54.0 + 436.8 + 1063.2) × 4 = 1554.0 × 4 = 6216.0 square inches, the area of that part of the section circumscribed by the sides of the vessel and the two extreme ordinates.

Again, $\frac{54 \times 6.2}{3} = \frac{334.8}{3} = 111.6$ square inches, the area of the two segments; and 6216.0 + 111.6 = 6327.6 square inches, the area of the whole section; then 6327.6 ÷ 231 = 27.392, the area in wine gallons.

To find the area of the fourth section.

Here 20.4 + 20.4 = 40.8, the sum of the extreme ordinates; (53.0 + 74.1 + 74.1 + 53.0) × 4 = 254.2 × 4 = 1016.8, four times, the sum of all the even ordinates (67.0 + 76.4 + 67.0) × 2 = 210.4 × 2 = 420.8, twice the sum of all the odd ordinates; then (40.8 + 1016.8 + 420.8) × 4 = 1478.4 × 4 = 5913.6 square inches, the area of that part of the section, circumscribed by the side of the vessel and the two extreme ordinates.

Again, $\frac{40.8 \times 4}{3} = \frac{163.2}{3} = 54.4$ square inches, the area of the two segments; and 5913.6 + 54.4 = 5968.0, square inches, the area of the whole section; then 5968.0 ÷ 231 = 25.835, the area in wine gallons.

A TABLE,

Shewing the Method of Inching the Wash-Back, given in this Example.

Wet Inches.	Contents in Gallons.	Wet Inches.	Contents in Gallons.	Wet Inches.	Contents in Gallons.	Wet Inches.	Contents in Gallons.
Dr p	15.000 30.884	12	353.025 29.185	23	670.474 27.392	34	967.115 25.835
2	45.884 30.884	13	382.210 29.185	24	697.866 27.392	35	992.950 25.835
3	76.768 30.884	14	411.395 29.185	25	725.258 27.392	36	1018.785 25.835
4	107.652 30.884	15	440.580 29.185	26	752.650 27.392	37	1044.620 25.835
5	138.536 30.884	16	469.765 29.185	27	780.042 27.392	38	1070.455 25.835
6	169.420 30.884	17	498.950 29.185	28	807.434 27.392	39	1096.290 25.835
7	200.304 30.884	18	528.135 29.185	29	834.826 27.392	40	1122.125 25.835
8	231.188 30.884	19	557.320 29.185	30	862.218 27.392	41	1147.960
9	262.072 30.884	20	586.505 29.185	31	889.610 25.835		
10	292.956 30.884	21	615.690 27.392	32	915.445 25.835		
11	323.840 29.185	22	643.082 27.392	33	941.280 25.835		

EXAM. 2.

To find the area of the first section.

Here 44.2 + 44.3 = 88.5, the sum of the extreme ordinates; (66.8 + 82.2 + 82.1 + 66.8) × 4 = 297.9 × 4 = 1191.6, four times the sum of all the even ordinates; (77.6 + 84.2 + 77.5) × 2 = 239.3 × 2 = 478.6, twice the sum of all the odd ordinates; then (88.5 + 478.6 + 1191.6) × 4 = 1758.7 × 4 = 7034.8 square inches, the area of that part of the section, circumscribed by the side of the vessel, and the two extreme ordinates.

Again, $\frac{88.5 \times 12}{3} = \frac{1062.0}{3} = 354$ square inches, the area of the two segments; and 354 + 7034.8 = 7388.8 square inches, the area of the whole section; then 7388.8 ÷ 231 = 31.986, the area in wine gallons.

To find the area of the second section.

Here 39.3 + 39.2 = 78.5, the sum of the extreme ordinates; (62.7 + 81.1 + 81.0 + 62.6) × 4 = 287.4 × 4 = 1149.6, four times the sum of all the even ordinates; (75.1 + 81.6 + 75.2) × 2 = 231.9 × 2 = 463.8, twice the sum of all the odd ordinates; then, (78.5 + 463.8 + 1149.6) × 4 = 1691.9 × 4 = 6767.6 square inches, the area of that part of the section, circumscribed by the side of the vessel and the two extreme ordinates.

Again, $\frac{78.5 \times 9.4}{3} = \frac{737.90}{3} = 245.966$, square inches, the area of the two segments; and 245.966 + 6767.6 = 7013.566 square inches, the area of the whole section; then 7013.566 ÷ 231 = 30.361, the area in wine gallons.

To find the area of the third section.

Here 32.5 + 32.6 = 65.1, the sum of the extreme ordinates; (59.1 + 78.5 + 78.4 + 59.2) × 4 = 275.2 × 4 = 1100.8, four times the sum of all the even ordinates; (72.7 + 79.0 + 77.2) × 2 = 228.9 × 2 = 457.8, twice the sum of all the odd ordinates; then (457.8 + 1100.8 + 65.1) × 4 = 1623.7 × 4 = 6494.8 square inches, the area of that part of the section, circumscribed by the sides of the vessel, and the two extreme ordinates.

Again, $\frac{65.1 \times 6.2}{3} = \frac{403.62}{3} = 134.54$ square inches, the area of the two segments; and 134.54 + 6494.8 = 6629.34 square inches, the area of the whole section; then 6629.34 ÷ 231 = 28.698, the area in wine gallons.

To find the area of the fourth section.

Here 25.8 + 25.7 = 51.5, the sum of the extreme ordinates; 55.8 + 75.8 + 75.7 + 55.8) × 4 = 263.1 × 4 = 1052.4, four times the sum of all the even ordinates; (69.5 + 76.4 + 69.5) × 2 = 215.4 × 2 = 430.8, twice the sum of all the odd ordinates; then (430.8 + 1052.4 + 51.5) × 4 = 1534.7 × 4 = 6138.8 square inches, the area of that part of the section, circumscribed by the sides of the vessel, and the two extreme ordinates.

Again, $\frac{51.5 \times 4}{3} = \frac{206.0}{3} = 68.66$ square inches, the area of the two segments; and 68.66 + 6138.8 = 6207.46 square inches, the area of the whole section; then 6207.46 ÷ 231 = 26.872, the area in wine gallons.

A TABLE

shewing the Method of Inching the Wash-Back, given in this Example.

Wet Inches.	Contents in Gallons.	Wet Inches.	Contents in Gallons.	Wet Inches.	Contents in Gallons.	Wet Inches.	Contents in Gallons.
Drip	15.000 31.986	12	365.221 30.361	23	695.866 28.698	34	1006.066 26 872
2	46.986 31.986	13	395.582 30.361	24	724.564 28.698	35	1032.938 26.872
3	78.972 31.986	14	425.943 30.361	25	753.262 28.698	36	1059.810 26.872
4	110.958 31.986	15	456.304 30.361	26	781.960 28 698	37	1086.682 26.872
5	142.944 31.986	16	486.665 30.361	27	810.658 28.698	38	1113.554 26.872
6	174.930 31.986	17	517.026 30.361	28	839.356 28.698	39	1140.426 26.872
7	206.916 31.986	18	547.387 30.361	29	868.054 28.698	40	1167.298 26.872
8	238.902 31.986	19	577.748 30.361	30	896.752 28.698	41	1194.170
	270.888 31.986	20	608.109 30.361	31	925.450 26.872		
	302.874 31.986	21	638.470 28.698	32	952.322 26.872		
	334.860 30 361	22	667.168 28.698	33	979.194 26.872		

SECTION VI.

THE METHOD OF GAUGING AND FIXING THE UTENSILS OF SOAP-MAKERS, STARCH-MAKERS, AND GLASS-MAKERS, AS PRACTISED IN THE EXCISE.

SOAP GAUGING.

PROBLEM.

To find the area and content of a rectangular hard-soap-frame, in pounds avoirdupois.

EXAM. 2.

Pounds.

```
  24.1 area.
   4.8 depth.
 -----
 1928
 964
------
115.68 content.
```

EXAM. 3.

Pounds.

```
  24.87 area.
    4.6 depth.
  -----
 14922
 9948
-------
114.402 content.
```

EXAM. 4.

Inches.
43 length.
14 breadth.

172
43

28.00)602(21.50, area in pounds hot.

27.14)602(22.18, area in pounds cold.

Then $21.5 \times 58 = 1247.0$, the content in pounds hot; and $22.18 \times 58 = 1286.44$, the content in pounds cold.

STARCH GAUGING.

PROBLEM I.

To gauge and fix a starch-vat in the form of a parallelopipedon.

EXAM. 2.

Bushels.	*Pounds.*
9.026 area.	225.6 5 area.
28.6 depth.	2 8.6 depth.
54 15 6	1353 9 0
722 08	18052 0
1805 2	45130
258.1436 content.	6453.5 9 0 content.

EXAM. 3.

Bushels.	*Pounds.*
10.69 8 area.	267.4 5 area.
26.4 depth.	2 6.4 depth.
42 79 2	1069 8 0
641 88	16047 0
213 9 6	53490
282.427 2 content.	7060.6 8 0 content.

EXAM. 4.

Here (225.8 × 156.6) ÷ 2828 = 35360.28 ÷ 2828 = 12.503, the area in bushels; and 12.503 × 25 = 312.575, the area in pounds, before fermentation.

Again, 35360.28 ÷ 2386 = 14.819, the area in bushels; and 14.819 × 25 = 370.475, the area in pounds, after fermentation.

PROBLEM II.

To gauge and fix a starch-vat, in the form of the frustum of a cone.

EXAM. 1.

To find the area of the second section.

Here (96.4 × 96.4) ÷ 3601 = 9292.96 ÷ 3601 = 2.580 bushels, the area before fermentation; and 9292.96 ÷ 3038 = 3.058 bushels, the area after fermentation.

To find the area of the third section.

Here (94.3 × 94.3) ÷ 3601 = 8892.49 ÷ 3601 = 2.469 bushels, the area before fermentation; and 8892.49 ÷ 3038 = 2.927 bushels, the area after fermentation.

To find the area of the fourth section.

Here (92.5 × 92.5) ÷ 3601 = 8556.25 ÷ 3601 = 2.376 bushels, the area before fermentation; and 8556.25 ÷ 3038 = 2.816 bushels, the area after fermentation.

EXAM. 2.

	Areas.		*Contents.*
rst section ...	2.677 × 10	=	26.77, First division.
cond section ..	2.580 × 10	=	25.80, Second division.
ird section ..	2.469 × 10	=	24.69, Third division.
urth section ..	2.376 × 8.6	=	20.43, Part of 4th divis.
ntent			97.69 Bushels.
			25
			488 45
			1953 8
tent			2442.25 Pounds.

EXAM. 3.

To find the area of the first section.

ere (124.3 × 124.3) ÷ 3601 = 15450.49 ÷ = 4.290 bushels, the area before fermentation; 15450.49 ÷ 3038 = 5.085 bushels, the area fermentation.

To find the area of the second section.

e (121.4 × 121.4) ÷ 3601 = 14737.96 ÷ = 4.109 bushels, the area before fermentation; 737.96 ÷ 3038 = 4.851 bushels, the area after tation.

To find the area of the third section.

(118.3 × 118.3) ÷ 3601 = 13994.89 ÷ 3.886 bushels, the area before fermentation; 94.89 ÷ 3038 = 4.606 bushels, the area after ation.

To find the area of the fourth section.

(115.5 × 115.5) ÷ 3601 = 13340.25 ÷ 3601 bushels, the area before fermentation; and ÷ 3038 = 4.391 bushels, the area after fern.

To find the area of the fifth section.

Here $(112.6 \times 112.6) \div 3601 = 12678.76 \div 3601 = 3.520$ bushels, the area before fermentation; and $12678.76 \div 3038 = 4.173$ bushels, the area after fermentation.

Note.—From the dimensions given in the question, and the areas of the different sections, found as above, we form the following Dimension Book.

DIMENSION BOOK.

A. B.'s Starch Vat, No. 2, gauged Feb. 18, 1822.

Divisions in Inches.	Depths from the Bottom.	Mean Diameters.	Areas in Bushels. Before Fermentation.	After Fermentation.
12	46	112.6	3.520	4.173
10	35	115.5	3.704	4.391
10	25	118.3	3.886	4.606
10	15	121.4	4.109	4.851
10	5	124.3	4.290	5.085

PROBLEM III.

'o gauge and fix a water-frame, in the form of a parallelopipedon.

EXAM. 1.

A TABLE,

hewing the Method of Tabulating the Water-Frame, given in this Example.

Dry Inches.	Contents in Green Starch Pounds.	Dry Inches.	Contents in Green Starch Pounds.
12	3976.740	22	1988.370
	198.837		198.837
13	3777.903	23	1789.533
	198.837		198.837
14	3579.066	24	1590.696
	198.837		198.837
15	3380.229	25	1391.859
	198.837		198.837
16	3181.392	26	1193.022
	198.837		198.837
17	2982.555	27	994.185
	198.837		198.837
18	2783.718	28	795.348
	198.837		198.837
19	2584.881	29	596.511
	198.837		198.837
20	2386.044	30	397.674
	198.837		198.837
21	2187.207	31	198.837
	198.837		198.837
22	1988.370	32	000.000

EXAM. 2.

Inches.
32.0 whole depth.
16.6 dry inches.
15.4 wet inches.

Then 198.837, the area in pounds, being multiplied by 15.4, the wet inches, we obtain 3062.0898 pounds the content required.

PROBLEM IV.

To gauge and fix a starch-box in the form of a parallelopipedon.

EXAM. 2.

Pounds.
23.3 3 area.
5.4 depth.
93 3 2
116 6 5
125.9 82 content.

EXAM. 3.

To find the area and content with the slide.

Inches.
64.4 length.
11.6 breadth.
386 4
644
Divisor 34.8)747.04(21.46lb. area.

And $21.46 \times 6.3 = 135.198$, the content in pounds.

To find the area and content without the slide.

Inches.	
64.4	length.
12.2	breadth.
128.8	
1288	
644	

Divisor 34.8)785.68(22.57 lbs. area.

And $22.57 \times 6.3 = 142.191$, the content in pounds.

GLASS GAUGING.

PROBLEM I.

To find the area and content of any circular pot, used in making glass.

EXAM. 2.

To find the area.

Inches.	
36.4	diameter.
36.4	ditto.
145 6	
2184	
1092	

Divisor 13.39)1324.96(98.95lb. the area in crown glass.

Also, $1324.96 \div 12.96 = 102.23$ lbs. the area in bottle glass.

To find the content.

Here 98.95 × 39.8 = 3938.210, the content in pounds of crown glass; and 100.23 × 39.8 = 4068.754, the content in Pounds of bottle glass.

EXAM. 3.

To find the area.

Inches.
38.7 diameter.
38.7 ditto.

```
   2709
  3096
 1161
```

Diviser 14.38)1497.69(104.15 lbs. the area in British plate glass.

Also, 1497.69 ÷ 14.32 = 104.58 lbs. the area in German sheet glass.

To find the content.

Here 104.15 × 39.8 = 4145.170, the content in pounds of British plate glass; and 104.58 × 39.8 = 4162.284, the content in pounds of German sheet glass.

PROBLEM II.

'o gauge and fix a pot in the form of a cylinder or the frustum of a cone, as practised in the Excise.

EXAM. 1.

A TABLE

hewing the Method of inching the Pot given in this Example.

Dry Inches.	Contents in C.	Q.	P.	Dry Inches.	Contents in C.	Q.	P.	Dry Inches.	Contents in C.	Q.	P.
Full	6 0	3 1	0.0 9.8	8	4 0	0 1	5.6 9.8	16	1 0	1 1	11.2 9.8
1	6 0	1 1	18.2 9.8	9	3 0	2 1	23.8 9.8	17	1 0	0 1	1.4 9.8
2	6 0	0 1	8.4 9.8	10	3 0	1 1	14.0 9.8	18	0 0	2 1	19.6 9.8
3	5 0	2 1	26.6 9.8	11	3 0	0 1	4.2 9.8	19	0 0	1 1	9.8 9.8
4	5 0	1 1	16.8 9.8	12	2 0	2 1	22.4 9.8	20	0	0	0.0
5	5 0	0 1	7.0 9.8	13	2 0	1 1	12.6 9.8				
6	4 0	2 1	25.2 9.8	14	2 0	0 1	2.8 9.8				
7	4 0	1 1	15.4 9.8	15	1 0	2 1	21.0 9.8				

A GLASS MAKER'S TABLE BOOK,

Formed from the preceding Table.

Dry inches.	Contents in C.	Q.	P.	Dry Inches.	Contents in C.	Q.	P.	Dry Inches.	Contents in C.	Q.	P.
Full	6	3	00	7	4	1	15	14	2	0	03
1	6	1	18	8	4	0	06	15	1	2	21
2	6	0	08	9	3	2	24	16	1	1	11
3	5	2	27	10	3	1	14	17	1	0	01
4	5	1	17	11	3	0	04	18	0	2	20
5	5	0	07	12	2	2	22	19	0	1	10
6	4	2	25	13	2	1	13	20	0	0	00

EXAM. 2.

First section and division.

Here 36.1 × 36.1 = 1303.21, the square of the diameter; and 1303.21 ÷ 12.96 = 100.55, the gross area. One-fifth of this area is = 20.11; then 100.55 − 20.11 = 80.44, the net area of the section; and 80.44 × 6 = 482.64 pounds, the net content of the division.

Second section and division.

Here 32.3 × 32.3 = 1043.29, the square of the diameter; and 1043.29 ÷ 12.96 = 80.50, the gross area. One-fifth of this area is = 16.10; then 80.50 − 16.10 = 64.40, the net area of the section; and 64.40 × 10 = 644.00 pounds, the net content of the division.

Third section and division.

Here 29.5 × 29.5 = 870.25, the square of the diameter; and 870.25 ÷ 12.96 = 67.14, the gross area. One-fifth of this area is = 13.42; then 67.14 − 13.42 = 53.72, the net area of the section; and 53.72 × 10 = 537.2, the net area of the section; and 53.72 × 10 = 537.20 pounds, the net content of the division.

Fourth section and division.

Here 25.5 × 25.5 = 650.25, the square of the diameter; and 650.25 ÷ 12.96 = 50.17, the gross area. One-fifth of this area is = 10.03; then 50.17 − 10.03 = 40.14, the net area of the section; and 40.14 × 7 = 2809.8 pounds, the net content of the division.

Note.—The net area of the last section is multiplied by 7, in order to make an allowance of 3 inches at the bottom of the vessel.

Areas and contents collected and reduced into hundred weights, quarters, and pounds.

Net Areas of the Sections in					Net Contents of the Divisions in				
No.	Pounds.	C.	Q.	P.	No.	Pounds.	C.	Q.	P.
1	80.44	0	2	24.44	1	482.64	4	1	6.64
2	64.40	0	2	08.40	2	644.00	5	3	0.00
3	53.72	0	1	25.72	3	537.20	4	3	5.20
4	40.14	0	1	12.14	4	280.98	2	2	0.98
Whole content of the Pot.						1944.82	17	1	12.82

A TABLE

Shewing the Method of inching the Pot, given in th Example.

Dry Inches.	Contents in C.	Q.	P.	Dry Inches.	Contents in C.	Q.	P.	Dry Inches.	Contents in C.	Q.	P.
Full	17 0	1 2	12.82 24.44	13	9 0	0 2	03.38 08.40	26 •	2 0	2 1	00.98 12.14
1	16 0	2 2	16.38 24.44	14	8 0	1 2	22.98 08.40	27	2 0	0 1	16.84 12.14
2	15 0	3 2	19.94 24.44	15	7 0	3 2	14.58 08.40	28	1 0	3 1	04.70 12.14
3	15 0	0 2	23.50 24.44	16 •	7 0	1 1	06.18 25.72	29	1 0	1 1	20.56 12.14
4	14 0	1 2	27.06 24.44	17	6 0	3 1	08.46 25.72	30	1 0	0 1	08.42 12.14
5	13 0	3 2	02.62 24.44	18	6 0	1 1	10.74 25.72	31	0 0	2 1	24.28 12.14
6 •	13 0	0 2	06.18 08.40	19	5 0	3 1	13.02 25.72	32	0 0	1 1	12.14 12.14
7	12 0	1 2	25.78 08.40	20	5 0	1 1	15.30 25.72	33	0	0	00.00
8	11 0	3 2	17.38 08.40	21	4 0	3 1	17.58 25.72				
9	11 0	1 2	08.98 08.40	22	4 0	1 1	19.86 25.72				
10	10 0	3 2	00.58 08.40	23	3 0	3 1	22.14 25.72				
11	10 0	0 2	20.18 08.40	24	3 0	1 1	24.42 25.72				
12	9 0	2 2	11.78 08.40	25	2 0	3 1	26.70 25.72				

CANDLES.

THE METHOD OF ESTIMATING THE WEIGHT OF CANDLES, BOTH BY THE PEN AND THE SLIDING RULE.

PROBLEM.

Given the number of candles in one pound, the number on one rod, and the number of rods, to find the whole weight in pounds.

EXAM. 2.

BY THE PEN.

$$\begin{array}{r} 25 \\ 28 \\ \hline 200 \\ 50 \\ \hline 12)\overline{700} \\ \hline 58\frac{1}{3} \end{array} \textit{ lbs. Ans.}$$

On A		On B.		On A.		On B.	
As 12	:	48	::	24	:	96	*Ans.*

EXAM. 3.

BY THE SLIDING RULE.

	On A.		On B.		On A.		On B.
As	16	:	30	::	32	:	60 lbs.
	18	:	34	::	36	:	68 lbs.
	Sum						128 *lbs. Ans.*

A TABLE

[illegible] the [illegible] of [illegible] the Pot, given in the Example.

Inches	Contents in C.	G.	P.	Inches	Contents in C.	G.	P.	Inches	Contents in C.	G.	P.
Full	[illegible]	[illegible]	[illegible]	13	[illegible]	[illegible]	[illegible]	26	[illegible]	[illegible]	[illegible]
1	[illegible]	[illegible]	[illegible]	14	[illegible]	[illegible]	[illegible]	27	[illegible]	[illegible]	[illegible]
2	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]	[illegible]	28	[illegible]	[illegible]	[illegible]
3	[illegible]	[illegible]	[illegible]	16	[illegible]	[illegible]	[illegible]	29	[illegible]	[illegible]	[illegible]
4	[illegible]	[illegible]	[illegible]	17	[illegible]	[illegible]	[illegible]	30	[illegible]	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	18	[illegible]	[illegible]	[illegible]	31	[illegible]	[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]	19	[illegible]	[illegible]	[illegible]	32	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	20	[illegible]	[illegible]	[illegible]	33	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]	21	[illegible]	[illegible]	[illegible]				
9	[illegible]	[illegible]	[illegible]	22	[illegible]	[illegible]	[illegible]				
10	[illegible]	[illegible]	[illegible]	23	[illegible]	[illegible]	[illegible]				
11	[illegible]	[illegible]	[illegible]	24	[illegible]	[illegible]	[illegible]				
12	[illegible]	[illegible]	[illegible]	25	[illegible]	[illegible]	[illegible]				

PROBLEM II.

To find the area and content of a cylindrical vessel, in Irish malt bushels, and liquid gallons.

EXAM. 2.

Here 82.4 × 82.4 = 6789.76, the square of the diameter; and 6789.76 ÷ 2773.1 = 2.448, the area in Irish malt bushels; also, 6789.76 ÷ 277.05 = 24.507, the area in Irish liquid gallons; then 2.448 × 52.6 = 128.7648, the content of the vessel, in Irish malt bushels; and 24.507 × 52.6 = 1289.0682, the content in Irish liquid gallons.

PROBLEM III.

To reduce Irish measure to English measure.

EXAM. 3.

Here 1.0128 × 2685 = 2719.368, English malt bushels.

EXAM. 4.

Here .9419 × 2864 = 2697.6016, English wine gallons.

EXAM. 5.

Here 56 × 32 = 1792, the number of gallons, according to the gauge in Ireland; and .7716 × 1792 = 1382.7072, the number of English ale gallons; then, 1382.7 ÷ 36 = 38 barrels, 14.7 gallons, the answer required.

PROBLEM IV.

To reduce English measure to Irish measure.

EXAM. 2.

Here 1.2959 × 2685 = 3479.4915 gallons, Irish measure.

EXAM. 3.

Here 1.0615 × 3634 = 3857.491 gallons, Irish measure.

SPECIFIC GRAVITY.

PROBLEM I.

To find the magnitude of a body from its weight.

EXAM. 2.

lb.	oz.	
186	12	gross weight.
45	9	tare of the cask.
141	3	neat weight of the spirits.

```
 oz.    lb.    oz.    cu. in.
As 922 : 141    3  ::  1728
          16            2259
         ----          -----
          849          15552
         141            8640
         ----          3456
         2259         3456
                     --------
Divisor 922)3903552(4233 cubic inches.
```

And 4233 ÷ 231 = 18.32, the content in wine gallons.

EXAM. 3.

	oz.		lb. oz.		cu. in.		cu. in.
As	1063	:	12 10	::	1728	:	15 24, the content

'equired.

EXAM. 4.

	oz.		lb. oz.		cu. in.		cu. in.
As	925	:	56 9	::	1728	:	1690, the content re-

uired.

PROBLEM II.

To find the weight of a body from its magnitude.

EXAM. 2.

	cu. in.		gal. cu. in.		oz.		oz.
	1728	:	36 × 282	::	1034	:	6074 = 3 cwt.

r. 15 lb. 10 oz. the weight required.

EXAM. 3.

'ere 12 × 12 × 63 = 144 × 63 = 9072 cubic feet, solidity of the stone; then, as 1 ft. : 9072 ft. :: 2700 24494400 oz.; and 24494400 ÷ 35840 = 683.4375 which is nearly equal to the burthen of an East ship.

The Method of Estimating

THE TONNAGE OF SHIPS.

CASE I.

When the vessel is laid dry.

EXAM. 2.

	Feet.	
Gross length	108.75	of the keel.
$29.5 \times \frac{3}{5} =$	17.70	the deduction.
True length	91.05	difference.
	29.5	breadth of the beam.
	45525	
	81945	
	18210	
	2685.975	first product.
	14.75	half the breadth.
	1342987 5	
	1880182 5	
	10743900	
	2685975	
	39618.13125	second product.

Then $39618.13125 \div 94 = 421.46948$ tons, the burthen required.

CASE II.

When the vessel is afloat.

EXAMPLE.

Here $150.75 \times 50.5 \times 25.25 = 7612.875 \times 25.25 = 192225.09375$; and $192225.09375 \div 94 = 2044.9478$ tons, the burthen required.

RULES FOR REDUCING SPIRITS OF ANY GIVEN STRENGTH, TO ANY REQUIRED STRENGTH.

PRELIMINARY OBSERVATIONS.

By Act of Parliament all spirits imported from reland or Scotland into England, of different strengths ıd different quantities, are reduced to sēven per ent. over hydrometer proof.

It is also enacted that all spirits made in Ireland, ıd exported from thence into England, at a strength ceeding 25 per cent. above proof, are seizable, and y spirits made in England or Scotland, and exported om thence into Ireland exceeding the above strength e also seizable.

No foreign spirits shall be imported and warehoused, exported in any boat less than 70 tons burthen, in any cask of less than 100 gallons; unless it pears to be occasioned from the leakage of a cask other accident, and without intention of fraud.

On the landing of such spirits, it must be reduced 7 per cent. over proof, but it shall not be forfeited any excess or deficiency of strength of any such irits being not more than 3 per cent. above or bev the strength of such spirits specified in the pert.

British plantation rum may be imported and wareused in casks of 60 gallons, or upwards.

Foreign wine not to be imported and warehoused casks less than 45 gallons each, or in vessels less n 60 tons burden.

All British spirits of a greater strength than 43 per t. above proof by Sykes's Hydrometer, shall be med spirits of wine.

Spirits manufactured and imported from Scotland England, of a greater strength than 7 per cent. ve proof, and not exceeding 10 per cent. above of, shall not be forfeited, but be charged with a ther duty proportioned to their surplus strength.

t would be quite superfluous to treat upon the use

and description of the Hydrometer here, as there is always a proper book of instructions given with the instrument.

1. *To calculate any given quantity of proof spirits in any other given quantity, either above, or below Hydrometer proof.*

RULE I.

When the spirits are above proof.

Multiply the given quantity by the rate per cent. above proof, divide the product by 100, add the quotient to the given quantity, and the sum will be the quantity of proof spirits required.

RULE II.

When the spirits are under proof.

Multiply the given quantity by the rate per cent. under proof, divide the product by 100, subtract the quotient from the given quantity, and the remainder will be the quantity of proof spirits required.

EXAMPLES.

1. If a puncheon of rum contain 124 gallons, at the strength of 15 per cent. over hydrometer proof, how many gallons of proof spirits does it contain.

124 gauged quantity.
15 rate per cent.
620
124
Divisor 100)1860
18.6 gallons over proof.
124 gauged quantity.
142.6 proof spirits required.

2 How many gallons of proof spirits does the foregoing puncheon contain, at the strength of 15 per cent. under proof.

$$\begin{array}{rl} 124 & \text{gauged quantity.} \\ 15 & \text{rate per cent.} \\ \hline 620 & \\ 124 & \\ \hline \text{Divisor } 100)1860 & \\ \hline 18.6 & \text{gallons under proof.} \end{array}$$

Then $124 - 18.6 = 105.4$ gallons of proof spirits.

3. Admit a cask of spirits to contain 132 gallons, it the strength of 20.5 per cent. over proof, required he number of gallons when reduced to proof spirits.

Here, $(132 \times 20.5) \div 100 = 2706 \div 100 = 27.06$ gallons above proof, then $132 + 27.06 = 159.06$ gallons of proof spirits required.

Note.—Proof spirits may be reduced to either above or below proof, by the preceding rules.

. *To reduce spirits of any given strength above 7 per cent. O. P., to the strength of 7 per cent. above Hydrometer proof.*

RULE.

As 107 is to 100 with the given rate per cent., so the given quantity, to the quantity required.

EXAMPLE.

A merchant imports 21010 gallons of spirits, at the rength of 20.5 per cent. above proof; how many allons when reduced to 7 per cent. above proof.

As $107 : 120.5 :: 21010 : 23660.79$ gallons of oof spirits required.

3. *To calculate whether the produce of any distillation exceeds the Distiller's credit of 19 gallons of spirits at 8 per cent. above proof, to 100 gallons of wort or wash distilled.*

RULE.

Reduce the whole quantity distilled, at different degrees of strength, to 8 per cent. above proof, divide the number of gallons of wash by 100, multiply the quotient by 19, and the product will be the number of gallons he is allowed to make, the difference between the quantity reduced to 8 per cent. above proof and the last number will be the excess or defect.

EXAMPLE.

Suppose a distiller made 2200 gallons of spirits at 8.5 per cent. O. P., 1300 gallons at 7 per cent. O. P., and 600 gallons of feints at 68 per cent. U. P.; from 18000 gallons of wash, required the excess or defect of his quantity allowed by act of Parliament.

$$\begin{aligned} 2200 \times 108.5 &= 238700 \\ 1300 \times 107 &= 139100 \\ 600 \times 32 &= \underline{\ 19200} \end{aligned}$$

Divisor $108)397000(3675.92$ gallons, made at 8 per cent. O. P.

Then $\frac{18000}{100} \times 19 = 180 \times 19 = 3420$ gallons, his wort is allowed to make;

And $3675.92 - 3420 = 255.92$ gallons above his credit, which must be charged with 9s. 2¼d. per gallon, in addition to all other duties. *(See the following note.)*

Note.—If any distiller of spirits in England, shall in the year ending on the 5th of July, in every year, make or produce from wort or wash made from malt, corn, grain, or tilts, or any mixture therewith, any quantity of spirits exceeding upon the average of his work in such year, the proportion of 19 gallons of spirits at

the strength of 8 per cent. above proof, for every 100 gallons of wort, or wash so distilled, he shall, in lieu of any penalty for the excess, pay duty for all spirits exceeding the proportion upon such average, after the rate of 9s. 2½d. for every gallon of such excess, over and above all other duties.

4. *To estimate a Rectifier's stock.*

RULE.

Reduce the whole of his stock at different degrees of strength (spirits of wine excepted) to 7 per cent. O. P., increase his stock at that strength by one-half thereof, also by twice the number of gallons of spirits of wine, and the sum will be the quantity of spirits for the rectifier's credit.

EXAMPLE.

A rectifier has 300 gallons of spirits at 7.5 per cent. O. P., 410 gallons at 15 per cent. O. P., 900 gallons at 25 per cent. O. P.; and 200 gallons of spirits of wine; what is his credit?

$$
\begin{array}{rl}
\text{Here } 300 \times 107.5 = & 32250 \\
410 \times 115 \ \ = & 47150 \\
900 \times 125 \ \ = & \underline{112500} \\
\text{Divisor } 107)\overline{191900}(& 1793.45 \text{ gallons at 7 per cent. O. P.} \\
& 896.72 \text{ one-half.} \\
200 \times 2 = & \underline{400} \quad \text{spirits of wine.} \\
\text{His credit} & 3090.17 \text{ gallons.}
\end{array}
$$

Note.—Any surplus found on balancing a rectifier's stock, is reduced to 7 per cent. above proof, by multiplying the given quantity by 2, and dividing the product by 3.

DUTIES.

An Alphabetical List of the DUTIES, ALLOWANCES, DRAWBACKS, *and* LICENCES, *charged on different Articles of the Growth, Produce, or Manufacture of* GREAT BRITAIN *or* IRELAND.

AUCTIONS.

	£	s.	d.
For every twenty shillings of the purchase-money, payable in virtue of any sale by auction, for the growers or first purchasers of any sheep or lamb's wool, the growth or produce of any part of the United Kingdom ————	0	0	2
For every twenty shillings of the purchase-money, payable in virtue of any sale at auction of estates, [illegible], [illegible] and [illegible], interest in the public funds, [illegible] ————	0	0	1
For every twenty shillings of the purchase-money, payable in virtue of any sale at auction of furniture, [illegible], [illegible], [illegible], horses, and carriages, and all other goods and chattels whatsoever, excepting [illegible] ————	0	1	3

For [illegible] in the exemption of duty, see Huie's Excise [illegible].

BEER.

	£	s.	d.
For every barrel of strong beer or ale, not being two-penny ale, of above sixteen shillings the barrel, brewed by any common brewer, victualler, or other person who shall sell, or tap out beer or ale	0	10	0
For every barrel of table beer of sixteen shillings the barrel, or under, brewed by any common brewer or other person who shall sell, or tap out beer or ale ...	0	2	3
For every barrel consisting of thirty-six gallons of two-penny ale, described in the seventh article of the treaty of Union with Scotland	0	4	2
For every barrel containing thirty-six gallons, English beer measure, of Irish beer, or ale, or mum, imported into Great Britain from Ireland	0	16	2
For every barrel containing thirty-two gallons, wine measure, of spruce beer, and all other kinds of beer, or ale, and for every such barrel of mum, imported from beyond the seas into Great Britain, not being Irish beer, ale, or mum, imported from Ireland......	2	0	0

By an act of 43 G. 3, c. 69, every common brewer is allowed 3 barrels in every 36, or $\frac{1}{12}$ part; whence 10s. — $\frac{1}{12}$ = 9s. 10d. the duty which must be charged upon every barrel of strong beer or ale; the same may be observed with table beer.

BRICKS AND TILES.

	£.	s.	d.
For every thousand of bricks, not exceeding any of the following dimensions, that is to say, ten inches long, 3 inches thick, and five inches wide	0	5	10
For every thousand of bricks, exceeding any of the foregoing dimensions	0	10	0
For every thousand of bricks, smoothed or polished, not exceeding ten inches long, three inches thick, and five inches wide	0	12	10
For every hundred of bricks, smoothed or polished, exceeding any of the foregoing dimensions	0	2	5
For every thousand of plain tiles	0	5	8
For every thousand of pan or ridge tiles	0	12	10
For every hundred of paving tiles, not exceeding ten inches square	0	2	5
For every hundred of paving tiles, exceeding ten inches square	0	4	10
For every thousand tiles, other than such as are before enumerated or described	0	4	10

All Irish bricks and tiles of the above denomination, imported into Great Britain from Ireland, are subject to the same duties.

CANDLES.

For every pound weight of tallow candles, and other candles whatsoever, except wax and spermaceti candles	0	0	1
For every pound weight of wax or spermaceti candles	0	0	3½

All Irish candles imported into Great Britain from Ireland, are subject to the same duties.

For the method of charging the duties on exciseable commodities by the pound weight avoirdupois, see the remark, page 78, Table 12, in the Gauging.

COFFEE.

For every pound weight of coffee or cocoa nuts, the produce of any British colony or plantation in America, or the West Indies in Africa, imported into Great Britain	0	1	0
For every pound weight of coffee, or cocoa nuts, imported into Great Britain by the East India Company	0	1	6
For every pound weight of all other coffee or cocoa nuts, imported into Great Britain..	0	2	6

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	3 [illegible]
[illegible]	[illegible]
[illegible]	1 [illegible] 0
[illegible]	3 13 6
[illegible]	0 [illegible]
[illegible]	0 [illegible]
[illegible]	1 [illegible] 0

[illegible] from Ireland.

[illegible] of Irish plate [illegible]	0 2 9
[illegible] flint glass and [illegible]	4 18 0
[illegible] stained, or paste [illegible]	4 7 0
[illegible] spread window glass [illegible] by the name of broad glass	1 10 0

	£.	s.	d.
For every hundred weight of all other Irish window glass, not being spread glass, whether flashed or otherwise manufactured, and commonly called crown glass or German sheet glass	3	13	6
For every dozen of reputed quart bottles of common green glass	0	1	6
For every hundred weight of vessels made use of in chemical laboratories, and of garden glasses, and of all other vessels or utensils of common bottle metal, manufactured in Ireland, common bottles excepted	0	4	0½
For every hundred weight of any sort or species of Irish glass, not herein before enumerated or described	3	3	0
For every hundred weight of plate glass and all other glass manufactures, imported from beyond the seas, not being flasks in which wine or oil shall or may be imported, nor foreign green glass bottles, nor Irish glass or glass manufactures	6	6	0

HIDES AND SKINS TANNED.

	£.	s.	d.
For every pound weight of hides of what kind soever, and of calve skins, kips, hog skins, dogs skins, and seal skins, which shall be tanned, and of sheep skins and lamb skins, which shall be tanned for gloves and bazils	0	0	1½
For every dozen of goat skins tanned with shomack, or otherwise, to resemble Spanish leather	0	4	0
For every dozen of sheep skins tanned for roans, being after the nature of Spanish leather	0	2	3
For every pound weight of all other skins, and of all parts and pieces of hides and skins, not herein before enumerated	0	0	6

TAWED.

	£.	s.	d.
For all hides of horses, mares, and geldings, which shall be dressed in alum and salt, or meal, or otherwise tawed; for every such hide	0	1	6
For all hides of steers, cows, or any other hides of what kind soever, those of horses, mares, and geldings excepted; for every such hide..................	0	3	0
For every pound weight of all calve skins, kips, and seal skins	0	0	1½
For every dozen of slink calve skins, dressed or tawed with the hair on	0	3	0

	£	s.	d.
For every dozen of slink calve skins, dressed or tawed without hair, and for every dozen of dogs skins, and seal skins, tawed or dressed	0	1	0
For every pound weight of buck and doe skins	0	0	6
For every dozen of goat skins and of beaver skins	0	2	0
For every pound weight of sheep and lamb skins	0	0	1½
For every pound weight of all other skins, and of all parts and pieces of hides and skins which shall be so dressed or tawed, not herein before enumerated	0	0	6

DRESSED IN OIL.

	£	s.	d.
For every pound weight of all buck, deer, and elk-skins dressed in oil	0	1	0
For every pound weight of all sheep and lamb skins	0	0	3
For every pound weight of all other skins, and of all hides, and parts and pieces of hides and skins which shall be dressed in oil	0	0	6
For every dozen of vellum	0	3	6
For every dozen of parchment	0	1	9

Hides and skins tanned, tawed, or dressed in oil, in Ireland, and imported from thence into Great Britain, unmanufactured, are subject to the same duties.

Imported from Ireland, manufactured.

	£	s.	d.
For every dozen of Irish vellum	0	3	6
For every dozen of Irish parchment	0	1	9
For every pound weight of tanned leather, manufactured and actually made into goods and wares in Ireland	0	0	1½
For every pound weight of Irish made boots and shoes, and gloves and other manufactures, made of tawed or dressed leather	0	0	1
For every pound weight of all buck, deer, and elk-skins, dressed in oil, and manufactured into goods and wares in Ireland	0	2	0
For every pound weight of all sheep and lamb-skins, dressed in oil, and manufactured into goods or wares in Ireland	0	0	3

HOPS.

	£	s.	d.
For every pound weight, of hops	0	0	2
For every pound weight, of Irish hops, imported from Ireland into Great Britain	0	0	2

MALT.

	£.	s.	d.
For every bushel of malt which shall be made in England from barley, or any other corn or grain	0	2	6
For every bushel of malt made in Scotland from barley or any other corn or grain	0	2	6
For every bushel of malt which shall be brought from Scotland into England without a certificate	0	2	6
For every bushel of malt unground, made in Ireland, and imported into England	0	3	6

MEAD OR METHEGLIN.

	£.	s.	d.
For every gallon of mead or metheglin, which shall be made for sale	0	4	6
For every gallon of mead or metheglin imported into Great Britain, not being Irish metheglin or mead, imported from Ireland	0	5	0
For every gallon of metheglin or mead imported from Ireland	0	1	6

PAPER.

	£.	s.	d.
For every pound weight, of paper of the first class or denomination, that is to say, all paper other than brown paper made of old ropes or cordage only, without separating or extracting the pitch or tar, or any part therefrom, and not being glazed paper for clothiers and hot presses, or sheathing paper, or button-paper, or button-board	0	0	3
For every pound weight, of paper of the second class or denomination, that is to say, all brown paper made of old ropes or cordage only, as before specified	0	0	1½
For every hundred weight of glazed paper for clothiers and hot-pressers, and of mill-boards and scale-boards	1	1	0
For every hundred weight of sheathing paper, button-paper, and button-board	1	1	0
For every hundred weight of paste-board made from paper wholly of the second class	0	14	0
For every hundred weight of paste board made wholly or in part from paper, mill-board, button-board, button-paper, glazed paper, or sheathing paper, other than paper of the second class or denomination	1	8	0

All glazed, or other paper couched, or pressed together, without the use of paste, with any sheet of paper of the same or any other class materials, is deemed subject, and liable, to the same duty as the first class paper.

Irish paper of all the above denominations, imported into Great Britain from Ireland, are subject to the same duties.

	£. s. d.
For every pound weight of books whether bound or unbound, and of maps, or prints imported into Great Britain from Ireland	0 0 3

PRINTED GOODS.

For every yard square of paper which shall be printed, painted, or stained to serve for hangings or other uses	0 0 1
For every yard square of Irish printed, painted, or stained papers, to serve for hangings or other uses imported into Great Britain from Ireland	0 0 [illegible]
For every yard in length, reckoning yard wide, of foreign calico, and of foreign muslin, printed, stained, painted, or dyed, except such as shall be dyed throughout of one colour only	0 0 [illegible]
For every yard in length, reckoning yard wide, of all linens, and of stuffs wholly made of cotton wool, commonly called British manufactory, and of British muslins, and of all fustians, velvets, velverets, dimities, and other figured stuffs, made of cotton and other materials mixed, or wholly made of cotton wool wove in Great Britain, and of all other stuffs whatsoever, which shall be printed, stained, painted, or dyed, except such as shall be dyed throughout of one colour only, and stuffs made of woollen, or whereof the greatest part in value shall be woollen	0 0 [illegible]
For every yard square of silks of whatsoever kind, printed, painted, stained, or dyed, except such as shall be dyed throughout of one colour only	0 [illegible]

Irish printed goods imported into Great Britain from Irel[illegible] subject to the same duties.

All calicoes printed, painted, or stained, shall pay as yar[illegible] if within, or not exceeding, one-eighth of a yard.

All ribbands and silks, printed, painted, or stained, thou[illegible] than half a yard in breadth, are declared to be within the [illegible]ing.

PEPPER.

For every pound weight of all pepper, not being cayenne, long pepper, or Guinea pepper	0 [illegible]

SALT.

For every bushel of salt, or rock salt, made at any salt work in England	0 [illegible]

Sixty-five pounds weight of rock salt shall be deemed a bushel f rock salt; and of every other kind of salt, not being rock ılt, fifty-six pounds weight shall be deemed a bushel.

	£.	s.	d.
or every ton of all mineral alkali called soda	1	10	0
or every bushel of foreign salt imported into Great Britain, not being Irish salt imported from Ireland	0	2	3
or every bushel of rock salt imported from Ireland into Great Britain, or from Scotland into England ...	0	2	0
or every bushel of coarse and impure rock salt delivered from any salt pit, mine, or warehouse, in lumps of not less than 20 lbs. weight, for the purpose of feeding, or mixing with the food of sheep and cattle, or steeping seed, or preserving hay, or being employed as manure for land	0	0	6
r every bushel of rock salt exported to parts beyond the seas, other than Ireland	0	0	1
r every hundred weight of salted beef, or pork or bacon, or other flesh brought from Scotland to England	0	0	6

SOAP.

every pound weight of hard cake soap, or ball oap	0	0	3
every pound weight of soft soap	0	0	1½

BRITISH SPIRITS.

every gallon of wort or wash, brewed or made om any malt, corn, grain, or tilts, or any mixture ıerewith, for extracting spirits in England for home onsumption	0	2	0
every gallon of cyder, or perry, or any sort of ritish materials, except such as are before mened	0	1	3
every gallon of wort or wash brewed from melasses sugar, or any mixture therewith, for extracting irits for home consumption	0	2	6
every gallon of foreign refused wine, or cyder or y other foreign materials, except melasses and sugar	0	3	6
every gallon of spirits extracted in England for me consumption, by any distiller from any wort wash, whereof the duty of two shillings for every llon shall not have been charged and paid, comted at the strength of seven per centum above oof	0	1	3½

	£.	s.	d.
For every gallon of spirits extracted in England for home consumption, from any wort or wash, made from melasses or sugar, or any mixture therewith, over and above the proportion of twenty one gallons of such spirits, the whole being computed at 8 per centum above hydrometer proof, for every 100 gallons of such wort or wash so distilled into spirits	0	11	0

MAIDSTONE DISTILLERY.

	£.	s.	d.
For every 120 gallons of wash produced by Sir William Bishop and George Bishop, at Maidstone, from a weight of malt or other grain including the bran, and not exceeding 112 pounds	2	17	4
For every gallon of wash in the possession of the said Sir William and George Bishop at any time when 30 gallons of any wash, so in their possession, shall be found to produce more than 2½ gallons of spirits at the strength of 1 in 6 under hydrometer proof ...	0	1	11

Imported into Scotland from England.

	£.	s.	d.
For every gallon of spirits extracted in England and imported into Scotland, of a strength not exceeding seven per centum above hydrometer proof, and so in proportion of any higher degree of strength, not exceeding the strength of ten per cent. above hydrometer proof	0	4	9½

Imported into England from Scotland.

	£.	s.	d.
For every gallon of spirits extracted in Scotland, and imported from thence into England, of a strength not exceeding seven per cent. above hydrometer proof, and so in proportion for any higher degree of strength not exceeding ten per cent. above hydrometer proof	0	10	6
For every gallon of wort or wash brewed or made from malt, corn, grain, or tilts, or any mixture therewith, for extracting spirits for home consumption in Scotland	0	0	7½
For every gallon of cyder or perry, or any other liquor, made from any sort of British materials, except such as before-mentioned	0	0	9
For every gallon of wort or wash made from melasses, or sugar, or any mixture therewith, for extracting spirits for consumption in Scotland	0	0	9
For every gallon of wash made from foreign refused wine or foreign cyder, or any other foreign materials, except melasses and sugar	0	0	9

	£.	s.	d.
'or every gallon of spirits of the strength of seven per centum above proof, made from malt, corn, grain, or tilts, or any mixture therewith, for extracting spirits for home consumption	0	0	8½
ɔr every gallon of spirits of the strength of one to ten over hydrometer proof, made from cyder, or perry, or any other liquor made from any sort of British materials, or any mixture therewith, for consumption in Scotland	0	2	1½
ır every gallon of spirits of the strength aforesaid, made from melasses or sugar, or any mixture therewith, for consumption in Scotland	0	5	7½
›r every gallon of spirits of the strength aforesaid, made from foreign or refused wine, or foreign cyder or wash, or wash prepared from any other foreign materials, for consumption in Scotland	0	6	9

Irish Spirits imported into Great Britain.

· every gallon of spirits extracted in Ireland, and nported from thence into England, at a strength ot exceeding seven per centum above hydrometer roof, and so in proportion for any greater degree f strength not exceeding twenty one per centum bove hydrometer proof	0	11	0
every gallon of spirits extracted in Ireland, and nported from thence into Scotland, at a strength ot exceeding seven per centum above hydrometer roof, and so in proportion for any greater degree ' strength not exceeding twenty-one per centum ›ove hydrometer proof	0	5	3½
every gallon of spirits extracted in Ireland, and ıported from thence into Scotland, and from :otland into England, at a strength not exceeding ven per centum above hydrometer proof	0	4	6

'oreign Spirits imported into Great Britain.

ɜvery gallon of single rum, spirits, or aqua vitæ, ɜ produce of the British colonies in America	0	10	4½
ɜvery gallon of rum, spirits, or aqua vitæ, above ɔof, the produce of the Brittsh colonies in America	1	0	0
:very gallon of single rum, spirits, or aqua vitæ, ported by the East India Company	0	15	5½¾
:very gallon of rum, spirits, or aqua vitæ above ›of, imported by the East India Company.........	1	7	3¼¾
:very gallon of single brandy, spirits, aquæ vitæ, strong waters of any kind, other than such rum, rits, or aqua vitæ as aforesaid, not being Irish :its imported from Ireland	0	17	0⅝

	£.	s.	d.
For every gallon of brandy, spirits, aqua vitæ or strong waters, above proof other than such brandy, rum, spirits, or aqua vitæ as aforesaid, imported into Great Britain, not being Irish spirits imported from Ireland	1	10	6

STARCH.

	£.	s.	d.
For every pound weight of starch of what kind soever, made in Great Britain	0	0	3
For every pound weight of Irish starch, or hair powder, imported from Ireland into Great Britain	0	0	3

STONE BOTTLES.

	£.	s.	d.
For every hundred weight of stone bottles, not exceeding two quarts measure, made in Great Britain	0	5	0
For every hundred weight of stone bottles, not exceeding two quarts measure, made in Ireland, and imported from thence into Great Britain	0	5	0
For every hundred weight of stone bottles, not exceeding two quarts measure, imported from any other place	0	5	0

SWEETS.

	£.	s.	d.
For every barrel of liquor made for sale, by infusion, fermentation, or otherwise, from fruit or sugar, or from fruit or sugar mixed with any other ingredients or materials, commonly called sweets, or made wines	2	9	0
For every barrel consisting of thirty-one gallons and a half English wine measure, of Irish sweets or other Irish liquor, made as aforesaid, commonly called sweets or made wines, imported from Ireland into Great Britain	2	9	0

TEA.

	£.	s.	d.
For all tea sold by the East India company, at or under two shillings per pound weight, to be computed upon the gross prices at which such tea shall be sold	96 per centum	0	0
For all tea sold by the East India company above two shillings per pound weight to be computed in like manner.	100 per centum	0	0

TOBACCO.

	£.	s.	d.
'or every pound weight of tobacco, the growth or production of his Majesty's colonies, plantations, or territories in America, or the United States of America, or any of the territories of the Emperor of Russia, or of the Ottoman, or Turkish empire, or from the East India Company, imported into Great Britain	0	3	0
'or every pound weight of tobacco, the growth or manufacture of the plantations or dominions of Spain or Portugal, imported into Great Britain ...	0	5	0
or every pound weight of all snuff (not being Irish snuff) imported into Great Britain	0	5	0

'obacco and Snuff imported into Great Britain from Ireland.

r every pound weight of unmanufactured tobacco, the growth or produce of Ireland, and of all Irish manufactured tobacco, imported into Great Britain	0	4	0
r every pound weight of Irish manufactured rappee snuff	0	3	6
r every pound weight of Irish manufactured brown Scotch snuff	0	3	4
every pound weight of Irish manufactured Scotch nuff, and of all other Irish manufactured snuff not efore particularly mentioned	0	4	9

VINEGAR.

every gallon, wine measure, of vinegar or acetous cid, or liquors prepared or preparing for vinegar, r acetous acid imported into Great Britain from reign parts	0	1	0
every gallon, wine measure, of vinegar or acetous id, or liquors prepared or preparing for vinegar acetous acid, made in Ireland, and imported from ence into Great Britain	0	0	4
every gallon, wine measure, of vinegar or acetous id, or liquors prepared or preparing for vinegar acetous acid made for sale in Great Britain	0	0	4

WINE.

every tun of French wine imported into Great itain	78	4	6
every tun of wine, the produce of his Majesty's tlements of the Cape of Good Hope, or of the ritories or dependencies thereof, imported into eat Britain	17	10	0

	£	s.	d.
For every tun of all other wines, imported into Great Britain	52	10	0

WIRE.

For every ounce troy of gilt wire	0	1	8
For every ounce troy of silver wire	0	1	2

Irish gilt and silver wire imported into Great Britain from Ireland are subjected to the same duties.

For every pound weight of Irish gold thread, gold lace, or gold fringe, made of plate wire, spun upon silk	0	15	4
For every pound weight of Irish silver thread, silver lace, or silver fringe, made of plate wire, spun upon silk	0	11	6

ALLOWANCES AND DRAWBACKS FOR HOME CONSUMPTION, OR MANUFACTURE.

PAPER.

For all glazed or other press-papers, consumed by clothiers and hot-presses in pressing of woollen cloths and stuffs.	The whole duty.
For all paper of the first class or denomination, used in the printing of any books in the Latin, Greek, Oriental, or Northern languages, or in the printing of bibles, testaments, psalm books, books of common prayer, and the confession of faith, and the larger and shorter catechism, within the two universities of Oxford and Cambridge, or within the universities of Scotland, by permission of the vice-chancellors and principals of the same respectively, or by the King's printers in England and Scotland respectively, in the printing of bibles, testaments, psalm books, books of common prayer of the Church of England, the book commonly called or known in Scotland by the name of the confession of faith, or the larger or shorter catechism of the Church of Scotland	The whole duty.

SOAP.

	£.	s.	d
or every pound weight of hard cake soap, or ball soap, consumed in making any cloths, serges, kerseys, bays, stockings, or other manufactures of sheep or lambs wool only, or manufactures whereof the greatest part of the value of the materials shall be wool, or in the finishing the said manufactures, or preparing the wool for the same.	0	0	2¼
ır every pound weight of soft soap, which shall be so consumed	0	0	1
ır every pound weight of hard cake soap, or ball soap, consumed in the whitening of new linen in the piece, for sale	0	0	2
r every pound weight of soft soap so consumed	0	0	0¾
r every pound weight of hard soap, consumed in preparing and finishing any manufactures from flax or cotton for sale, except such as shall be used in whitening new linen in the piece, for sale	0	0	1½
r every pound weight of soft soap, consumed as foresaid	0	0	0¾

STARCH.

	£.	s.	d
: every pound weight of starch used in preparing ınd finishing any manufacturers from flax or :otton for sale, except such as shall be consumed n finishing new linen in the piece, for sale	0	0	1½
' every pound weight of starch used in finishing ıew linen in the piece, for sale	0	0	3

WINE.

' all sorts of wines shipped for the use of admirals, aptains, or other commissioned officers of his Aajesty's navy, and for the use of commissioned fficers of the royal marines, or of persons acting as uch, for their actual consumption on board such f his Majesty's ships as they shall serve in, at uch ports, and in such quantities, and in such ıanner, as is provided by law	All the duties.

DRAWBACKS PAYABLE ON BRITISH GOODS,

For which all the Duties imposed in respect thereof shall have been duly paid, and which shall be duly exported to foreign parts as merchandize.

BEER.

	£. s. d.
For every barrel of strong beer or ale, above sixteen shillings per barrel, not being two-penny ale, described in the seventh article of the treaty of union with Scotland	0 14 2

The drawbacks on exportation to Ireland are the same as the countervailing duties payable upon commodities imported from thence into Great Britain.

BRICKS AND TILES.

For all the brick and tiles .... All the duties.

CANDLES.

For all candles .... All the duties.

CHOCOLATE.

For every pound weight of chocolate 0 2 0

CYDER AND PERRY.

For every hogshead of cyder or perry .... All the duties.

GLASS.

For every hundred weight of unground or unpolished plate glass, which shall be exported in rectangular plates of perfect glass, of the dimensions of six inches in length and four inches in breadth at the least, and of the thickness of one quarter of an inch, and not more than half an inch	4 18 0
For every square foot of plate glass	0 6 6½

	£.	s.	d.
For every hundred weight of flint glass wares, vessels, or utensils, or of phial glass wares, vessels, or utensils, which shall be exported to Ireland, Guernsey, Jersey, Alderney, Sark, or Man	4	18	0
'or every hundred weight of the above species of glass exported beyond the seas, other than the above places	6	3	0
or every hundred weight of spread window glass ...	1	10	0
or every hundred weight of window glass, not being spread glass, whether flashed or otherwise, commonly called crown, or German sheet glass, which shall be exported in whole tables, half tables, or quarter tables	3	13	6
›r every hundred weight of panes of window glass, not being spread glass, whether flashed or otherwise, commonly called crown, or German sheet glass, which shall be exported beyond the seas, other than Ireland, Guernsey, Jersey, Alderney, Sark, or Man, such panes being in rectangular igures, not less than six inches in length by four nches in breadth	4	18	0
r every hundred weight of common bottles, not being phials, and of vessels made use of in chemical laboratories, and of garden glasses, and of ll other vessels of common bottle metal.	0	8	1

'he drawbacks on glass exported to Ireland, are the same as duty on Irish glass imported.

HIDES.

every pound weight of all hides and skins, and parts and eces of hides and skins, tanned or tawed, two-thirds of the reective duty paid.

very pound weight of all hides and skins, and rts and pieces of hides and skins, tanned and rried	0	0	2
very pound weight of all leather tanned or taw-, and which shall be manufactured and made o boots, shoes, saddles, or gloves	0	0	3
every pound weight of all leather tanned or ed, and which shall be manufactured and made o goods and wares, other than boots, shoes, dles, or gloves	0	0	2
ll goat skins tanned with sumack, or otherwise, esemble Spanish leather, and all sheep skins ned for roans, being after the nature of Spanish her	the whole of the duty.		
ery pound weight of boots, or shoes, the upperhers, vamps, and boot-legs of which are made Morocco, Spanish leather, or kid skins	0	0	4

	£. s. d.
For every pound weight of all buck, deer, or elk skins dressed in oil, whether manufactured and made into goods and wares or not	0 1 0
For all other hides and skins and parts and pieces of other hides and skins dressed in oil	the whole of the duty.
For every pound weight of all other hides and skins dressed in oil, and which shall be manufactured into goods and wares, except sheep and lamb skins dressed in oil, and made into goods and wares other than gloves	0 0 6
For every pound weight of sheep and lamb skins dressed in oil, and which shall be manufactured into goods and wares other than gloves	0 0 4

HOPS.

For every pound weight of hops exported to the Isle of Man	the whole duty.

The drawback for hops exported to Ireland, is the same as the duty on Irish hops imported.

MALT.

For every bushel of malt made in England, and exported from thence to Ireland	0 2 6
For every bushel of malt made in Scotland, and imported from thence to Ireland	0 2 6

PAPER.

For all paper which shall be duly exported as merchandize to foreign parts ..,......	All the duties.

All brown paper, made of old ropes or cordage only, without separating or extracting the pitch or tar therefrom, and without any mixture of other materials therewith, shall be deemed paper of the second class or denomination; and all other paper, glazed paper for clothiers and hot-pressers excepted, shall be deemed paper of the first class or denomination.

PRINTED GOODS.

For every yard square of paper printed, painted, or stained, for hangings or other uses; and for all linens, stuffs, fustians, velvets, velverets, dimities, figured stuffs wholly made of cotton wool, calicoes and muslins; and for all silks printed stained, painted, or dyed	All the duties.

SOAP.

For every pound weight of hard cake soap, or ball soap, or of soft soap	All the duties.

SPIRITS.

	£. s. d.
For every gallon of spirits, extracted from corn or grain in England, and exported to Ireland, at the strength of seven per centum above hydrometer proof, and so in proportion for any greater degree of strength, not exceeding that of twenty one per cent. above hydrometer proof	0 0 6

STARCH.

or every pound weight of starch	The whole duty.

STONE BOTTLES.

or every hundred weight of stone bottles	0 5 0

TEA.

or all teas exported from the warehouses in which the same shall have been lodged upon the importation thereof to Ireland, his Majesty's plantations, or settlements in America, the United States of America, the Islands of Jersey or Guernsey, Gibraltar, or any port or place on the continent of Europe, where there shall be a British Consul resident, or to Africa	All the duties.

TOBACCO.

r every pound weight of shag, roll, or carrot tobacco, manufactured at any of the ports of Great Britain, into which tobacco may be lawfully imported, or within two miles thereof, from tobacco for which the duties shall have been paid and exported by the manufacturer thereof, from such ports o foreign parts	0 3 6

VINEGAR.

every gallon, wine measure, of vinegar or acetous cid, not being under proof, which shall be brewed, r made in Great Britain for sale, or made in Ireland, and imported from thence into Great Britain, nd exported from thence into foreign parts	0 0 4

WINE.

	£.	s.	d.
For every tun of wine of the produce of his Majesty's settlements of the Cape of of Good Hope, or of the territories or dependencies thereof	16	9	0
For every tun of French wine	74	0	6
For every tun of any other wines imported into Great Britain	49	7	5

WIRE.

	£.	s.	d.
For every pound weight of gold thread, gold lace, or gold fringe, made of plate wire, spun upon silk, such plate wire being made of gilt wire	0	15	4
For every pound weight of silver thread, silver lace, or silver fringe, made of plate wire, spun upon silk, such plate wire being made of silver wire	0	11	6

The Act of Geo. IV. chap. 30, repeals the countervailing duties upon commodoties imported from Ireland into Great Britain, and exported from Great Britain into Ireland, but did not appear before the foregoing tables were struck off the press.

1. All articles liable to equal duties of Excise, in Great Britain and Ireland, may be imported without duty, and exported without drawback, between the two countries.

2. Articles imported, and liable to the highest rate of duty in the importing country, shall pay only the excess of duty.

3. Articles exported, and liable to the highest rate of duty in the exporting country, shall be allowed a drawback in the excess of duty only.

4. Articles liable to duty in the exporting country, shall pay the whole duty on import, and no drawback shall be allowed on export.

5. Articles liable to duty in the exporting country, and not in the importing country, shall receive the whole drawback on exportation, and shall not be liable to duty on importation.

6. Articles liable to duty, but warehoused duty free, may be exported from warehouses in one country, and shall be liable to duty on import into the other.

EXCISE LICENCES.

All Excise Licences must be taken out annually; and every Maker, Manufacturer, Trader, Dealer, or Retailer must, before he begins to manufacture, deal in, or retail the undermentioned commodities, pay for every such Licence as follows, viz.

AUCTIONEERS.

	£.	s.	d.
;very auctioneer	0	12	0

BEER.

	£.	s.	d.
very common brewer of table-beer, not being a common brewer of strong beer	2	0	0
very common brewer of strong beer, or brewer of intermediate beer, if the quantity of beer brewed by him, within the year ending 5th July each year, previous to taking out the licence, shall not exceed 1000 barrels	2	5	0
the same shall exceed 1000, and not exceed 2000 barrels	3	0	0
the same shall exceed 2000, and not exceed 5000 barrels	7	10	0
the same shall exceed 5000, and not exceed 7500 barrels	11	5	0
the same shall exceed 7500, and not exceed 10,000 barrels	15	0	0
the same shall exceed 10,000, and not exceed 20,000 barrels	30	0	0
the same shall exceed 20,000, and not exceed 30,000 barrels	45	0	0
the same shall exceed 30,000, and not exceed 40,000 barrels	60	0	0
If the same shall exceed 40,000 barrels	75	0	0
ery person who shall first become a common brewer of strong beer, or brewer of intermediate beer, pay for a licence, *2l. 5s.* and within ten days after the 5th July next, after taking out such licence, such additional sums, *called a surcharge*, as with the *2l. 5s.* shall amount to the duties herein directed to be paid, according to the number of barrels of strong beer brewed within the preceding year	2	5	0 and a surcharge.

BEER RETAILERS.

	£	s.	d.
Every person who shall sell beer or ale by retail, or who shall sell cyder or perry, to be drunk or consumed in his, her, or their house or premises, shall pay for a licence, if the dwelling-house in which such person taking out such licence, shall not, together with the offices, courts, yards, and gardens therewith occupied, be rated, under the authority of any act for granting duties on inhabited houses, at a rent of 15*l.* or upwards	2	2	0
If rated at 15*l.* or upwards, and under 20*l.*	3	3	0
If rated at 20*l.* or upwards	4	4	0
Every retailer of intermediate beer	1	1	0

CANDLES.

Every dealer in, or seller of, wax or spermaceti candles	1	1	0
Every maker of wax or spermaceti candles for sale	12	0	0
Every chandler or maker of candles, other than wax or spermaceti candles	2	0	0

COFFEE, TEA, COCOA-NUTS AND CHOCOLATE.

Every person trading in, vending, or selling coffee, tea, pepper, cocoa-nuts or chocolate	0	11	0

GLASS-MAKERS.

Every glass-maker, for each and every glass-house ...	20	0	0
Every maker of flint, or phial glass for each lear for annealing of flint or phial glass, besides the licence for each glass-house	100	0	0

LEATHER.

Every tanner within the bills of mortality	10	0	0
Every other tanner	5	0	0
Every tawer	2	0	0
Every dresser of hides and skins in oil	4	0	0
Every currier	4	0	0
Every maker of vellum or parchment	2	0	0

MALTSTERS.

	£.	s.	d.
Every maltster or maker of malt, if the quantity of malt made by him within the year ending 5th July in each year, previous to taking out the licence, shall not exceed 50 quarters	0	7	6
If the same shall exceed 50, and not exceed 100 quarters	0	15	0
If the same shall exceed 100, and not exceed 150 quarters	1	2	6
If the same shall exceed 150, and not exceed 200 quarters	1	10	0
If the same shall exceed 200, and not exceed 250 quarters	1	17	6
If the same shall exceed 250, and not exceed 300 quarters	2	5	0
If the same shall exceed 300, and not exceed 350 quarters	2	12	6
If the same shall exceed 350, and not exceed 400 quarters	3	0	0
If the same shall exceed 400, and not exceed 450 quarters	3	7	6
If the same shall exceed 450, and not exceed 500 quarters	3	15	0
If the same shall exceed 500, and not exceed 550 quarters	4	2	6
Or if the same shall exceed 550 quarters	4	10	0
Every person who shall first become a maltster, or maker of malt for sale, shall pay for a licence, 7*s.* 6*d.* and within ten days after the 5th of July next, after taking out such licence, such additional sum as with the said 7*s.* 6*d.* shall amount to the duty herein as before directed to be paid, according to the quantity of malt made within the preceding year	0	7	6 and a surcharge.

MEAD AND METHEGLIN.

Every maker of metheglin or mead for sale	2	0	0

OXYGENATED MURIATIC ACID.

Every maker of oxygenated muriatic acid, or oxymuriate of lime	5	0	0

PAPER.

Every maker of paper or pasteboard, and every paper stainer	4	0	0

PLATE.

	£.	s.	d.
Every person trading in, vending, or selling any gold or silver plate, or any goods or wares in which any quantity of gold, exceeding two pennyweights, and under two ounces in weight, or any quantity of silver exceeding five pennyweights, and under thirty ounces in weight, in any one separate and distinct ware or piece of goods, which shall be so manufactured	4	18	0
Every person trading in, vending, or selling any gold or silver plate, or any goods or wares in which any quantity of gold of the weight of two ounces or upwards, or any quantity of silver weighing thirty ounces or upwards, in any one separate and distinct ware or piece of goods, which shall be so manufactured	11	10	0
Every pawnbroker trading in, vending, or selling gold or silver plate	11	10	0
Every refiner of gold or silver	11	10	0

PRINTERS.

	£.	s.	d.
Every printer, painter, or stainer of silks	20	0	0
Every calico printer, and every printer, painter, or stainer of linens, cottons, or stuffs	20	0	0

SOAP.

	£.	s.	d.
Every maker of soap for sale	4	0	0

SPIRITS.

	£.	s.	d.
Every retailer of distilled spirituous liquors or strong waters, not being a retailer of plain aqua vitæ only, distilled from British materials in Scotland, shall pay for a licence, if the dwelling-house in which such retailer shall reside, or retail such spirituous liquors at the time of taking out such licence, shall not together with the offices, courts, yards, and gardens therewith occupied, be rated under the authority of any act for granting duties on inhabited houses at a rent of 15l. per annum, or upwards	5	5	0
If rated at 15l. and under 20l.	6	6	0
If at 20l., and under 25l.	8	5	0
If at 25l., and under 30l.	8	17	0
If at 30l., and under 40l.	9	9	0
If at 40l., and under 50l.	10	1	0
If rated at 50l. and upwards	10	13	0

	£.	s.	d.
ːvery person within the limits of any royal borough, burgh of barony or regality in any part of Scotland, or in any place in any other part of Scotland, other than within the Highlands of Scotland, limited and described in the act of parliament, shall retail any spirits made from malt, corn, grain, barley, beer, big, or other British materials, and commonly known by the name of aqua vitæ	4	0	0
very person who shall retail such spirits within the several counties and districts of the Highlands of Scotland, the royal burghs, burghs of barony or regality therein excepted	2	0	0
ery dealer in brandy, or other spirituous liquors, or strong waters, not being a retailer in any part of Great Britain, or not being a wholesale seller of or dealer in plain aqua vitæ only, distilled from malt, corn, or grain, or other British materials in Scotland	10	0	0
ery person in Scotland, who shall by wholesale sell or deal in spirits made from malt, corn, grain, barley, beer, bigg, or other British materials, not being a licensed distiller, rectifier, compounder, or etailer of spirits	6	0	0
ːry distiller or maker of low wines or spirits in ːngland	20	0	0
ry distiller or maker of low wines or spirits in cotland	10	0	0
ry rectifier of spirits in England	10	0	0
ry rectifier of spirits in Scotland	5	0	0

STARCH.

ːy starch-maker	10	0	0

SWEETS.

y maker of sweets, or made-wines, other than ead, for sale	10	0	0
y retailer of British made-wines or sweets	4	8	0

TOBACCO.

y manufacturer of tobacco or snuff, if the toːco and snuff-work weighed by such person for nufacture, within the year ending on the tenth October previous to taking the licence, shall not ːeed 20,000 pounds weight	3	0	0
e same shall have exceeded 20,000, and shall exceed 30,000 lbs.	4	10	0

	£.	s.	d.
If the same shall have exceeded 30,000, and shall not have exceeded 40,000 lbs.	6	0	0
If the same shall have exceeded 40,000, and shall not have exceeded 50,000 lbs.	7	10	0
If the same shall have exceeded 50,000, and shall not have exceeded 60,000 lbs.	9	0	0
If the same shall have exceeded 60,000, and shall not have exceeded 70,000 lbs.	10	10	0
If the same shall have exceeded 70,000, and shall not have exceeded 80,000 lbs.	12	0	0
If the same shall have exceeded 80,000, and shall not have exceeded 90,000 lbs.	13	10	0
If the same shall have exceeded 90,000, and shall not have exceeded 100,000 lbs.	15	0	0
If the same shall have exceeded 100,000, and shall not have exceeded 120,000 lbs.	18	0	0
If the same shall have exceeded 120,000, and shall not have exceeded 150,000 lbs.	22	10	0
If the same shall have exceeded 150,000 lbs. weight	30	0	0
Every person who shall first become a manufacturer of tobacco or snuff shall pay for a licence the sum of 5l., and within ten days after the tenth day of October next, after taking out such licence, such additional sum as with the said 5l. shall amount to the duty herein directed to be paid, according to the quantity of tobacco and snuff-work weighed for manufacture within the preceding year	5	0	0 and a surcharge.
Every dealer in, or seller of tobacco or snuff, within the limits of the chief offices of Excise in London, or Edinburgh	0	10	0
Every dealer in, or seller of tobacco or snuff, in any other part of Great Britain	0	5	0

VINEGAR.

Every maker of vinegar for sale	20	0	0

WINE.

Every retailer of foreign wine in England, who shall have an excise licence for retailing spirituous liquors	4	8	0
Every retailer of foreign wine in England, who shall not have an excise licence for retailing spirituous liquors, or a licence for retailing beer, ale, or other exciseable liquors	10	8	0
Every retailer of foreign wine in England, who shall have taken out a licence for retailing beer, ale, and other exciseable liquors, but shall not have an excise licence for retailing spirituous liquors	8	8	0

	£.	s.	d.
Every retailer of foreign wine in Scotland, who shall have an excise licence for retailing spirituous liquors	2	13	4
Every retailer of foreign wine in Scotland, who shall not have an excise licence for retailing spirituous liquors, or a licence for retailing beer, ale, or other exciseable liquors	6	13	4
Every retailer of foreign wine in Scotland, having a licence for retailing beer, ale, or other exciseable liquors, but not having an excise licence for retailing spirituous liquors	5	6	8

WIRE.

Every wiredrawer, or other person, who shall draw, or cause to be drawn, any gilt or silver wire, commonly called big wire	4	0	0

By Act of Geo. IV. Chap. 51, Every brewer of intermediate beer is subject to the duty of 5s. per barrel, containing 36 gallons, without any allowance whatever, other than that of the heat.

A Precedent of Vouchers and Abstracts for the Assistance of Young Officers and Pupils, in making up their Accounts.

BEER

VOUCHER.

——— Collection.

——— District.

—— Division, 5th Round, 1823.

From January 5th, 1823.

To February 23rd, 1823.

N. B. The above represents the back of the Beer Voucher, which will serve as a sufficient precedent for all the rest.

—*Collection.*—*District.*—*Division.* *5th Round,* 1828.

Names.	Date of Charge.	Common Brewers. Barrels of X.	T.	X.	T.	Victuallers. Gallons of X.	T.	Barrels of X.	T.
	Jan. 15	21¼	6¼						
	29	24¾	7						
	Feb. 14	20½	6½						
A. B.				66½	19¾				
	Jan. 11	18	10¼						
	18	17½							
	Feb. 14	16¼	14¾						
	23	20	11¼						
C. D.				71¾	36¼				
	Odds					2.5	3.0		
	Jan. 11					104			
	23					102.5	18		
	Feb. 17					105.5	20.5		
	19					106			
E. F.						420.5	41.5		
	Odds					4.0	7.5		
						416.5	34	12¼	1
	Odds					3.5	4.5		
	Jan. 16					212.5	78.5		
	28					224			
	Feb. 10					211.5	70		
G. H.						652	153		
	Odds					6.0			
						646	153	19	4½
By Drinks								2¼	1½
Total				138¼	56¼			33½	7

The Amount of

Common Brewers .... { Strong, Table,

Victuallers .... { Strong, Table,

Cash,

Voucher, from January 5th, to February 23rd, 1823.

Duty.		
£.	s.	d.
85	19	5

Voucher is,
hundred and thirty-eight barrels, and one firkin.
y-six barrels, and one firkin.
rty-three barrels, and two firkins.
en barrels.
ity-five pounds, nineteen shillings, and fivepence.

—Collection.—District.—Division. *5th Round,* 1821
Voucher, from January 5th, to February 23rd, 1823.

Names.	Date of Charge.	Hds. of Cyder at 1*l.* 10*s.*	Total Hds. of Cyder.	Duty. £.	*s.*	*d.*
	Jan. 17	$3\frac{1}{4}$				
	Feb. 5	$2\frac{3}{4}$				
	13	$3\frac{1}{2}$				
A. B			$9\frac{1}{2}$			
	Jan. 14	$1\frac{3}{4}$				
	Feb. 1	2				
	16	$1\frac{1}{2}$				
C. D.			$5\frac{1}{4}$			
Total			$14\frac{3}{4}$	22	2	6

The amount of this Voucher is,
Fourteen hogsheads and three-fourths of cyder.
Cash.—Twenty-two pounds, two shillings, and sixpence.

-Collection.—District.—Division. 5th *Round*, 1823.
'oucher. from January 5th to February 23rd, 1823.

ames.	Date of Charge.	Malt at 2s. 6d. per bushel. Gross.	Floor.	Net.	Duty. £.	s.	d.
	Jan. 7	120.4					
	11	119.6					
	15	112.1					
	19	123.7					
	24	121.5					
	28		174.8				
	Total	597.3	174.8				
	Net	477.84					
	Floor	87 40					
	Odds	.64					
B.		565.		565			
		surcharge	4.8				
	Jan. 11	104.2					
	16		170.1				
	23	111					
	Feb. 3	118.8					
	11		194.6				
	Total	334	369.5				
	Net	267.20					
	Floor	184.75					
	Odds	.26					
		452		452			
				1017	127	2	6

ımount of this Voucher is,
:housand and seventeen net bushels of malt.
—One hundred and twenty-seven pounds, two
, and sixpence.

[illegible]—[illegible]—[illegible] [illegible] Round, 1823
[illegible] from January 5th, to February 23rd, 1823

Names	Date and Amount of each Charge.		Pounds of Tallow Candles at 1d.	Duty.		
	Date.	lbs.		£	s.	d.
	Jan. [illegible]	[illegible]				
	[illegible]	[illegible]				
	[illegible]	[illegible]				
	[illegible]	[illegible]				
	Feb. [illegible]	[illegible]				
[illegible]		[illegible]	1789			
	Jan. [illegible]	[illegible]				
	[illegible]	[illegible]				
	[illegible]	[illegible]				
	[illegible]	[illegible]				
	Feb. [illegible]	[illegible]				
[illegible]		1405	1405			
Total			3194	13	6	2

The amount of this Voucher is,

Three thousand, one hundred and ninety-four pounds weight of tallow candles.

Cash.—Thirteen pounds, six shillings, and twopence.

-Collection.—District.—Division. 5th Round, 1823.
Voucher, from January 5th, to February 23rd, 1823.

Names.	Date and Amount of each Charge.		Pounds of Hard Soap at $2\frac{1}{4}$d.	Duty.		
	Date.	lbs.		£.	s.	d.
	Jan. 11	2400				
	21	2000				
	Feb. 16	2600				
A. B.		7000	7000			
	Jan. 18	3700				
	26	3400				
	Feb. 16	3100				
C. D.		10200	10200			
Total			17200	161	5	6

The amount of this Voucher is,
Seventeen thousand, two hundred pounds of hard ›ap.
Cash.—One hundred and sixty-one pounds, five ıillings, and sixpence.

—*Collection.—District.—Division.* *5th Round*, 1823.

Names.	Date of Charge.	1st. Class at 3*d*. per lb.		2nd. Class at 1½*d*. per lb.		Glazed at 1*l*. 1*s*. per cwt.		
		Reams.	Weight.	Reams.	Weight.	Parcels.	Dozens.	Weight.
	Jan. 11	84	1260	26	782			
	21			08	210	04	24	96
	Feb. 5	14	276			08	48	196
	Total	98	1536	34	992	12	72	292
	Allowed		30.72		19.84			5.84
	Net		1505.28		972.16			286.16
	Odds		82		14			12.90
A. B.	Charge	98	1506	34	972	12	72	280
	Jan. 6	28	510	12	380			
	12	16	272			12	72	291
	28			20	674	06	36	150
	Feb. 10	40	604	19	702			
	Total	84	1386	51	1756	18	108	441
	Allowed		27.72		35.12			8.82
	Net		1358.28		1720.88			432.18
	Odds		64		64			7.06
C. D.	Charged	84	1358	51	1721	18	108	440
Total								

The amount of

1st Class ....

2nd Class ....

Glazed ....

Cash ....

ucher, from January 5th, to February 23rd, 1823.

	Total.								Duty.		
1st Class.		2nd Class.		Glazed.							
	Weight.	Reams.	Weight.	Parcels	Dozens	Cwt.	qrs.	£.	s.	d.
8	1506	84	972	12	72	2	2			
4	1358	51	1721	18	108	3	3			
2	2864	85	2693	30	180	6	1	59	3	10½

Voucher is,

e hundred and eighty-two reams, weighing two housand, eight hundred and sixty-four pounds.

hty-five reams, weighing two thousand, six hunred and ninety-three pounds.

irty parcels, one hundred and eighty dozens, weighing six cwt. one quarter.

ty-nine pounds, three shillings, and tenpence halfenny.

—Collection.—District.—Division. *5th Round,* 1823.

Names.	Occupation.	Date of Charge.	Tanned. at 1½ per lb. No.	Tanned. at 1½ per lb. Wt.	Tawed. Horse Hides at 1*s.* 6*d.*	Tawed. Slink with at 3*s.* p. doz	Tawed. Shp. &c. at 1¼*d.* p. lb. No.	Tawed. Shp. &c. at 1¼*d.* p. lb. Wt.
		Jan. 17	24	316				
		Feb. 5	36	811				
A. B.	Tanner.	13	12	480				
		Jan. 17			3	5	15	46
		Feb. 1					10	34
C. D.	Tawer.	16			2	14		
		Jan. 11						
		21						
E. F.	Oil Dr.	29						
		Jan. 11						
		23						
G. H.	P. M.	29						
Total			72	1607	5	19	25	80

The amount of

Tanned ....

Tawed.................

Dressed in Oil ....

Cash ....

'oucher, from January 5th, to February 23rd, 1823.

'ieces at d. p lb.		Dressed in Oil.				Duty.		
		Sheep at 3d. per. lb.		Vellum at 3s. 6d.	Parchment at 1s. 9d.			
lo.	Wt.	No.	Wt.	per doz.	per dozen.	£.	s.	d.
6	32							
10	84							
		108	221					
		410	842					
		305	601					
				8	240			
				10				
					416			
16	116	823	1664	18	656	97	18	8½

iis Voucher is,

eventy-two hides and skins, weighing one thousand, six hundred and seven pounds.

ive horse hides, nineteen slink with hair on, twenty-five sheep, &c. weighing eighty pounds, with sixteen pieces, weighing one hundred and sixteen pounds.

ight hundred and twenty-three sheep, &c. weighing one thousand, six hundred and sixty-four pounds, eighteen dozen of vellum, and six hundred and fifty-six dozen of parchment.

Jinety-seven pounds, eighteen shillings, and eight-pence halfpenny.

Note 1.—When there are two or more traders of one occupation, ie voucher should be titled with a total column of each species, n order to be compared with the survey-book. See paper oucher.

2. ... ous directions for making up the principal ... is practised in the Excise; we shall not ... and by referring him to the preceding ... be enabled to make any voucher that

Note. In the preceding Precedent of Account [illegible] from the [illegible] to the net charge [illegible] charges in his survey books [illegible]

Names.		Malt at 2*s*.6*d*.per Bushel.		Duty. £.	*s.*	*d.*
A. B.	....	565	....	70	12	6
C. D.	....	452	....	56	10	0
Malt	....	1017	....	127	2	6
		Tallow Candles at 1*d*.				
A. B.	....	1789	....	7	9	1
C. D.	....	1405	....	5	17	1
Candles	....	3194	....	13	6	2
		Soap at 2½*d*. per lb.				
A. B.	....	7000	....	65	12	6
C. D.	....	10200	....	95	12	6
Soap	....	17200	....	161	5	0

	Paper. 1st Class at 3*d*. per lb.	2nd Class at 1½*d*. per lb.	Glazed at 1*l*. 1*s*. per cwt. Cwt.	qr.	£.	*s.*	*d.*
A. B.	1506	972	2	2	27	10	6
C. D.	1358	1721	3	3	31	13	4½
Paper	2864	2693	6	1	59	3	10½

		Hides.									Duty.		
		Tanned.	Tawed.				Dressed in Oil.						
		at 1½d. per lb.	Horse Hides at 1s. 6d.	Slinks with at 3s. per doz.	Sheep, &c. at 1½d. per lb.	Pieces at 6d. per lb.	Sheep, &c. at 3d. per lb.	Vellum at 3s. 6d. per doz.	Parchment at 1s. 9d. per doz.		£.	s.	d.
A. B.	Tanner	1607									10	0	10½
C. D.	Tawer		5	19	80	116					6	10	10
E. F.	Oil Dr.						1664				20	16	0
G. H.	P. M.							18	656		60	11	0
Hides		1607	5	19	80	116	1664	18	656		97	18	8½

Note.—In the preceding Precedent of Accounts, we have generally shewn the method of reducing the goods from the gross to the net charge, as it may tend to assist the young officer in the process of making up the charges in his survey-books; but only the gross and net quantities are absolutely necessary in real practice.

——— District, }
——— Division, } 5th Round, 1823.

From January 5th, to February 23rd, 1823.

	Beer.			Cyder.			Malt.			Candles.			Soap.			Paper.			Hides.		
	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
Arrears	74	14	2	10	2	6	255	7	6	—	—	—				58	13	7½	107	9	7
Amounts	85	19	5	22	2	6	127	2	6	13	6	2	161	5	0	59	3	10½	97	18	8½
Total	160	13	7	32	5	0	382	10	0	13	6	2	161	5	0	117	17	6	205	8	3½
Receipts Arrears																					
Compare	160	13	7	32	5	0	382	10	0	13	6	2	161	5	0	117	17	6	205	8	3½

ERRATA.

Page 6. Exam. 12.

Here $\frac{16 \times 22 \times 36}{18 \times 25 \times 48} = \frac{12672}{21600}$, *the equivalent fraction; then,* $12672 \div 21600 = .5866$, *Ans.*

Page 80. Exam. 6.

For 12303, *read* 123093; *and for* 32.646, *read* 32.425.

Page 83. Exam. 4.

Here $96.8 \times 96.8 \times 84.6 = 9370.24 \times 84.6 = 792722.304$; *and* $792722.304 \div 2738 = 289.526$, *the content in malt bushels.*

Page 94. Prob. XVI. Exam. 2.

For 5385.79, *read* 5385.75; *for* 4411.76, *read* 4411.8; *and for* 65.668, *read* 65.667.

Page 94. Exam. 3.

For 41069.92, *read* 41070.
Page 222, line 8, *for* .0023867, *read* .002366.
5, Exam. 4, *for* .9929583, *read* .0029583.
227, 4, Exam. 2, *for* 0.4375, *read* .04375; and *for* 32.43479375, *read* 32.43459375.

Page 271. Exam. 2.

Here $\frac{48 \times 24}{12} = \frac{1152}{12} = 96$ *lbs. Ans.*

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